Problems

1. Prove $G_k(z)$ satisfies the modular transformation property for $k \geq 3$.
2. Use the transformation property to prove that there are no non-zero modular forms of odd weight on $\mathrm{SL}_2(\mathbb{Z})$. Verify that $G_{2k+1}(z) = 0$ for $k \geq 1$.
3. Prove that $E_6(i) = 0$.
4. For which weights k will $E_k(\omega) = 0$ where $\omega = e^{2\pi i/3}$?
5. Prove that for each $n \geq 1$

$$\sigma_7(n) = \sigma_3(n) + 120 \sum_{i=1}^{n-1} \sigma_3(i)\sigma_3(n-i).$$

6. Prove $\tau(n) \equiv \sigma_{11}(n) \pmod{691}$ using $E_6^2(z)$, $E_{12}(z)$ and $\Delta(z)$.
7. Compute the first 1000 coefficients of $\tau(n)$ and make a conjecture about what you observe.