

THE COMPUTER PROGRAMME (SEE PAGE 26)

The programme was written to divide the first of the following two power series by the second

$$u_0 + u_1 x + u_2 x^2 + \dots + u_n x^n + \dots,$$

$$1 + v_1 x + v_2 x^2 + \dots + v_n x^n + \dots,$$

both sets of coefficients being integral. Denoting the quotient power series by

$$w_0 + w_1 x + w_2 x^2 + \dots + w_n x^n + \dots,$$

we have, equating coefficients of powers of  $x$  in the first of these series with those in the product of the second and third, and transposing,

$$w_0 = u_0,$$

$$w_1 = u_1 - (v_1 w_0),$$

$$w_2 = u_2 - (v_1 w_1 + v_2 w_0),$$

.....

$$w_n = u_n - (v_1 w_{n-1} + v_2 w_{n-2} + \dots + v_n w_0),$$

.....

Thus  $w_0, w_1, w_2, \dots, w_n, \dots$  are integral and may be successively found by means of these relations.

We omit the actual programme since its notation is peculiar to "pegasus" and content ourselves with the following observations. The calculation of the  $w_n$  is basically a simple process and indeed the only sub-routines used were a "read"

and a "print" routine. As each  $w_n$  is found it is both stored and printed; the process terminates at some predetermined value of  $n$  (142 in our case), which number forms part of the data. The computer was set to stop immediately if "overflow" occurred at any stage, but in fact this did not happen. The total computer-time taken for the six divisions was well under an hour.

## NOTATION

The pages of definition are indicated.

$f(z)$	1
$[ ]$	1
$< >$	14
Part 1 ( $a = 13$ )	
$\alpha, \beta, \gamma, \alpha', \beta', \gamma'$	1
$a, b, c, a', b', c'$	2
$A, B, C, K$	3
$F$	6
$\phi(s)$	7, 8
$l, m, n, l', m', n'$	14
$\phi'(s)$	16, 17
$N_b, N_{b\circ}$	22
$R_{b\circ}(d)$	24, 25, 26
$U, V$	29
$N'_{b\circ}$	36
$R_d$	36
$t_d$	36
$T_d$	37, 38
$t_{b\circ}(d), T_{b\circ}(d)$	39

Part 2 (a = 17)

$a_1, a_2, \dots, a_9$	47
$a_1, a_2, \dots, a_8$	48
F	50
$b_1, b_2, b_3, b_4$	50
$\lambda, \mu$	52
$c_1, c_2$	53
$\delta$	57
$m_1, m_2, n_1, n_2$	58
<u>Part 3 (a = 12)</u>	
$a_1, a_2, \dots, a_9$	62
$a_1, a_2, \dots, a_8$	63
F	65
$b_1, b_2, b_3, c_1, c_2, c_3, d_1, d_2, d_3$	65
$\lambda, \mu$	70
$m_1, m_2, m_3$	74
$\delta$	76
<u>Part 4 (a = 11)</u>	
$\phi(s)$	80
<u>Part 5</u>	
$\eta(\tau), g = g(\tau), h = h(\tau)$	82
$c(l, m)$	83
$A = A(\tau), b = b(\tau)$	83
$d(l, m)$	86

REFERENCES

1. A. O. L. Atkin, "Proof of a conjecture of Ramanujan" (unpublished).
2. ——— and S. M. Hussain, "Some properties of partitions (2)", *Trans. Amer. Math. Soc.* vol. 89 (1958) pp. 184-200.
3. ——— and P. Swinnerton-Dyer, "Some properties of partitions", *Proc. London Math. Soc.* (3) vol. 4 (1954) pp. 84-106.
4. L. R. Ford, "Automorphic Functions", 2nd ed. (Chelsea Publishing Co., New York, 1951).
5. H. Gupta, C. E. Gwyther and J. C. P. Miller, "Tables of Partitions", *Royal Soc. Math. Tables* vol. 4 (Cambridge University Press, 1958) pp. 89-121.
6. S. M. Hussain, "Studies in Partition Theory", Ph. D. thesis (University of Durham, 1954).
7. O. Kolberg, "Some identities involving the partition function", *Math. Scand.* vol. 5 (1957) pp. 77-92.
8. J. Lehner, "Further congruence properties of the Fourier coefficients of the modular invariant  $j(\tau)$ ", *Amer. J. Math.* vol. 71 (1949), pp. 373-386.
9. M. Newman, "Remarks on some modular identities", *Trans. Amer. Math. Soc.* vol. 73 (1952) pp. 313-320.
10. ———, "On the existence of identities for the coefficients of certain modular forms", *J. London Math. Soc.* vol. 31 (1956) pp. 350-359.
11. ———, "A table of the coefficients of the powers of  $\eta(\tau)$ ", *Proc. Kon. Nederl. Akad. Wetensch.*, Ser. A, vol. 59 =

11. cont.  
Indag. Math. vol. 18 (1956) pp. 204-216.

12. S. Ramanujan, "Some properties of  $p(n)$ , the number of partitions of  $n$ ", Proc. Cambridge Philos. Soc. vol. 19 (1919) pp. 207-210 = "Collected Papers of Srinivasa Ramanujan" (Cambridge University Press, 1927) pp. 210-213.
13. —, "Notebooks of Srinivasa Ramanujan" vol. 1 (Tata Institute of Fundamental Research, Bombay, 1957).
14. J. Tannery and J. Molk, "Eléments de la Théorie des Fonctions Elliptiques" vol. 2 (Gauthier-Villars and Son, Paris, 1896).
15. G. N. Watson, "Ramanujan's Vermutung über Zerfällungsanzahlen", J. Reine Angew. Math. vol. 179 (1938) pp. 97-128.
16. H. Weber, "Lehrbuch der Algebra" vol. 3 (Brunswick, 1908).
17. H. S. Zuckerman, "Identities analogous to Ramanujan's identities involving the partition function", Duke Math. J. vol. 5 (1939) pp. 88-110.

