

$$\sum^5 = 1$$

(LFT)

Suppose $n \equiv j \pmod{5}$
Then

$$n = 5k + j \quad (k \in \mathbb{Z})$$

and

$$\begin{aligned} \sum^n &= \sum^{5k+j} \\ &= (\sum^5)^k \sum^j = \sum^j \end{aligned}$$

So

$$\sum^n = 1, 3, 3^2, 3^3 \text{ or } 3^4$$

according to whether

$$n \equiv 0, 1, 2, 3, \text{ or } 4 \pmod{5}$$