```
> with(qseries):
> with(ETA):

> gpA1:=[1,2,4,3,2,-5,50,5,25,-2,100,-3]:
> epA1:=gp2etaprod(gpA1);
```

$$epA1 := \frac{\eta(\tau)^2\,\eta(4\,\tau)^3\,\eta(50\,\tau)^5}{\eta(2\,\tau)^5\,\eta(25\,\tau)^2\,\eta(100\,\tau)^3} \tag{1}$$

```
> A1q:=etaprodtoqseries(epA1,1000):

> gpa1:=[1,2,10,4,2,-4,5,-2]:
> epa1:=gp2etaprod(gpa1);
```

$$epa1 := \frac{\eta(\tau)^2\,\eta(10\,\tau)^4}{\eta(2\,\tau)^4\,\eta(5\,\tau)^2} \tag{2}$$

```
> aq1:=etaprodtoqseries(epa1,2000):
> gprho:=[2,2,20,4,4,-4,10,-2]:
> eprho:=gp2etaprod(gprho);
```

$$eprho := \frac{\eta(2\,\tau)^2\,\eta(20\,\tau)^4}{\eta(4\,\tau)^4\,\eta(10\,\tau)^2} \tag{3}$$

```
> G1a:=etaprodtoqseries(eprho,1001):
> G1:=convert(series(1-G1a,q,1020),polynom):
> etamake(G1,q,100);
```

$$\frac{\eta(10\,\tau)^3\,\eta(2\,\tau)}{\eta(20\,\tau)\,\eta(4\,\tau)^3} \tag{4}$$

```
> U01:=sift(A1q,q,5,0,2000):

> etacombo:=findlincombo(U01,[seq( aq1^k,k=-2..5),seq( aq1^k*(G1),
  k=-2..5)],[seq( etamake((aq1)^k,q,100),k=-2..5),seq( etamake(
  (aq1)^k*G1,q,100),k=-2..5)],q,0);
```

$$nx = , 16$$

$$\#\,of\,terms\,, 37$$

$$\begin{aligned} etacombo := &-\frac{53\,\eta(\tau)^2\,\eta(10\,\tau)^4}{\eta(2\,\tau)^4\,\eta(5\,\tau)^2} + \frac{350\,\eta(10\,\tau)^8\,\eta(\tau)^4}{\eta(5\,\tau)^4\,\eta(2\,\tau)^8} - \frac{1050\,\eta(10\,\tau)^{12}\,\eta(\tau)^6}{\eta(5\,\tau)^6\,\eta(2\,\tau)^{12}} \\ &+ \frac{1375\,\eta(10\,\tau)^{16}\,\eta(\tau)^8}{\eta(5\,\tau)^8\,\eta(2\,\tau)^{16}} - \frac{625\,\eta(10\,\tau)^{20}\,\eta(\tau)^{10}}{\eta(5\,\tau)^{10}\,\eta(2\,\tau)^{20}} + \frac{13\,\eta(10\,\tau)^3\,\eta(2\,\tau)}{\eta(20\,\tau)\,\eta(4\,\tau)^3} \\ &- \frac{75\,\eta(10\,\tau)^7\,\eta(\tau)^2}{\eta(20\,\tau)\,\eta(5\,\tau)^2\,\eta(4\,\tau)^3\,\eta(2\,\tau)^3} + \frac{175\,\eta(10\,\tau)^{11}\,\eta(\tau)^4}{\eta(20\,\tau)\,\eta(5\,\tau)^4\,\eta(4\,\tau)^3\,\eta(2\,\tau)^7} \\ &- \frac{125\,\eta(10\,\tau)^{15}\,\eta(\tau)^6}{\eta(20\,\tau)\,\eta(5\,\tau)^6\,\eta(4\,\tau)^3\,\eta(2\,\tau)^{11}} \end{aligned} \tag{5}$$

```
> provemodfuncGAMMA0UpETAid();
-------------------------------------------------------------
provemodfuncGAMMA0UpETAid(EP,p,etacombo,N)
      EP = one eta-product
       p = prime
etacombo = sum of modular functions on Gamma[0](N)
           Each term in the sum is a eta-quotient to base N.
       N = Positive integer multiple of p

   This function PROVES the id U[p](EP) = etacombo
   global vars (can be used for error-checking):
   qcheck, modfunccheck, totcheck, _ORDS, jptmp, jpqd, eptmp,
   gltmp, EPRODL, GETAL, COFS, conpres, CONTERMS, mintottmp

-------------------------------------------------------------
> provemodfuncGAMMA0UpETAid(epA1,5,etacombo,20);
*** There were NO errors.
*** o EP is an MF on Gamma[0](100)
*** o Each term in the etacombo is a  modular function on
      Gamma0(20).
*** o We also checked that the total order of
      each term etacombo was zero.
*** To prove the identity U[5](EP)=etacombo we need to show
    that v[oo](ID) > 9    This means checking up to q^(10).
Do you want to prove the identity? (yes/no)
You entered yes.
We verify the identity to O(q^(49)).
We find that LHS - RHS is
```

$$O(q^{49})$$

```
RESULT: The identity holds to O(q^(49)).
CONCLUSION: This proves the identity since we had only
            to show that v[oo](ID) > 9.
> provemodfuncGAMMA0UpETAidBATCH(epA1,5,etacombo,20);
*** There were NO errors.
*** o EP is an MF on Gamma[0](100)
*** o Each term in the etacombo is a  modular function on
      Gamma0(20).
*** o We also checked that the total order of
      each term etacombo was zero.
*** To prove the identity U[5](EP)=etacombo we need to show
    that v[oo](ID) > 9    This means checking up to q^(10).
We find that LHS - RHS is
```

$$O(q^{49})$$

$$[1, -9, O(q^{49})] \tag{6}$$

```
> noprint:=true:
> provemodfuncGAMMA0UpETAidBATCH(epA1,5,etacombo,20);
```

$$[1, -9, O(q^{49})] \tag{7}$$