

(2B)

$$\zeta = e^{2\pi i/5} \text{ satisfies}$$

$$P(\zeta) = 0$$

$$\text{where } P(z) = 1 + z + z^2 + z^3 + z^4$$

This is the MINIMAL POLYNOMIAL

of ζ ; i.e. the unique monic polynomial with rational coefficients of which $z = \zeta$ is a root.

EX Let p be prime. The minimal polynomial of $\zeta_p = e^{2\pi i/p}$

$$= \cos\left(\frac{2\pi}{p}\right) + i \sin\left(\frac{2\pi}{p}\right)$$

is

$$\Phi_p(z) = 1 + z + z^2 + \dots + z^{p-1}$$