## Definition of $f(q)$

$$f(q) = 1 + \frac{q}{(1-2\cos\frac{2\pi}{5}q+q^2)}$$

$$+ \frac{q^4}{(1-2\cos\frac{2\pi}{5}q+q^2)(1-2\cos\frac{2\pi}{5}2q^2+q^4)}$$

$$+ \cdots$$

$$= 1 + \frac{q}{(1-e^{2\pi i/5}q)(1-e^{-2\pi i/5}q)}$$

$$+ \frac{q^4}{(1-e^{2\pi i/5}q)(1-e^{-2\pi i/5}q)(1-e^{2\pi i/5}q^2)(1-e^{2\pi i/5}q^2)}$$

$$+ \cdots$$

$$= 1 + \sum_{n=1}^{\infty} \frac{q^{n^2}}{(e^{2\pi i/5}q;q)_n (e^{-2\pi i/5}q;q)_n}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{q^{n^2}}{(\zeta q;q)_n (\zeta^{-1}q;q)_n}$$

where $\zeta = e^{2\pi i/5} = \cos(\frac{2\pi}{5}) + i\sin(\frac{2\pi}{5})$.