SOME CONGRUENCE PROPERTIES
OF THE
PARTITION FUNCTION

by

Francis G. Garvan

Submitted for the degree of

Master of Science

University of New South Wales, 1982.

CONTENTS

INTRODUCTION

In this thesis we obtain some identities and congruence properties involving the partition function using elementary methods.

Recently Hirschhorn and Hunt have published a simple proof of Ramanujan's conjecture for powers of 5. In the first chapter we show how their methods can be extended to prove Watson's partition congruences for powers of 7. We establish appropriate generating formulae from which Watson's results follow easily. Our proofs are more straightforward than those of Watson. They are elementary, depending only on classical identities of Euler and Jacobi. Watson's proofs rely on the modular equation of seventh order. We also need the modular equation but we derive it using the elementary techniques of O. Kolberg.

Hirschhorn and Hunt have also obtained two new partition congruences modulo powers of 5. In the second chapter we obtain three more similar congruences. In fact we prove that similar congruences exist for all higher powers of 5. This was known to Atkin although he never published a proof. The methods we use are analogous to those of Atkin and O'Brien who have proved the existence of similar congruences modulo powers of 13.

In the third chapter we obtain analogous results modulo powers of 7. We give eight congruence relations. The first two were known to Watson whilst the remaining six appear to be new.

In the fourth chapter we show how Hirschhorn and Hunt's methods can be extended to the functions $p_{-k}(n)$, to obtain congruence relations due to Atkin. We give a detailed discussion of Atkin's proof.

In the final chapter we show how the methods of the first chapter can be extended to obtain a special case of a theorem due to Newman, from which we are able to give an elementary proof of $p(11n+6) \equiv 0 \pmod{11}$. The ideas we use have come from papers by Atkin and Swinnerton-Dyer and Atkin and Hussain. We also give an elementary derivation of Fine's modular equation of eleventh order and hence obtain an identity of the Ramanujan type which also yields a proof of $p(11n+6) \equiv 0 \pmod{11}$.

I would like to take this opportunity to thank my supervisor, Dr. Michael Hirschhorn, for his constant help and encouragement.