```
> restart;
> gc();
> currentdir();
```
"C:\cygwin\home\fgarvan\maple\mypackages\thetaids\examples2" **(1)**

```
> currentdir("C:\\cygwin\\home\\fgarvan\\maple\\mypackages\\thetaids\\examples2");
```
"C:\cygwin\home\fgarvan\maple\mypackages\thetaids\examples2" **(2)**

```
> currentdir();
```
"H:\maple\mypackages\thetaids\examples2" **(3)**

```
> read allprogs:
> with(qseries):
> read moreprogs:
"END"
> xprint:=false: proveit:=true:
```
==

p=5

```
> read ramdata;
```
$RAMTYPE1 := [[11, 1, -1], [16, 1, -1], [6, 1, -1], [7, 2, -1], [8, 3, -1], [9, 4, -1], [36, 1, -1]]$

$RAMTYPE2 := [[1, 4, 1], [1, 4, -1], [1, 9, 1], [2, 3, 1], [1, 14, 1], [1, 24, 1]]$

$RAMTYPE3 := [[3, 7, 1, 21, 1, -1], [2, 13, 1, 26, 1, -1], [1, 39, 1, 13, 3, -1], [1, 34, 1, 17, 2, -1], [2, 33, 1, 66, 1, -1], [3, 22, 1, 11, 6, -1]]$ **(4)**

```
> nops(RAMTYPE1)+nops(RAMTYPE2)+nops(RAMTYPE3);
```
19 **(5)**

```
> G:=j->1/GetaL(qr(5),5,j):H:=j->1/GetaL(qnr(5),5,j):
> GM:=j->1/MGetaL(qr(5),5,j): HM:=j->1/MGetaL(qnr(5),5,j):

> GE:=j->-GetaLEXP(qr(5),5,j):HE:=j->-GetaLEXP(qnr(5),5,j):
> GE(1),HE(1);
```
$-\frac{1}{60}, \frac{11}{60}$ **(6)**

```
> isolve(GE(a)+HE(b)=0);
```
$\{a = 11\ \_Z1, b = \_Z1\}$ **(7)**

```
> findtype1(6);
*** There were NO errors.  Each term was modular function on
    Gamma1(30). Also -mintotord=8. To prove the identity
    we need to  check up to O(q^(10)).
    To be on the safe side we check up to O(q^(68)).
*** The identity below is PROVED!
[6, 1, -1]
```
$$\_G(6)\ \_H(1) - \_G(1)\ \_H(6) = \frac{\eta(6\tau)\,\eta(\tau)}{\eta(3\tau)\,\eta(2\tau)}$$

$[[6, 1, -1]]$ **(8)**

```
> myramtype1:=findtype1(36); #actually checked to 500
*** There were NO errors.  Each term was modular function on
    Gamma1(30). Also -mintotord=8. To prove the identity
```

```
    we need to  check up to O(q^(10)).
    To be on the safe side we check up to O(q^(68)).
*** The identity below is PROVED!
[6, 1, -1]
```

$$\_G(6)\ \_H(1) - \_G(1)\ \_H(6) = \frac{\eta(6\tau)\,\eta(\tau)}{\eta(3\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(55). Also -mintotord=40. To prove the identity
    we need to  check up to O(q^(42)).
    To be on the safe side we check up to O(q^(150)).
*** The identity below is PROVED!
[11, 1, -1]
```

$$\_G(11)\ \_H(1) - \_G(1)\ \_H(11) = 1$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(70). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(188)).
*** The identity below is PROVED!
[7, 2, -1]
```

$$\_G(7)\ \_H(2) - \_G(2)\ \_H(7) = \frac{\eta(14\tau)\,\eta(\tau)}{\eta(7\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(80). Also -mintotord=64. To prove the identity
    we need to  check up to O(q^(66)).
    To be on the safe side we check up to O(q^(224)).
*** The identity below is PROVED!
[16, 1, -1]
```

$$\_G(16)\ \_H(1) - \_G(1)\ \_H(16) = \frac{\eta(4\tau)^2}{\eta(8\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=128. To prove the identity
    we need to  check up to O(q^(130)).
    To be on the safe side we check up to O(q^(368)).
*** The identity below is PROVED!
[8, 3, -1]
```

$$\_G(8)\ \_H(3) - \_G(3)\ \_H(8) = \frac{\eta(24\tau)\,\eta(6\tau)\,\eta(4\tau)\,\eta(\tau)}{\eta(12\tau)\,\eta(8\tau)\,\eta(3\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(180). Also -mintotord=288. To prove the identity
    we need to  check up to O(q^(290)).
    To be on the safe side we check up to O(q^(648)).
*** The identity below is PROVED!
[9, 4, -1]
```

$$\_G(9)\ \_H(4) - \_G(4)\ \_H(9) = \frac{\eta(36\tau)\,\eta(6\tau)^2\,\eta(\tau)}{\eta(18\tau)\,\eta(12\tau)\,\eta(3\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(180). Also -mintotord=288. To prove the identity
    we need to  check up to O(q^(290)).
    To be on the safe side we check up to O(q^(648)).
*** The identity below is PROVED!
```

```
[36, 1, -1]
```

$$\_G(36)\ \_H(1) - \_G(1)\ \_H(36) = \frac{\eta(9\tau)\,\eta(6\tau)^2\,\eta(4\tau)}{\eta(18\tau)\,\eta(12\tau)\,\eta(3\tau)\,\eta(2\tau)}$$

myramtype1 := [[6, 1, -1], [11, 1, -1], [7, 2, -1], [16, 1, -1], [8, 3, -1], [9, 4, -1], [36, 1, -1]] **(9)**

```
> myramtype1set:=convert(myramtype1,set);
```

myramtype1set := {[6, 1, -1], [7, 2, -1], [8, 3, -1], [9, 4, -1], [11, 1, -1], [16, 1, -1], [36, 1, -1]} **(10)**

```
> nops(myramtype1);
```

7 **(11)**

```
> nops(RAMTYPE1);
```

7 **(12)**

```
> evalb(convert(myramtype1,set) = convert(RAMTYPE1,set));
```

true **(13)**

```
> myramtype2:=findtype2(24); #actually checked up to 500
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 4, -1]
```

$$\_G(1)\ \_G(4) - \_H(1)\ \_H(4) = \frac{\eta(10\tau)^5}{\eta(20\tau)^2\,\eta(5\tau)^2\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 4, 1]
```

$$\_G(1)\ \_G(4) + \_H(1)\ \_H(4) = \frac{\eta(2\tau)^4}{\eta(4\tau)^2\,\eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(30). Also -mintotord=8. To prove the identity
    we need to  check up to O(q^(10)).
    To be on the safe side we check up to O(q^(68)).
*** The identity below is PROVED!
[2, 3, 1]
```

$$\_G(2)\ \_G(3) + \_H(2)\ \_H(3) = \frac{\eta(3\tau)\,\eta(2\tau)}{\eta(6\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(45). Also -mintotord=24. To prove the identity
    we need to  check up to O(q^(26)).
    To be on the safe side we check up to O(q^(114)).
*** The identity below is PROVED!
[1, 9, 1]
```

$$\_G(1)\ \_G(9) + \_H(1)\ \_H(9) = \frac{\eta(3\,\tau)^2}{\eta(9\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(70). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(188)).
*** The identity below is PROVED!
[1, 14, 1]
```

$$\_G(1)\ \_G(14) + \_H(1)\ \_H(14) = \frac{\eta(7\,\tau)\ \eta(2\,\tau)}{\eta(14\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=128. To prove the identity
    we need to  check up to O(q^(130)).
    To be on the safe side we check up to O(q^(368)).
*** The identity below is PROVED!
[1, 24, 1]
```

$$\_G(1)\ \_G(24) + \_H(1)\ \_H(24) = \frac{\eta(12\,\tau)\ \eta(8\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(24\,\tau)\ \eta(6\,\tau)\ \eta(4\,\tau)\ \eta(\tau)}$$

$$myramtype2 := [[1, 4, -1], [1, 4, 1], [2, 3, 1], [1, 9, 1], [1, 14, 1], [1, 24, 1]] \quad \textbf{(14)}$$

```
> evalb(convert(myramtype2,set) = convert(RAMTYPE2,set));
```

$$true \quad \textbf{(15)}$$

```
> findtype3(130);
*** There were NO errors.  Each term was modular function on
    Gamma1(105). Also -mintotord=192. To prove the identity
    we need to  check up to O(q^(194)).
    To be on the safe side we check up to O(q^(402)).
*** The identity below is PROVED!
[3, 7, 1, 21, 1, -1]
```

$$\frac{\_G(3)\ \_G(7) + \_H(3)\ \_H(7)}{\_G(21)\ \_H(1) - \_H(21)\ \_G(1)} = 1$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=224. To prove the identity
    we need to  check up to O(q^(226)).
    To be on the safe side we check up to O(q^(464)).
*** The identity below is PROVED!
[1, 24, 1, 12, 2, -1]
```

$$\frac{\_G(1)\ \_G(24) + \_H(1)\ \_H(24)}{\_G(12)\ \_H(2) - \_H(12)\ \_G(2)} = \frac{\eta(8\,\tau)\ \eta(3\,\tau)}{\eta(24\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(130). Also -mintotord=240. To prove the identity
    we need to  check up to O(q^(242)).
    To be on the safe side we check up to O(q^(500)).
*** The identity below is PROVED!
[2, 13, 1, 26, 1, -1]
```

$$\frac{\_G(2)\ \_G(13) + \_H(2)\ \_H(13)}{\_G(26)\ \_H(1) - \_H(26)\ \_G(1)} = 1$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(170). Also -mintotord=448. To prove the identity
    we need to  check up to O(q^(450)).
    To be on the safe side we check up to O(q^(788)).
```

```
*** The identity below is PROVED!
[1, 34, 1, 17, 2, -1]
```

$$\frac{\_G(1)\ \_G(34)+\_H(1)\ \_H(34)}{\_G(17)\ \_H(2)-\_H(17)\ \_G(2)}=\frac{\eta(17\,\tau)\ \eta(2\,\tau)}{\eta(34\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(195). Also -mintotord=768. To prove the identity
    we need to  check up to O(q^(770)).
    To be on the safe side we check up to O(q^(1158)).
*** The identity below is PROVED!
[1, 39, 1, 13, 3, -1]
```

$$\frac{\_G(1)\ \_G(39)+\_H(1)\ \_H(39)}{\_G(13)\ \_H(3)-\_H(13)\ \_G(3)}=\frac{\eta(13\,\tau)\ \eta(3\,\tau)}{\eta(39\,\tau)\ \eta(\tau)}$$

```
"n=", 50
 abs(mintotord)=-1008, which is too large
 Try increasing the global var qthreshold.
[1, 54, 1, 27, 2, -1]
```

$$\frac{\_G(1)\ \_G(54)+\_H(1)\ \_H(54)}{\_G(27)\ \_H(2)-\_H(27)\ \_G(2)}=\frac{\eta(27\,\tau)\ \eta(18\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(54\,\tau)\ \eta(9\,\tau)\ \eta(6\,\tau)\ \eta(\tau)}$$

```
 abs(mintotord)=-1152, which is too large
 Try increasing the global var qthreshold.
[7, 8, 1, 56, 1, -1]
```

$$\frac{\_G(7)\ \_G(8)+\_H(7)\ \_H(8)}{\_G(56)\ \_H(1)-\_H(56)\ \_G(1)}=\frac{\eta(28\,\tau)\ \eta(2\,\tau)}{\eta(14\,\tau)\ \eta(4\,\tau)}$$

```
 abs(mintotord)=-1600, which is too large
 Try increasing the global var qthreshold.
[3, 22, 1, 11, 6, -1]
```

$$\frac{\_G(3)\ \_G(22)+\_H(3)\ \_H(22)}{\_G(11)\ \_H(6)-\_H(11)\ \_G(6)}=\frac{\eta(33\,\tau)\ \eta(2\,\tau)}{\eta(66\,\tau)\ \eta(\tau)}$$

```
 abs(mintotord)=-1600, which is too large
 Try increasing the global var qthreshold.
[2, 33, 1, 66, 1, -1]
```

$$\frac{\_G(2)\ \_G(33)+\_H(2)\ \_H(33)}{\_G(66)\ \_H(1)-\_H(66)\ \_G(1)}=\frac{\eta(22\,\tau)\ \eta(3\,\tau)}{\eta(11\,\tau)\ \eta(6\,\tau)}$$

```
 abs(mintotord)=-2688, which is too large
 Try increasing the global var qthreshold.
[4, 21, 1, 12, 7, -1]
```

$$\frac{\_G(4)\ \_G(21)+\_H(4)\ \_H(21)}{\_G(12)\ \_H(7)-\_H(12)\ \_G(7)}=\frac{\eta(42\,\tau)\ \eta(28\,\tau)\ \eta(12\,\tau)\ \eta(7\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(84\,\tau)\ \eta(21\,\tau)\ \eta(14\,\tau)\ \eta(6\,\tau)\ \eta(4\,\tau)\ \eta(\tau)}$$

```
 abs(mintotord)=-2688, which is too large
 Try increasing the global var qthreshold.
[1, 84, 1, 28, 3, -1]
```

$$\frac{\_G(1)\ \_G(84)+\_H(1)\ \_H(84)}{\_G(28)\ \_H(3)-\_H(28)\ \_G(3)}=\frac{\eta(42\,\tau)\ \eta(28\,\tau)\ \eta(12\,\tau)\ \eta(7\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(84\,\tau)\ \eta(21\,\tau)\ \eta(14\,\tau)\ \eta(6\,\tau)\ \eta(4\,\tau)\ \eta(\tau)}$$

```
 abs(mintotord)=-3072, which is too large
 Try increasing the global var qthreshold.
[3, 32, 1, 96, 1, -1]
```

$$\frac{\_G(3)\ \_G(32)+\_H(3)\ \_H(32)}{\_G(96)\ \_H(1)-\_H(96)\ \_G(1)}=\frac{\eta(48\tau)\ \eta(12\tau)\ \eta(8\tau)\ \eta(2\tau)}{\eta(24\tau)\ \eta(16\tau)\ \eta(6\tau)\ \eta(4\tau)}$$

```
"n=", 100
 abs(mintotord)=-5760, which is too large
 Try increasing the global var qthreshold.
[7, 18, 1, 14, 9, -1]
```

$$\frac{\_G(7)\ \_G(18)+\_H(7)\ \_H(18)}{\_G(14)\ \_H(9)-\_H(14)\ \_G(9)}=\frac{\eta(63\tau)\ \eta(42\tau)\ \eta(3\tau)\ \eta(2\tau)}{\eta(126\tau)\ \eta(21\tau)\ \eta(6\tau)\ \eta(\tau)}$$

```
 abs(mintotord)=-5760, which is too large
 Try increasing the global var qthreshold.
[2, 63, 1, 126, 1, -1]
```

$$\frac{\_G(2)\ \_G(63)+\_H(2)\ \_H(63)}{\_G(126)\ \_H(1)-\_H(126)\ \_G(1)}=\frac{\eta(42\tau)\ \eta(18\tau)\ \eta(7\tau)\ \eta(3\tau)}{\eta(21\tau)\ \eta(14\tau)\ \eta(9\tau)\ \eta(6\tau)}$$

[[3, 7, 1, 21, 1, -1], [1, 24, 1, 12, 2, -1], [2, 13, 1, 26, 1, -1], [1, 34, 1, 17, 2, -1], [1, 39, 1, 13, 3, -1], [1, 54, 1, 27, 2, -1], [7, 8, 1, 56, 1, -1], [3, 22, 1, 11, 6, -1], [2, 33, 1, 66, 1, -1], [4, 21, 1, 12, 7, -1], [1, 84, 1, 28, 3, -1], [3, 32, 1, 96, 1, -1], [7, 18, 1, 14, 9, -1], [2, 63, 1, 126, 1, -1]] **(16)**

```
> myramtype3:=%;
```

myramtype3 := [[3, 7, 1, 21, 1, -1], [1, 24, 1, 12, 2, -1], [2, 13, 1, 26, 1, -1], [1, 34, 1, 17, 2, -1], [1, 39, 1, 13, 3, -1], [1, 54, 1, 27, 2, -1], [7, 8, 1, 56, 1, -1], [3, 22, 1, 11, 6, -1], [2, 33, 1, 66, 1, -1], [4, 21, 1, 12, 7, -1], [1, 84, 1, 28, 3, -1], [3, 32, 1, 96, 1, -1], [7, 18, 1, 14, 9, -1], [2, 63, 1, 126, 1, -1]] **(17)**

```
> nops(myramtype3);
```

14 **(18)**

```
> evalb(convert(myramtype3,set) = convert(RAMTYPE3,set));
```

false **(19)**

```
> myramtype3set:=convert(myramtype3,set): RAMTYPE3SET:=convert
  (RAMTYPE3,set):
> nops(myramtype3set intersect RAMTYPE3SET);
```

6 **(20)**

```
> myramtype3set intersect RAMTYPE3SET;
```

{[1, 34, 1, 17, 2, -1], [1, 39, 1, 13, 3, -1], [2, 13, 1, 26, 1, -1], [2, 33, 1, 66, 1, -1], [3, 7, 1, 21, 1, -1], [3, 22, 1, 11, 6, -1]} **(21)**

```
> NEWTYPE3:= myramtype3set minus RAMTYPE3SET;
```

NEWTYPE3 := {[1, 24, 1, 12, 2, -1], [1, 54, 1, 27, 2, -1], [1, 84, 1, 28, 3, -1], [2, 63, 1, 126, 1, -1], [3, 32, 1, 96, 1, -1], [4, 21, 1, 12, 7, -1], [7, 8, 1, 56, 1, -1], [7, 18, 1, 14, 9, -1]} **(22)**

```
> nops(%);
```

8 **(23)**

```
> ##<--- HERE --->
> findtype4(24); #checked up to 130
"n=", 5
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=128. To prove the identity
    we need to  check up to O(q^(130)).
    To be on the safe side we check up to O(q^(368)).
*** The identity below is PROVED!
```

```
[6, 1, -1]
```

$$\_GM(6)\ \_HM(1) - \_GM(1)\ \_HM(6) = \frac{\eta(24\tau)\,\eta(6\tau)^3\,\eta(4\tau)^3\,\eta(\tau)}{\eta(12\tau)^3\,\eta(8\tau)\,\eta(3\tau)\,\eta(2\tau)^3}$$

```
"n=", 10
"n=", 15
"n=", 20
```

$$[[6, 1, -1]] \tag{24}$$

```
> findtype5(130);
*** There were NO errors.  Each term was modular function on
    Gamma1(80). Also -mintotord=64. To prove the identity
    we need to  check up to O(q^(66)).
    To be on the safe side we check up to O(q^(224)).
*** The identity below is PROVED!
[4, 1, 1]
```

$$\_GM(1)\ \_GM(4) + \_HM(1)\ \_HM(4) = \frac{\eta(4\tau)^2}{\eta(8\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=128. To prove the identity
    we need to  check up to O(q^(130)).
    To be on the safe side we check up to O(q^(368)).
*** The identity below is PROVED!
[3, 2, 1]
```

$$\_GM(2)\ \_GM(3) + \_HM(2)\ \_HM(3) = \frac{\eta(12\tau)^3\,\eta(8\tau)\,\eta(3\tau)\,\eta(2\tau)^3}{\eta(24\tau)\,\eta(6\tau)^3\,\eta(4\tau)^3\,\eta(\tau)}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
"n=", 90
"n=", 100
"n=", 110
"n=", 120
"n=", 130
```

$$[[4, 1, 1], [3, 2, 1]] \tag{25}$$

```
> #<-- HERE --->
> findtype6(24);  #checked up 120
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)\ \_HM(1) - \_GM(1)\ \_H(1) = \frac{2\,\eta(20\tau)^2}{\eta(10\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
```

```
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, 1]
```

$$\_G(1)\,\_HM(1) + \_GM(1)\,\_H(1) = \frac{2\,\eta(4\,\tau)^2}{\eta(2\,\tau)^2}$$

```
"n=", 10
"n=", 20
```

$$[[1, 1, -1], [1, 1, 1]] \tag{26}$$

```
> findtype7(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(180). Also -mintotord=288. To prove the identity
    we need to  check up to O(q^(290)).
    To be on the safe side we check up to O(q^(648)).
*** The identity below is PROVED!
[9, 1, -1]
```

$$\_GM(1)\,\_G(9) - \_HM(1)\,\_H(9) = \frac{\eta(18\,\tau)^2\,\eta(12\,\tau)\,\eta(\tau)}{\eta(36\,\tau)\,\eta(9\,\tau)\,\eta(6\,\tau)\,\eta(2\,\tau)}$$

```
"n=", 10
"n=", 20
```

$$[[9, 1, -1]] \tag{27}$$

```
> read moreprogs:
"END"
```

```
> TT1:=300: TT2:=600:
> xprint:=false:
> findtype8(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(15). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(34)).
*** The identity below is PROVED!
[3, -1]
```

$$\_G(1)^3\,\_H(3) - \_H(1)^3\,\_G(3) = \frac{3\,\eta(15\,\tau)^3}{\eta(5\,\tau)\,\eta(3\,\tau)\,\eta(\tau)}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
WARNING: There were 2 ebasethreshold problems.
         See the global array EBL.
```

$$[[3, -1]] \tag{28}$$

```
> EBL;
```

$$[\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2),\ \_G(1)^2\,\_H(2) + \_H(1)^2\,\_G(2)] \tag{29}$$

```
> series(jac2series(G(1)^2*H(2)-H(1)^2*G(2),300)/q^(4/3),q,300):
> jacprodmake(%,q,250);
```

$$\frac{2\,JAC(0, 10, \infty)^5}{JAC(2, 10, \infty)\,JAC(3, 10, \infty)\,JAC(4, 10, \infty)^2\,JAC(5, 10, \infty)} \tag{30}$$

```
> findtype9();
*** There were NO errors.  Each term was modular function on
    Gamma1(5). Also -mintotord=2. To prove the identity
    we need to  check up to O(q^(4)).
    To be on the safe side we check up to O(q^(12)).
*** The identity below is PROVED!
[11, 1, 1]
```

$$\_G(1)^{11}\,\_H(1) - \_H(1)^{11}\,\_G(1) - 1 = \frac{11\,\eta(5\tau)^6}{\eta(\tau)^6}$$

$$[[11, 1, 1]] \tag{31}$$

```
> xprint:=false:read moreprogs:
> findtype10(76*2);
"END"
"n=", 50
 abs(mintotord)=-2160, which is too large
 Try increasing the global var qthreshold.
[19, 4, -1, 76, 1, 1]
```

$$\frac{\_G(19)\,\_H(4) - \_H(19)\,\_G(4)}{\_G(76)\,\_HM(1) + \_H(76)\,\_GM(1)} = \frac{\eta(76\tau)\,\eta(2\tau)}{\eta(38\tau)\,\eta(4\tau)}$$

```
 abs(mintotord)=-2400, which is too large
 Try increasing the global var qthreshold.
[28, 3, -1, 12, 7, 1]
```

$$\frac{\_G(28)\,\_H(3) - \_H(28)\,\_G(3)}{\_G(12)\,\_HM(7) + \_H(12)\,\_GM(7)} = \frac{\eta(21\tau)\,\eta(14\tau)^2\,\eta(6\tau)\,\eta(4\tau)\,\eta(\tau)}{\eta(42\tau)\,\eta(28\tau)\,\eta(7\tau)\,\eta(3\tau)\,\eta(2\tau)^2}$$

```
 abs(mintotord)=-2400, which is too large
 Try increasing the global var qthreshold.
[12, 7, -1, 28, 3, 1]
```

$$\frac{\_G(12)\,\_H(7) - \_H(12)\,\_G(7)}{\_G(28)\,\_HM(3) + \_H(28)\,\_GM(3)} = \frac{\eta(84\tau)\,\eta(21\tau)\,\eta(14\tau)\,\eta(6\tau)^2\,\eta(\tau)}{\eta(42\tau)^2\,\eta(12\tau)\,\eta(7\tau)\,\eta(3\tau)\,\eta(2\tau)}$$

```
"n=", 100
"n=", 150
```

$$[[19, 4, -1, 76, 1, 1], [28, 3, -1, 12, 7, 1], [12, 7, -1, 28, 3, 1]] \tag{32}$$

```
> read moreprogs:
> findtype11(84*3);
"END"
"n=", 50
"n=", 100
"n=", 150
"n=", 200
"n=", 250
```

$$[\,] \tag{33}$$

==
============

p=8

```
> G:=j->1/GetaL([1],8,j): H:=j->1/GetaL([3],8,j):
> GM:=j->1/MGetaL([1],8,j): HM:=j->1/MGetaL([3],8,j):
> GE:=j->-GetaLEXP([1],8,j): HE:=j->-GetaLEXP([3],8,j):
> GE(1),HE(1);
```

$$-\frac{11}{48}, \frac{13}{48} \tag{34}$$

```
> myramtype1:=findtype1(15);
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=6. To prove the identity
    we need to  check up to O(q^(8)).
    To be on the safe side we check up to O(q^(54)).
*** The identity below is PROVED!
[3, 1, -1]
```

$$\_G(3)\ \_H(1) - \_G(1)\ \_H(3) = \frac{\eta(12\,\tau)^2\,\eta(\tau)}{\eta(24\,\tau)\,\eta(8\,\tau)\,\eta(3\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=6. To prove the identity
    we need to  check up to O(q^(8)).
    To be on the safe side we check up to O(q^(54)).
*** The identity below is PROVED!
[3, 1, 1]
```

$$\_G(3)\ \_H(1) + \_G(1)\ \_H(3) = \frac{\eta(6\,\tau)^2\,\eta(4\,\tau)^2\,\eta(2\,\tau)}{\eta(12\,\tau)\,\eta(8\,\tau)^2\,\eta(3\,\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(40). Also -mintotord=20. To prove the identity
    we need to  check up to O(q^(22)).
    To be on the safe side we check up to O(q^(100)).
*** The identity below is PROVED!
[5, 1, -1]
```

$$\_G(5)\ \_H(1) - \_G(1)\ \_H(5) = \frac{\eta(20\,\tau)\,\eta(10\,\tau)\,\eta(2\,\tau)}{\eta(40\,\tau)\,\eta(8\,\tau)\,\eta(5\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(56). Also -mintotord=36. To prove the identity
    we need to  check up to O(q^(38)).
    To be on the safe side we check up to O(q^(148)).
*** The identity below is PROVED!
[7, 1, -1]
```

$$\_G(7)\ \_H(1) - \_G(1)\ \_H(7) = \frac{\eta(28\,\tau)\,\eta(4\,\tau)}{\eta(56\,\tau)\,\eta(8\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(72). Also -mintotord=60. To prove the identity
    we need to  check up to O(q^(62)).
    To be on the safe side we check up to O(q^(204)).
*** The identity below is PROVED!
[9, 1, -1]
```

$$\_G(9)\ \_H(1) - \_G(1)\ \_H(9) = \frac{\eta(36\,\tau)\,\eta(6\,\tau)^2\,\eta(4\,\tau)}{\eta(72\,\tau)\,\eta(12\,\tau)\,\eta(8\,\tau)\,\eta(3\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=144. To prove the identity
    we need to  check up to O(q^(146)).
    To be on the safe side we check up to O(q^(384)).
*** The identity below is PROVED!
[5, 3, -1]
```

$$\_G(5)\ \_H(3) - \_G(3)\ \_H(5) = \frac{\eta(60\tau)\,\eta(15\tau)\,\eta(10\tau)\,\eta(6\tau)\,\eta(4\tau)\,\eta(\tau)}{\eta(40\tau)\,\eta(30\tau)\,\eta(24\tau)\,\eta(5\tau)\,\eta(3\tau)\,\eta(2\tau)}$$

$$myramtype1 := [[3, 1, -1], [3, 1, 1], [5, 1, -1], [7, 1, -1], [9, 1, -1], [5, 3, -1]] \tag{35}$$

```
> myramtype2:=findtype2(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(8). Also -mintotord=1. To prove the identity
    we need to  check up to O(q^(3)).
    To be on the safe side we check up to O(q^(17)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)^2 - \_H(1)^2 = \frac{\eta(4\tau)^6}{\eta(8\tau)^4\,\eta(2\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(8). Also -mintotord=1. To prove the identity
    we need to  check up to O(q^(3)).
    To be on the safe side we check up to O(q^(17)).
*** The identity below is PROVED!
[1, 1, 1]
```

$$\_G(1)^2 + \_H(1)^2 = \frac{\eta(2\tau)^6}{\eta(8\tau)^2\,\eta(4\tau)\,\eta(\tau)^3}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=6. To prove the identity
    we need to  check up to O(q^(8)).
    To be on the safe side we check up to O(q^(54)).
*** The identity below is PROVED!
[1, 3, -1]
```

$$\_G(1)\ \_G(3) - \_H(1)\ \_H(3) = \frac{\eta(12\tau)^2\,\eta(6\tau)\,\eta(2\tau)^2}{\eta(24\tau)^2\,\eta(4\tau)\,\eta(3\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=6. To prove the identity
    we need to  check up to O(q^(8)).
    To be on the safe side we check up to O(q^(54)).
*** The identity below is PROVED!
[1, 3, 1]
```

$$\_G(1)\ \_G(3) + \_H(1)\ \_H(3) = \frac{\eta(4\tau)^2\,\eta(3\tau)}{\eta(24\tau)\,\eta(8\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(40). Also -mintotord=20. To prove the identity
    we need to  check up to O(q^(22)).
    To be on the safe side we check up to O(q^(100)).
*** The identity below is PROVED!
[1, 5, 1]
```

$$\_G(1)\ \_G(5)+\_H(1)\ \_H(5)=\frac{\eta(10\,\tau)\ \eta(4\,\tau)\ \eta(2\,\tau)}{\eta(40\,\tau)\ \eta(8\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(72). Also -mintotord=60. To prove the identity
    we need to  check up to O(q^(62)).
    To be on the safe side we check up to O(q^(204)).
*** The identity below is PROVED!
[1, 9, 1]
```

$$\_G(1)\ \_G(9)+\_H(1)\ \_H(9)=\frac{\eta(18\,\tau)\ \eta(12\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(72\,\tau)\ \eta(9\,\tau)\ \eta(8\,\tau)\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=144. To prove the identity
    we need to  check up to O(q^(146)).
    To be on the safe side we check up to O(q^(384)).
*** The identity below is PROVED!
[1, 15, 1]
```

$$\_G(1)\ \_G(15)+\_H(1)\ \_H(15)=\frac{\eta(30\,\tau)\ \eta(20\,\tau)\ \eta(12\,\tau)\ \eta(5\,\tau)\ \eta(3\,\tau)\ \eta(2\,\tau)}{\eta(120\,\tau)\ \eta(15\,\tau)\ \eta(10\,\tau)\ \eta(8\,\tau)\ \eta(6\,\tau)\ \eta(\tau)}$$

"n=", 50

$$myramtype2 := [[1,1,-1],[1,1,1],[1,3,-1],[1,3,1],[1,5,1],[1,9,1],[1,15,1]] \qquad \textbf{(36)}$$

```
> findtype3(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=10. To prove the identity
    we need to  check up to O(q^(12)).
    To be on the safe side we check up to O(q^(58)).
*** The identity below is PROVED!
[1, 3, -1, 3, 1, -1]
```

$$\frac{\_G(1)\ \_G(3)-\_H(1)\ \_H(3)}{\_G(3)\ \_H(1)-\_H(3)\ \_G(1)}=\frac{\eta(8\,\tau)\ \eta(6\,\tau)\ \eta(2\,\tau)^2}{\eta(24\,\tau)\ \eta(4\,\tau)\ \eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=8. To prove the identity
    we need to  check up to O(q^(10)).
    To be on the safe side we check up to O(q^(56)).
*** The identity below is PROVED!
[1, 3, -1, 3, 1, 1]
```

$$\frac{\_G(1)\ \_G(3)-\_H(1)\ \_H(3)}{\_G(3)\ \_H(1)+\_H(3)\ \_G(1)}=\frac{\eta(12\,\tau)^3\ \eta(8\,\tau)^2\ \eta(2\,\tau)}{\eta(24\,\tau)^2\ \eta(6\,\tau)\ \eta(4\,\tau)^3}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=12. To prove the identity
    we need to  check up to O(q^(14)).
    To be on the safe side we check up to O(q^(60)).
*** The identity below is PROVED!
[1, 3, 1, 3, 1, -1]
```

$$\frac{\_G(1)\ \_G(3)+\_H(1)\ \_H(3)}{\_G(3)\ \_H(1)-\_H(3)\ \_G(1)}=\frac{\eta(4\,\tau)^2\ \eta(3\,\tau)^2}{\eta(12\,\tau)^2\ \eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=10. To prove the identity
    we need to  check up to O(q^(12)).
```

To be on the safe side we check up to O(q^(58)).
*** The identity below is PROVED!
[1, 3, 1, 3, 1, 1]

$$\frac{\_G(1)\ \_G(3) + \_H(1)\ \_H(3)}{\_G(3)\ \_H(1) + \_H(3)\ \_G(1)} = \frac{\eta(12\,\tau)\ \eta(8\,\tau)\ \eta(3\,\tau)^2}{\eta(24\,\tau)\ \eta(6\,\tau)^2\ \eta(2\,\tau)}$$

*** There were NO errors. Each term was modular function on Gamma1(40). Also -mintotord=32. To prove the identity we need to check up to O(q^(34)).
To be on the safe side we check up to O(q^(112)).
*** The identity below is PROVED!
[1, 5, 1, 5, 1, -1]

$$\frac{\_G(1)\ \_G(5) + \_H(1)\ \_H(5)}{\_G(5)\ \_H(1) - \_H(5)\ \_G(1)} = \frac{\eta(5\,\tau)\ \eta(4\,\tau)}{\eta(20\,\tau)\ \eta(\tau)}$$

*** There were NO errors. Each term was modular function on Gamma1(72). Also -mintotord=120. To prove the identity we need to check up to O(q^(122)).
To be on the safe side we check up to O(q^(264)).
*** The identity below is PROVED!
[3, 3, -1, 9, 1, -1]

$$\frac{\_G(3)^2 - \_H(3)^2}{\_G(9)\ \_H(1) - \_H(9)\ \_G(1)} = \frac{\eta(72\,\tau)\ \eta(12\,\tau)^7\ \eta(8\,\tau)}{\eta(36\,\tau)\ \eta(24\,\tau)^4\ \eta(6\,\tau)^3\ \eta(4\,\tau)}$$

*** There were NO errors. Each term was modular function on Gamma1(72). Also -mintotord=108. To prove the identity we need to check up to O(q^(110)).
To be on the safe side we check up to O(q^(252)).
*** The identity below is PROVED!
[3, 3, 1, 9, 1, -1]

$$\frac{\_G(3)^2 + \_H(3)^2}{\_G(9)\ \_H(1) - \_H(9)\ \_G(1)} = \frac{\eta(72\,\tau)\ \eta(8\,\tau)\ \eta(6\,\tau)^4}{\eta(36\,\tau)\ \eta(24\,\tau)^2\ \eta(4\,\tau)\ \eta(3\,\tau)^2}$$

*** There were NO errors. Each term was modular function on Gamma1(72). Also -mintotord=96. To prove the identity we need to check up to O(q^(98)).
To be on the safe side we check up to O(q^(240)).
*** The identity below is PROVED!
[1, 9, 1, 9, 1, -1]

$$\frac{\_G(1)\ \_G(9) + \_H(1)\ \_H(9)}{\_G(9)\ \_H(1) - \_H(9)\ \_G(1)} = \frac{\eta(18\,\tau)\ \eta(12\,\tau)^2\ \eta(3\,\tau)^2\ \eta(2\,\tau)}{\eta(36\,\tau)\ \eta(9\,\tau)\ \eta(6\,\tau)^2\ \eta(4\,\tau)\ \eta(\tau)}$$

*** There were NO errors. Each term was modular function on Gamma1(120). Also -mintotord=256. To prove the identity we need to check up to O(q^(258)).
To be on the safe side we check up to O(q^(496)).
*** The identity below is PROVED!
[3, 5, -1, 15, 1, 1]

$$\frac{\_G(3)\ \_G(5) - \_H(3)\ \_H(5)}{\_G(15)\ \_H(1) + \_H(15)\ \_G(1)} = \frac{\eta(60\,\tau)\ \eta(4\,\tau)}{\eta(20\,\tau)\ \eta(12\,\tau)}$$

*** There were NO errors. Each term was modular function on Gamma1(120). Also -mintotord=224. To prove the identity we need to check up to O(q^(226)).

```
    To be on the safe side we check up to O(q^(464)).
*** The identity below is PROVED!
[3, 5, 1, 15, 1, -1]
```

$$\frac{\_G(3)\ \_G(5)+\_H(3)\ \_H(5)}{\_G(15)\ \_H(1)-\_H(15)\ \_G(1)}=\frac{\eta(120\,\tau)\,\eta(20\,\tau)\,\eta(12\,\tau)\,\eta(8\,\tau)}{\eta(60\,\tau)\,\eta(40\,\tau)\,\eta(24\,\tau)\,\eta(4\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=192. To prove the identity
    we need to  check up to O(q^(194)).
    To be on the safe side we check up to O(q^(432)).
*** The identity below is PROVED!
[1, 15, -1, 5, 3, 1]
```

$$\frac{\_G(1)\ \_G(15)-\_H(1)\ \_H(15)}{\_G(5)\ \_H(3)+\_H(5)\ \_G(3)}=\frac{\eta(60\,\tau)^2\,\eta(40\,\tau)^2\,\eta(24\,\tau)^2\,\eta(10\,\tau)\,\eta(6\,\tau)\,\eta(4\,\tau)^2}{\eta(120\,\tau)^2\,\eta(30\,\tau)\,\eta(20\,\tau)^2\,\eta(12\,\tau)^2\,\eta(8\,\tau)^2\,\eta(2\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=288. To prove the identity
    we need to  check up to O(q^(290)).
    To be on the safe side we check up to O(q^(528)).
*** The identity below is PROVED!
[1, 15, 1, 5, 3, -1]
```

$$\frac{\_G(1)\ \_G(15)+\_H(1)\ \_H(15)}{\_G(5)\ \_H(3)-\_H(5)\ \_G(3)}$$

$$=\frac{\eta(40\,\tau)\,\eta(30\,\tau)^2\,\eta(24\,\tau)\,\eta(20\,\tau)\,\eta(12\,\tau)\,\eta(5\,\tau)^2\,\eta(3\,\tau)^2\,\eta(2\,\tau)^2}{\eta(120\,\tau)\,\eta(60\,\tau)\,\eta(15\,\tau)^2\,\eta(10\,\tau)^2\,\eta(8\,\tau)\,\eta(6\,\tau)^2\,\eta(4\,\tau)\,\eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(168). Also -mintotord=528. To prove the identity
    we need to  check up to O(q^(530)).
    To be on the safe side we check up to O(q^(864)).
*** The identity below is PROVED!
[3, 7, 1, 21, 1, -1]
```

$$\frac{\_G(3)\ \_G(7)+\_H(3)\ \_H(7)}{\_G(21)\ \_H(1)-\_H(21)\ \_G(1)}=\frac{\eta(168\,\tau)\,\eta(28\,\tau)\,\eta(21\,\tau)\,\eta(8\,\tau)}{\eta(84\,\tau)\,\eta(56\,\tau)\,\eta(24\,\tau)\,\eta(7\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(168). Also -mintotord=528. To prove the identity
    we need to  check up to O(q^(530)).
    To be on the safe side we check up to O(q^(864)).
*** The identity below is PROVED!
[1, 21, 1, 7, 3, -1]
```

$$\frac{\_G(1)\ \_G(21)+\_H(1)\ \_H(21)}{\_G(7)\ \_H(3)-\_H(7)\ \_G(3)}=\frac{\eta(56\,\tau)\,\eta(24\,\tau)\,\eta(4\,\tau)\,\eta(3\,\tau)}{\eta(168\,\tau)\,\eta(12\,\tau)\,\eta(8\,\tau)\,\eta(\tau)}$$

```
 abs(mintotord)=-1632, which is too large
 Try increasing the global var qthreshold.
[1, 39, 1, 13, 3, -1]
```

$$\frac{\_G(1)\ \_G(39)+\_H(1)\ \_H(39)}{\_G(13)\ \_H(3)-\_H(13)\ \_G(3)}=\frac{\eta(104\,\tau)\,\eta(78\,\tau)\,\eta(24\,\tau)\,\eta(13\,\tau)\,\eta(3\,\tau)\,\eta(2\,\tau)}{\eta(312\,\tau)\,\eta(39\,\tau)\,\eta(26\,\tau)\,\eta(8\,\tau)\,\eta(6\,\tau)\,\eta(\tau)}$$

```
"n=", 50
 abs(mintotord)=-3680, which is too large
 Try increasing the global var qthreshold.
[1, 55, 1, 11, 5, -1]
```

$$\frac{\_G(1)\ \_G(55) + \_H(1)\ \_H(55)}{\_G(11)\ \_H(5) - \_H(11)\ \_G(5)} = \frac{\eta(110\tau)\,\eta(88\tau)\,\eta(40\tau)\,\eta(11\tau)\,\eta(5\tau)\,\eta(2\tau)}{q^{55/3}\,\eta(55\tau)\,\eta(22\tau)\,\eta(10\tau)\,\eta(8\tau)\,\eta(\tau)}$$

[[1, 3, -1, 3, 1, -1], [1, 3, -1, 3, 1, 1], [1, 3, 1, 3, 1, -1], [1, 3, 1, 3, 1, 1], [1, 5, 1, 5, 1, -1], [3, 3, -1, 9, 1, -1], [3, 3, 1, 9, 1, -1], [1, 9, 1, 9, 1, -1], [3, 5, -1, 15, 1, 1], [3, 5, 1, 15, 1, -1], [1, 15, -1, 5, 3, 1], [1, 15, 1, 5, 3, -1], [3, 7, 1, 21, 1, -1], [1, 21, 1, 7, 3, -1], [1, 39, 1, 13, 3, -1], [1, 55, 1, 11, 5, -1]] **(37)**

```
> etamake(series(jac2series( (G(1)*G(55)+H(1)*H(55))/(G(11)*H(5)-H(11)*G(5)),1000)*q^(35/3),q,1000),q,800);
```

$$\frac{q^{35/3}\,\eta(110\tau)\,\eta(88\tau)\,\eta(40\tau)\,\eta(11\tau)\,\eta(5\tau)\,\eta(2\tau)}{\eta(440\tau)\,\eta(55\tau)\,\eta(22\tau)\,\eta(10\tau)\,\eta(8\tau)\,\eta(\tau)} \quad \textbf{(38)}$$

```
> findtype4(60);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
"n=", 25
"n=", 30
"n=", 35
"n=", 40
"n=", 45
"n=", 50
"n=", 55
"n=", 60
```

[ ] **(39)**

```
> findtype5(60);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
```

[ ] **(40)**

```
> xprint:=false:read moreprogs:
"END"
> TT1;
```

300 **(41)**

```
> findtype6(120);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
"n=", 90
"n=", 100
"n=", 110
"n=", 120
WARNING: There were 20 ebasethreshold problems.
         See the global array EBL.
```

$$[\,]$$ (42)

```
> ebasethreshold;
```

$$1000$$ (43)

```
> findtype7(120);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
"n=", 90
"n=", 100
"n=", 110
"n=", 120
WARNING: There were 22 ebasethreshold problems.
         See the global array EBL.
```

$$[\,]$$ (44)

```
> read moreprogs:
"END"
> findtype8(10);
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=10. To prove the identity
    we need to  check up to O(q^(12)).
    To be on the safe side we check up to O(q^(58)).
*** The identity below is PROVED!
[3, -1]
```

$$\_G(1)^3\,\_H(3) - \_H(1)^3\,\_G(3) = \frac{3\,\eta(24\,\tau)^2\,\eta(6\,\tau)\,\eta(4\,\tau)\,\eta(2\,\tau)^3}{\eta(8\,\tau)^4\,\eta(3\,\tau)\,\eta(\tau)^2}$$

```
"n=", 10
WARNING: There were 2 ebasethreshold problems.
         See the global array EBL.
```

$$[[3, -1]]$$ (45)

```
> EBL;
```

$$[\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2),\ \_G(1)^2\,\_H(2) + \_H(1)^2\,\_G(2)]$$ (46)

```
> jacprodmake(series(jac2series(G(1)^2*H(2)-H(1)^2*G(2),300)/q^
  (13/12),q,300),q,250);
```

$$\frac{2\,JAC(0, 16, \infty)^5\,JAC(4, 16, \infty)}{JAC(1, 16, \infty)\,JAC(3, 16, \infty)\,JAC(5, 16, \infty)\,JAC(6, 16, \infty)\,JAC(7, 16, \infty)\,JAC(8, 16, \infty)}$$ (47)

```
> jac2getaprod(%);
```

$$\frac{2\,\eta_{16,4}(\tau)}{\eta_{16,1}(\tau)\,q^{13/12}\,\eta_{16,3}(\tau)\,\eta_{16,5}(\tau)\,\eta_{16,6}(\tau)\,\eta_{16,7}(\tau)\,\eta_{16,8}(\tau)}$$ (48)

```
> FIND4F(17,10,300);
> FIND5F(2,10,300);
```

$$\text{"COND: "}, -\frac{3}{5}\,a + \frac{3}{5}$$ (49)

```
> [seq([ithprime(j),modp(ithprime(j),5)],j=1..20)];
```

[[2, 2], [3, 3], [5, 0], [7, 2], [11, 1], [13, 3], [17, 2], [19, 4], [23, 3], [29, 4], [31, 1], [37, 2], [41, 1], [43, 3], [47, 2], [53, 3], [59, 4], [61, 1], [67, 2], [71, 1]] **(50)**

```
> FIND5F(61,10,300);
> findtype9();
*** There were NO errors.  Each term was modular function on
    Gamma1(8). Also -mintotord=1. To prove the identity
    we need to  check up to O(q^(3)).
    To be on the safe side we check up to O(q^(17)).
*** The identity below is PROVED!
[13, 11, 0]
```

$$\_G(1)^{13}\,\_H(1)^{11} - \_H(1)^{13}\,\_G(1)^{11} = \frac{\eta(4\,\tau)^{6}\,\eta(2\,\tau)^{10}}{\eta(8\,\tau)^{4}\,\eta(\tau)^{12}}$$

[[13, 11, 0]] **(51)**

```
> findtype10(84*3);
"n=", 50
"n=", 100
"n=", 150
"n=", 200
"n=", 250
```

[ ] **(52)**

```
> findtype11(300);
"n=", 50
"n=", 100
"n=", 150
"n=", 200
"n=", 250
"n=", 300
```

[ ] **(53)**

==

p=8 Version 2

```
> GetaL([4],8,1);
```

$$\frac{JAC(4,8,\infty)}{q^{1/3}\,JAC(0,8,\infty)}$$ **(54)**

```
> GetaB(4,8,1);
```

$$\frac{JAC(4,8,\infty)}{JAC(0,8,\infty)}$$ **(55)**

```
> jac2prod(%);
```

$$``(q\sim^{4}, q\sim^{8})_{\infty}^{2}$$ **(56)**

```
> G:=j->1/GetaL([1],8,j)/sqrt(GetaL([4],8,j)): H:=j->1/GetaL([3],8,
  j)/sqrt(GetaL([4],8,j)):
> GM:j->1/MGetaL([1,4],8,j): HM:=j->1/MGetaL([3,4],8,j):

> GE:=j->-GetaLEXP([1],8,j)-1/2*GetaLEXP([4],8,j);
```

$$GE := j \rightarrow -GetaLEXP([1],8,j) - \frac{1}{2}\,GetaLEXP([4],8,j)$$ **(57)**

```
> HE:=j->-GetaLEXP([3],8,j)-1/2*GetaLEXP([4],8,j);
```

$$HE := j \rightarrow -GetaLEXP([3], 8, j) - \frac{1}{2}\, GetaLEXP([4], 8, j) \quad (58)$$

```
> GE(a),HE(b);
```

$$-\frac{1}{16}\, a,\ \frac{7}{16}\, b \quad (59)$$

```
> series(jac2series(G(7)*H(1)-H(7)*G(1),300),q,300):
> series((jac2series( G(1)^7*H(1)-H(1)^7*G(1),300)-1),q,300):
> etamake(%,q,280);
```

$$\frac{7\, \eta(8\tau)^4\, \eta(2\tau)^2}{\eta(4\tau)^2\, \eta(\tau)^4} \quad (60)$$

```
> series(jac2series(G(1)*H(1),300)/q^(3/8),q,40);
```

$$1 + q + q^2 + 2q^3 + 4q^4 + 5q^5 + 6q^6 + 9q^7 + 13q^8 + 17q^9 + 21q^{10} + 28q^{11} + 39q^{12} + 49q^{13} + 60q^{14} + 78q^{15} + 101q^{16} + 125q^{17} + 153q^{18} + 192q^{19} + 241q^{20} + 295q^{21} + 357q^{22} + 438q^{23} + 540q^{24} + 652q^{25} + 781q^{26} + 946q^{27} + 1145q^{28} + 1368q^{29} + 1627q^{30} + 1945q^{31} + 2324q^{32} + 2754q^{33} + 3249q^{34} + 3845q^{35} + 4550q^{36} + 5348q^{37} + 6265q^{38} + 7356q^{39} + O(q^{40}) \quad (61)$$

```
> etamake(%,q,38);
```

$$\frac{\eta(8\tau)^2\, \eta(2\tau)}{q^{3/8}\, \eta(4\tau)^2\, \eta(\tau)} \quad (62)$$

```
> findtype1(12);
Error, (in JACP2jaclist) chk<>0
> series(S2,q,10);
```

$$1 - q\text{~}^{5/3} - q\text{~}^{8/3} + 2q\text{~}^3 - q\text{~}^{11/3} + 3q\text{~}^4 - q\text{~}^{14/3} + q\text{~}^5 - 2q\text{~}^{17/3} + 3q\text{~}^6 - 2q\text{~}^{20/3} + 6q\text{~}^7 - 2q\text{~}^{23/3} + 8q\text{~}^8 - 3q\text{~}^{26/3} + 7q\text{~}^9 - 4q\text{~}^{29/3} + O(q\text{~}^{10}) \quad (63)$$

==

p=10

```
> G:=j->1/GetaL([1],10,j): H:=j->1/GetaL([3],10,j):
> GM:=j->1/MGetaL([1],10,j): HM:=j->1/MGetaL([3],10,j):
> GE:=j->-GetaLEXP([1],10,j): HE:=j->-GetaLEXP([3],10,j):
> GE(1),HE(1);
```

$$-\frac{23}{60},\ \frac{13}{60} \quad (64)$$

```
> F1:=G(1)*q^(23/60);
```

$$F1 := \frac{JAC(0, 10, \infty)}{JAC(1, 10, \infty)} \quad (65)$$

```
> F2:=GM(1)*q^(23/60);
```

$$F2 := \frac{JAC(0, 20, \infty)\, JAC(1, 10, \infty)}{JAC(2, 20, \infty)\, JAC(0, 10, \infty)} \quad (66)$$

```
> series(subs(q=-q,jac2series(F1,300))-jac2series(F2,300),q,10);
```

$$O(q^{10}) \quad (67)$$

```
> myramtype1:=findtype1(6); #actually checked up to 150
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=40. To prove the identity
    we need to  check up to O(q^(42)).
    To be on the safe side we check up to O(q^(160)).
*** The identity below is PROVED!
[6, 1, -1]
```

$$\_G(6)\ \_H(1) - \_G(1)\ \_H(6) = \frac{\eta(30\tau)^3\,\eta(12\tau)\,\eta(5\tau)\,\eta(4\tau)}{\eta(60\tau)^2\,\eta(15\tau)\,\eta(10\tau)^2\,\eta(6\tau)}$$

$$myramtype1 := [[6, 1, -1]] \tag{68}$$

```
> myramtype2:=findtype2(9); #actually checked up to 100
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=40. To prove the identity
    we need to  check up to O(q^(42)).
    To be on the safe side we check up to O(q^(160)).
*** The identity below is PROVED!
[2, 3, -1]
```

$$\_G(2)\ \_G(3) - \_H(2)\ \_H(3) = \frac{\eta(15\tau)\,\eta(12\tau)\,\eta(10\tau)^3\,\eta(4\tau)}{\eta(30\tau)^2\,\eta(20\tau)^2\,\eta(5\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(90). Also -mintotord=96. To prove the identity
    we need to  check up to O(q^(98)).
    To be on the safe side we check up to O(q^(276)).
*** The identity below is PROVED!
[1, 9, -1]
```

$$\_G(1)\ \_G(9) - \_H(1)\ \_H(9) = \frac{\eta(45\tau)\,\eta(30\tau)^2\,\eta(18\tau)\,\eta(5\tau)\,\eta(3\tau)\,\eta(2\tau)}{\eta(90\tau)^2\,\eta(15\tau)\,\eta(10\tau)^2\,\eta(9\tau)\,\eta(\tau)}$$

$$myramtype2 := [[2, 3, -1], [1, 9, -1]] \tag{69}$$

```
> findtype3(120);
"n=", 50
"n=", 100
```

$$[\,] \tag{70}$$

```
> xprint:=true:read moreprogs:
"END"
> findtype4(120);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
"n=", 25
"n=", 30
"n=", 35
"n=", 40
"n=", 45
"n=", 50
"n=", 55
"n=", 60
"n=", 65
"n=", 70
```

```
"n=", 75
"n=", 80
"n=", 85
"n=", 90
"n=", 95
"n=", 100
"n=", 105
"n=", 110
"n=", 115
"n=", 120
```

$$[\,] \quad (71)$$

```
> findtype5(120);
*** There were NO errors.  Each term was modular function on
    Gamma1(80). Also -mintotord=64. To prove the identity
    we need to  check up to O(q^(66)).
    To be on the safe side we check up to O(q^(224)).
*** The identity below is PROVED!
[4, 1, -1]
```

$$\_GM(1)\ \_GM(4) - \_HM(1)\ \_HM(4) = \frac{\eta(40\,\tau)\ \eta(16\,\tau)\ \eta(10\,\tau)\ \eta(4\,\tau)^3\ \eta(\tau)}{\eta(80\,\tau)\ \eta(20\,\tau)\ \eta(8\,\tau)^2\ \eta(5\,\tau)\ \eta(2\,\tau)^2}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
"n=", 90
"n=", 100
"n=", 110
"n=", 120
```

$$[[4, 1, -1]] \quad (72)$$

```
> xprint:=false:
> findtype6(80);
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)\ \_HM(1) - \_GM(1)\ \_H(1) = \frac{2\,\eta(20\,\tau)^2}{\eta(10\,\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, 1]
```

$$\_G(1)\ \_HM(1) + \_GM(1)\ \_H(1) = \frac{2\,\eta(4\,\tau)^2}{\eta(10\,\tau)\ \eta(2\,\tau)}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
```

[[1, 1, -1], [1, 1, 1]] **(73)**

```
> findtype7(80);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
"n=", 70
"n=", 80
```

[ ] **(74)**

```
> findtype8(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[2, -1]
```

$$\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2) = \frac{2\,\eta(20\,\tau)^2\,\eta(5\,\tau)\,\eta(2\,\tau)}{\eta(10\,\tau)^3\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[2, 1]
```

$$\_G(1)^2\,\_H(2) + \_H(1)^2\,\_G(2) = \frac{2\,\eta(5\,\tau)\,\eta(4\,\tau)^2}{\eta(10\,\tau)^2\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(30). Also -mintotord=16. To prove the identity
    we need to  check up to O(q^(18)).
    To be on the safe side we check up to O(q^(76)).
*** The identity below is PROVED!
[3, -1]
```

$$\_G(1)^3\,\_H(3) - \_H(1)^3\,\_G(3) = \frac{3\,\eta(30\,\tau)\,\eta(15\,\tau)\,\eta(6\,\tau)\,\eta(5\,\tau)^2\,\eta(2\,\tau)^3}{\eta(10\,\tau)^5\,\eta(3\,\tau)\,\eta(\tau)^2}$$

```
"n=", 10
"n=", 20
```

[[2, -1], [2, 1], [3, -1]] **(75)**

```
> findtype9();
```

[ ] **(76)**

```
> findtype10(100);
"n=", 50
"n=", 100
```

$$[\,] \qquad (77)$$

```
> findtype11(100);
"n=", 50
"n=", 100
```

$$[\,] \qquad (78)$$

==

p=12

```
> phi(12);
```

$$4 \qquad (79)$$

```
> G:=j->1/GetaL([1],12,j): H:=j->1/GetaL([5],12,j):
> GM:=j->1/MGetaL([1],12,j): HM:=j->1/MGetaL([5],12,j):
> GE:=j->-GetaLEXP([1],12,j): HE:=j->-GetaLEXP([5],12,j):
> GE(1),HE(1);
```

$$-\frac{13}{24}, \frac{11}{24} \qquad (80)$$

```
> myramtype1:=findtype1(20);
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(52)).
*** The identity below is PROVED!
[2, 1, -1]
```

$$\_G(2)\ \_H(1) - \_G(1)\ \_H(2) = \frac{\eta(6\,\tau)\ \eta(4\,\tau)\ \eta(\tau)}{\eta(12\,\tau)^2\ \eta(2\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(52)).
*** The identity below is PROVED!
[2, 1, 1]
```

$$\_G(2)\ \_H(1) + \_G(1)\ \_H(2) = \frac{\eta(4\,\tau)\ \eta(3\,\tau)^2}{\eta(12\,\tau)^2\ \eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(36). Also -mintotord=12. To prove the identity
    we need to  check up to O(q^(14)).
    To be on the safe side we check up to O(q^(84)).
*** The identity below is PROVED!
[3, 1, -1]
```

$$\_G(3)\ \_H(1) - \_G(1)\ \_H(3) = \frac{\eta(18\,\tau)\ \eta(2\,\tau)}{\eta(36\,\tau)\ \eta(12\,\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(36). Also -mintotord=12. To prove the identity
    we need to  check up to O(q^(14)).
    To be on the safe side we check up to O(q^(84)).
*** The identity below is PROVED!
```

```
[3, 1, 1]
```

$$\_G(3)\ \_H(1) + \_G(1)\ \_H(3) = \frac{\eta(9\tau)^2\,\eta(6\tau)^5\,\eta(4\tau)}{\eta(18\tau)^2\,\eta(12\tau)^3\,\eta(3\tau)^2\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(48). Also -mintotord=24. To prove the identity
    we need to  check up to O(q^(26)).
    To be on the safe side we check up to O(q^(120)).
*** The identity below is PROVED!
[4, 1, -1]
```

$$\_G(4)\ \_H(1) - \_G(1)\ \_H(4) = \frac{\eta(16\tau)\,\eta(3\tau)}{\eta(48\tau)\,\eta(12\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=40. To prove the identity
    we need to  check up to O(q^(42)).
    To be on the safe side we check up to O(q^(160)).
*** The identity below is PROVED!
[5, 1, -1]
```

$$\_G(5)\ \_H(1) - \_G(1)\ \_H(5) = \frac{\eta(15\tau)\,\eta(10\tau)\,\eta(6\tau)\,\eta(4\tau)}{\eta(60\tau)\,\eta(12\tau)^2\,\eta(5\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(72). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(192)).
*** The identity below is PROVED!
[3, 2, -1]
```

$$\_G(3)\ \_H(2) - \_G(2)\ \_H(3) = \frac{\eta(72\tau)\,\eta(12\tau)\,\eta(9\tau)\,\eta(8\tau)\,\eta(6\tau)\,\eta(\tau)}{\eta(36\tau)^2\,\eta(24\tau)^2\,\eta(3\tau)\,\eta(2\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(72). Also -mintotord=60. To prove the identity
    we need to  check up to O(q^(62)).
    To be on the safe side we check up to O(q^(204)).
*** The identity below is PROVED!
[6, 1, -1]
```

$$\_G(6)\ \_H(1) - \_G(1)\ \_H(6) = \frac{\eta(9\tau)\,\eta(8\tau)}{\eta(72\tau)\,\eta(12\tau)}$$

myramtype1 := [[2, 1, -1], [2, 1, 1], [3, 1, -1], [3, 1, 1], [4, 1, -1], [5, 1, -1], [3, 2, -1], [6, 1, -1]] **(81)**

```
> myramtype2:=findtype2(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(12). Also -mintotord=2. To prove the identity
    we need to  check up to O(q^(4)).
    To be on the safe side we check up to O(q^(26)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)^2 - \_H(1)^2 = \frac{\eta(6\tau)^3\,\eta(2\tau)^3}{\eta(12\tau)^4\,\eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(12). Also -mintotord=2. To prove the identity
```

```
    we need to  check up to O(q^(4)).
    To be on the safe side we check up to O(q^(26)).
*** The identity below is PROVED!
[1, 1, 1]
```

$$\_G(1)^2+\_H(1)^2=\frac{\eta(4\tau)\,\eta(3\tau)^4\,\eta(2\tau)}{\eta(12\tau)^3\,\eta(6\tau)\,\eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(24). Also -mintotord=8. To prove the identity
    we need to  check up to O(q^(10)).
    To be on the safe side we check up to O(q^(56)).
*** The identity below is PROVED!
[1, 2, -1]
```

$$\_G(1)\ \_G(2)-\_H(1)\ \_H(2)=\frac{\eta(8\tau)^2\,\eta(3\tau)^2}{\eta(24\tau)^2\,\eta(12\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(36). Also -mintotord=18. To prove the identity
    we need to  check up to O(q^(20)).
    To be on the safe side we check up to O(q^(90)).
*** The identity below is PROVED!
[1, 3, -1]
```

$$\_G(1)\ \_G(3)-\_H(1)\ \_H(3)=\frac{\eta(18\tau)\,\eta(9\tau)\,\eta(4\tau)\,\eta(2\tau)}{\eta(36\tau)^2\,\eta(12\tau)\,\eta(\tau)}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=40. To prove the identity
    we need to  check up to O(q^(42)).
    To be on the safe side we check up to O(q^(160)).
*** The identity below is PROVED!
[1, 5, -1]
```

$$\_G(1)\ \_G(5)-\_H(1)\ \_H(5)=\frac{\eta(30\tau)\,\eta(20\tau)\,\eta(3\tau)\,\eta(2\tau)}{\eta(60\tau)^2\,\eta(12\tau)\,\eta(\tau)}$$

$$myramtype2:=[[1,1,-1],[1,1,1],[1,2,-1],[1,3,-1],[1,5,-1]] \qquad \mathbf{(82)}$$

```
> findtype3(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(120). Also -mintotord=256. To prove the identity
    we need to  check up to O(q^(258)).
    To be on the safe side we check up to O(q^(496)).
*** The identity below is PROVED!
[1, 10, -1, 5, 2, -1]
```

$$\frac{\_G(1)\ \_G(10)-\_H(1)\ \_H(10)}{\_G(5)\ \_H(2)-\_H(5)\ \_G(2)}=\frac{\eta(60\tau)^2\,\eta(24\tau)^2\,\eta(5\tau)\,\eta(2\tau)}{\eta(120\tau)^2\,\eta(12\tau)^2\,\eta(10\tau)\,\eta(\tau)}$$

$$[[1,10,-1,5,2,-1]] \qquad \mathbf{(83)}$$

```
> findtype4(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(48). Also -mintotord=24. To prove the identity
    we need to  check up to O(q^(26)).
    To be on the safe side we check up to O(q^(120)).
*** The identity below is PROVED!
[2, 1, -1]
```

$$\_GM(2)\ \_HM(1) - \_GM(1)\ \_HM(2) = \frac{\eta(48\tau)\,\eta(8\tau)^3\,\eta(6\tau)\,\eta(\tau)}{\eta(24\tau)^3\,\eta(16\tau)\,\eta(4\tau)\,\eta(2\tau)}$$

```
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

[[2, 1, -1]] **(84)**

```
> findtype5(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(48). Also -mintotord=24. To prove the identity
    we need to  check up to O(q^(26)).
    To be on the safe side we check up to O(q^(120)).
*** The identity below is PROVED!
[2, 1, -1]
```

$$\_GM(1)\ \_GM(2) - \_HM(1)\ \_HM(2) = \frac{\eta(16\tau)\,\eta(6\tau)\,\eta(\tau)}{\eta(48\tau)\,\eta(12\tau)\,\eta(2\tau)}$$

```
"n=", 10
"n=", 20
```

[[2, 1, -1]] **(85)**

```
> xprint:=false:
> findtype6(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)\ \_HM(1) - \_GM(1)\ \_H(1) = \frac{2\,\eta(20\tau)^2}{\eta(10\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(20). Also -mintotord=4. To prove the identity
    we need to  check up to O(q^(6)).
    To be on the safe side we check up to O(q^(44)).
*** The identity below is PROVED!
[1, 1, 1]
```

$$\_G(1)\ \_HM(1) + \_GM(1)\ \_H(1) = \frac{2\,\eta(4\tau)^2}{\eta(10\tau)\,\eta(2\tau)}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
WARNING: There were ebasethreshold problems.
```

[[1, 1, -1], [1, 1, 1]] **(86)**

```
> findtype7(60);
"n=", 10
"n=", 20
"n=", 30
```

```
"n=", 40
"n=", 50
"n=", 60
```

$$[\,] \quad (87)$$

```
> FIND4F(2,10,300);
```

$$\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2) = \frac{2\,\eta(24\,\tau)^3\,\eta(6\,\tau)\,\eta(4\,\tau)^2\,\eta(3\,\tau)}{\eta(12\,\tau)^5\,\eta(8\,\tau)\,\eta(\tau)} \quad (88)$$

```
> FIND4F(3,10,300);
```

$$\_G(1)^3\,\_H(3) - \_H(1)^3\,\_G(3) = \frac{3\,\eta(36\,\tau)^2\,\eta(9\,\tau)\,\eta(6\,\tau)\,\eta(4\,\tau)\,\eta(3\,\tau)\,\eta(2\,\tau)^2}{\eta(18\,\tau)\,\eta(12\,\tau)^5\,\eta(\tau)^2} \quad (89)$$

```
> FIND4F(13,10,300);
> FIND5F(2,10,300);
```

"COND: ", $-a+1$ **(90)**

```
> FIND5F(11,10,300);
> findtype9();
*** There were NO errors.  Each term was modular function on
    Gamma1(12). Also -mintotord=2. To prove the identity
    we need to  check up to O(q^(4)).
    To be on the safe side we check up to O(q^(26)).
*** The identity below is PROVED!
[11, 13, 0]
```

$$\_G(1)^{11}\,\_H(1)^{13} - \_H(1)^{11}\,\_G(1)^{13} = -\frac{\eta(3\,\tau)^{11}\,\eta(2\,\tau)^{14}}{\eta(12\,\tau)^4\,\eta(6\,\tau)^8\,\eta(\tau)^{13}}$$

$$[[11, 13, 0]] \quad (91)$$

```
> findtype10(60);
"n=", 50
```

$$[\,] \quad (92)$$

```
> findtype11(100);
"n=", 50
"n=", 100
```

$$[\,] \quad (93)$$

==
==============

p=13

```
> G:=j->1/GetaL(qr(13),13,j): H:=j->1/GetaL(qnr(13),13,j):

> GM:=j->1/MGetaL(qr(13),13,j): HM:=j->1/MGetaL(qnr(13),13,j):
> GE:=j->-GetaLEXP(qr(13),13,j): HE:=j->-GetaLEXP(qnr(13),13,j):
> GE(1),HE(1);
```

$$-\frac{1}{4},\ \frac{3}{4} \quad (94)$$

```
> ;
```

$$-\frac{1}{4},\ \frac{3}{4} \quad (95)$$

```
> myramtype1:=findtype1(48);
*** There were NO errors.  Each term was modular function on
```

```
    Gamma1(39). Also -mintotord=24. To prove the identity
    we need to  check up to O(q^(26)).
    To be on the safe side we check up to O(q^(102)).
*** The identity below is PROVED!
[3, 1, -1]
```

$$\_G(3)\ \_H(1) - \_G(1)\ \_H(3) = 1$$

$$myramtype1 := [[3, 1, -1]] \tag{96}$$

```
> myramtype2:=findtype2(48);
```

$$myramtype2 := [\ ] \tag{97}$$

```
> findtype3(48);
*** There were NO errors.  Each term was modular function on
    Gamma1(26). Also -mintotord=18. To prove the identity
    we need to  check up to O(q^(20)).
    To be on the safe side we check up to O(q^(70)).
*** The identity below is PROVED!
[1, 2, 1, 2, 1, -1]
```

$$\frac{\_G(1)\ \_G(2) + \_H(1)\ \_H(2)}{\_G(2)\ \_H(1) - \_H(2)\ \_G(1)} = \frac{\eta(13\,\tau)^2\,\eta(2\,\tau)^2}{\eta(26\,\tau)^2\,\eta(\tau)^2}$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(130). Also -mintotord=432. To prove the identity
    we need to  check up to O(q^(434)).
    To be on the safe side we check up to O(q^(692)).
*** The identity below is PROVED!
[2, 5, 1, 10, 1, -1]
```

$$\frac{\_G(2)\ \_G(5) + \_H(2)\ \_H(5)}{\_G(10)\ \_H(1) - \_H(10)\ \_G(1)} = 1$$

```
*** There were NO errors.  Each term was modular function on
    Gamma1(182). Also -mintotord=864. To prove the identity
    we need to  check up to O(q^(866)).
    To be on the safe side we check up to O(q^(1228)).
*** The identity below is PROVED!
[1, 14, 1, 7, 2, -1]
```

$$\frac{\_G(1)\ \_G(14) + \_H(1)\ \_H(14)}{\_G(7)\ \_H(2) - \_H(7)\ \_G(2)} = \frac{\eta(91\,\tau)\,\eta(26\,\tau)\,\eta(7\,\tau)\,\eta(2\,\tau)}{\eta(182\,\tau)\,\eta(14\,\tau)\,\eta(13\,\tau)\,\eta(\tau)}$$

$$[[1, 2, 1, 2, 1, -1], [2, 5, 1, 10, 1, -1], [1, 14, 1, 7, 2, -1]] \tag{98}$$

```
> findtype4(48);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
"n=", 25
"n=", 30
"n=", 35
"n=", 40
"n=", 45
```

$$[\ ] \tag{99}$$

```
> findtype5(48);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
```

(100)

$$[\,]\tag{100}$$

```
> findtype6(48);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
```

$$[\,]\tag{101}$$

```
> findtype7(48);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
```

$$[\,]\tag{102}$$

```
> findtype8(24);
"n=", 10
"n=", 20
```

$$[\,]\tag{103}$$

```
> FIND4F(3,10,300);
> FIND5F(2,10,100);
> FIND5F(3,10,100);
```

$$\_G(1)^3\,\_H(1) - \_H(1)^3\,\_G(1) = 1 + \frac{3\,\eta(13\,\tau)^2}{\eta(\tau)^2}\tag{104}$$

```
> FIND5F(5,10,100);
> findtype9();
*** There were NO errors.  Each term was modular function on
    Gamma1(13). Also -mintotord=6. To prove the identity
    we need to  check up to O(q^(8)).
    To be on the safe side we check up to O(q^(32)).
*** The identity below is PROVED!
[3, 1, 1]
```

$$\_G(1)^3\,\_H(1) - \_H(1)^3\,\_G(1) - 1 = \frac{3\,\eta(13\,\tau)^2}{\eta(\tau)^2}$$

$$[[3, 1, 1]]\tag{105}$$

```
> findtype10(84*2);
"n=", 50
"n=", 100
"n=", 150
```

$$[\,]\tag{106}$$

```
> findtype11(100);
"n=", 50
"n=", 100
```

$$[\,]\tag{107}$$

===

p=15

```
> G:=j->1/GetaL([1,4],15,j): H:=j->1/GetaL([2,7],15,j):
> GE:=j->-GetaLEXP([1,4],15,j): HE:=j->-GetaLEXP([2,7],15,j):
> GM:=j->1/MGetaL([1,4],15,j): HM:=j->1/MGetaL([2,7],15,j):
```

```
> GE(1),HE(1);
```

$$-\frac{17}{30}, \frac{7}{30} \qquad (108)$$

```
> checkL([1,4],15);
```

1, {1, 4, 11, 14}, {2, 7, 8, 13}
2, {2, 7, 8, 13}, {1, 4, 11, 14}
4, {1, 4, 11, 14}, {2, 7, 8, 13}
7, {2, 7, 8, 13}, {1, 4, 11, 14}
8, {2, 7, 8, 13}, {1, 4, 11, 14}
11, {1, 4, 11, 14}, {2, 7, 8, 13}
13, {2, 7, 8, 13}, {1, 4, 11, 14}
14, {1, 4, 11, 14}, {2, 7, 8, 13} **(109)**

```
> myramtype1:=findtype1(60);
"n=", 50
```

myramtype1 := [ ] **(110)**

```
> myramtype2:=findtype2(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(168)).
*** The identity below is PROVED!
[1, 4, -1]
```

$$\_G(1)\ \_G(4) - \_H(1)\ \_H(4) = \frac{\eta(30\tau)^2\,\eta(12\tau)\,\eta(10\tau)\,\eta(3\tau)\,\eta(2\tau)}{\eta(60\tau)^2\,\eta(15\tau)^2\,\eta(4\tau)\,\eta(\tau)}$$

```
"n=", 50
```

myramtype2 := [[1, 4, -1]] **(111)**

```
> findtype3(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(90). Also -mintotord=120. To prove the identity
    we need to  check up to O(q^(122)).
    To be on the safe side we check up to O(q^(300)).
*** The identity below is PROVED!
[2, 3, -1, 6, 1, -1]
```

$$\frac{\_G(2)\ \_G(3) - \_H(2)\ \_H(3)}{\_G(6)\ \_H(1) - \_H(6)\ \_G(1)} = \frac{\eta(90\tau)\,\eta(15\tau)^3\,\eta(10\tau)\,\eta(6\tau)}{\eta(45\tau)\,\eta(30\tau)^3\,\eta(5\tau)\,\eta(3\tau)}$$

```
"n=", 50
```

[[2, 3, -1, 6, 1, -1]] **(112)**

```
> findtype4(60);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
"n=", 25
"n=", 30
"n=", 35
"n=", 40
"n=", 45
"n=", 50
```

```
"n=", 55
"n=", 60
```

$$[\,]\qquad (113)$$

```
> findtype5(60);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
```

$$[\,]\qquad (114)$$

```
> findtype6(60);
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(168)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)\ \_HM(1) - \_GM(1)\ \_H(1) = \frac{2\,\eta(60\,\tau)^2\,\eta(10\,\tau)\,\eta(6\,\tau)^3\,\eta(4\,\tau)}{\eta(30\,\tau)^4\,\eta(12\,\tau)\,\eta(2\,\tau)^2}$$

```
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
WARNING: There were ebasethreshold problems.
```

$$[[1, 1, -1]]\qquad (115)$$

```
> findtype7(60);
"n=", 10
"n=", 20
"n=", 30
"n=", 40
"n=", 50
"n=", 60
WARNING: There were ebasethreshold problems.
```

$$[\,]\qquad (116)$$

```
> findtype8(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(30). Also -mintotord=12. To prove the identity
    we need to  check up to O(q^(14)).
    To be on the safe side we check up to O(q^(72)).
*** The identity below is PROVED!
[2, 1]
```

$$\_G(1)^2\ \_H(2) + \_H(1)^2\ \_G(2) = \frac{2\,\eta(10\,\tau)^2\,\eta(6\,\tau)\,\eta(3\,\tau)^2}{\eta(15\,\tau)^3\,\eta(2\,\tau)\,\eta(\tau)}$$

```
"n=", 10
"n=", 20
```

$$[[2, 1]]\qquad (117)$$

```
> FIND4F(11,10,300);
> FIND5F(2,10,300);
```

$$"COND: ", -\frac{4}{5}a + \frac{4}{5} \quad (118)$$

```
> FIND5F(31,10,300);
> GE(1),HE(1);
```

$$-\frac{17}{30}, \frac{7}{30} \quad (119)$$

```
> findtype9();
```

[ ] (120)

```
> findtype10(160);
"n=", 50
"n=", 100
"n=", 150
```

[ ] (121)

```
> findtype11(100);
"n=", 50
"n=", 100
```

[ ] (122)

==
===============

p=16

```
> phi(16);
```

8 (123)

```
> checkL([1,7],16);
```

1, {1, 7, 9, 15}, {3, 5, 11, 13}
3, {3, 5, 11, 13}, {1, 7, 9, 15}
5, {3, 5, 11, 13}, {1, 7, 9, 15}
7, {1, 7, 9, 15}, {3, 5, 11, 13}
9, {1, 7, 9, 15}, {3, 5, 11, 13}
11, {3, 5, 11, 13}, {1, 7, 9, 15}
13, {3, 5, 11, 13}, {1, 7, 9, 15}
15, {1, 7, 9, 15}, {3, 5, 11, 13} (124)

==
================

p=17

```
> G:=j->1/GetaL(qr(17),17,j): H:=j->1/GetaL(qnr(17),17,j):
> GM:=j->1/MGetaL(qr(17),17,j): HM:=j->1/MGetaL(qnr(17),17,j):
> GE:=j->-GetaLEXP(qr(17),17,j): HE:=j->-GetaLEXP(qnr(17),17,j):
> GE(1),HE(1);
```

$$-\frac{2}{3}, \frac{4}{3} \quad (125)$$

```
> xprint:=false:
> myramtype1:=findtype1(2); #actually checked up to 120
*** There were NO errors.  Each term was modular function on
    Gamma1(34). Also -mintotord=16. To prove the identity
    we need to  check up to O(q^(18)).
    To be on the safe side we check up to O(q^(84)).
```

```
*** The identity below is PROVED!
[2, 1, -1]
```

$$\_G(2)\ \_H(1) - \_G(1)\ \_H(2) = 1$$

$$myramtype1 := [[2, 1, -1]] \tag{126}$$

```
> R2:=series(jac2series(G(1)^2*H(1)-G(1)*H(1)^2,500),q,500):
> E17:=series(q^2*etaq(q,17,3000)^3/etaq(q,1,3000)^3,q,3000):
> P17:=sift(E17,q,17,0,2999):
```

```
> findnonhom([(R2^2-1)/8,E17],q,4,1);
```

$$\#\ of\ terms\ ,37$$

$$-----RELATIONS----of\ order---,4$$

$$\left\{-\frac{1}{4}X_2^3 + X_1^4 + X_2^4 - \frac{1}{16}X_2^2 - X_1X_2^2 + X_1^3X_2 - \frac{7}{4}X_1^2X_2^2 - X_1X_2^3\right\} \tag{127}$$

```
> _EnvExplicit:=true:
```

```
> galois(%,x1);
```

"4T5", {"S(4)"}, "-", 24, {"(1 4)", "(2 4)", "(3 4)"} **(128)**

```
> myramtype2:=findtype2(120);
"n=", 50
"n=", 100
```

$$myramtype2 := [\ ] \tag{129}$$

```
> xprint:=false;
> findtype3(24);
```

$$xprint := false$$

[ ] **(130)**

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

[ ] **(131)**

```
> findtype5(24);
"n=", 10
"n=", 20
```

[ ] **(132)**

```
> findtype6(24);
"n=", 10
"n=", 20
WARNING: There were ebasethreshold problems.
```

[ ] **(133)**

```
> findtype7(24);
"n=", 10
"n=", 20
```

[ ] **(134)**

```
> findtype8(24);
"n=", 10
"n=", 20
```

[ ] **(135)**

```
> FIND4F(17,10,300);
> FIND5F(5,10,300);
> findtype9();
```
[ ] (136)

```
> findtype10(120);
"n=", 50
"n=", 100
```
[ ] (137)

==
================

p=26

8 (138)

```
> phi(26);
```
12 (139)

```
> primroot(26);
```
7 (140)

```
> [seq(modp(7^(2*i),26),i=0..5)];
```
[1, 23, 9, 25, 3, 17] (141)

```
> [seq(modp(7^(2*i+1),26),i=0..5)];
```
[7, 5, 11, 19, 21, 15] (142)

```
> G:=j->1/GetaL([1,3,9],26,j): H:=j->1/GetaL([7,5,11],26,j):
> GE:=j->-GetaLEXP([1,3,9],26,j): HE:=j->-GetaLEXP([7,5,11],26,j):
> GM:=j->1/MGetaL([1,3,9],26,j): HM:=j->1/MGetaL([7,5,11],26,j):
> GE(1),HE(1);
```
$$-\frac{7}{4}, \frac{5}{4}$$ (143)

```
> findtype1(24);
```
[ ] (144)

```
> findtype2(24);
```
[ ] (145)

```
> findtype3(24);
```
[ ] (146)

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```
[ ] (147)

```
> findtype5(24);
"n=", 10
"n=", 20
```
[ ] (148)

```
> TT1:=300:TT2:=600:
> findtype6(24);
"n=", 10
"n=", 20
```
[ ] (149)

```
> TT1,TT2;
```
(150)

200, 600 **(150)**

```
> findtype7(24);
"n=", 10
"n=", 20
```

[ ] **(151)**

```
> findtype8(24);
"n=", 10
"n=", 20
```

[ ] **(152)**

```
> TT1,TT2;
```

100, 400 **(153)**

```
> FIND4F(13,10,300);
> FIND5F(5,10,300);
> findtype9();
```

[ ] **(154)**

===

p=29

12 **(155)**

```
> G:=j->1/GetaL(qr(29),29,j): H:=j->1/GetaL(qnr(29),29,j):
> GM:=j->1/MGetaL(qr(29),29,j): HM:=j->1/MGetaL(qnr(29),29,j):
> GE:=j->-GetaLEXP(qr(29),29,j): HE:=j->-GetaLEXP(qnr(29),29,j):
> GE(1),HE(1);
```

$$-\frac{11}{12}, \frac{25}{12}$$ **(156)**

```
> TT1,TT2;TT1:=100: TT2:=400:
```

300, 600 **(157)**

```
> findtype1(24);
```

[ ] **(158)**

```
> findtype2(24);
```

[ ] **(159)**

```
> findtype3(24);
```

[ ] **(160)**

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

[ ] **(161)**

```
> findtype5(24);
"n=", 10
"n=", 20
```

[ ] **(162)**

```
> TT1:=300: TT2:=600:
> findtype6(24);
"n=", 10
"n=", 20
```

[ ] **(163)**

```
> findtype7(24);
"n=", 10
"n=", 20
WARNING: There were 14 ebasethreshold problems.
         See the global array EBL.
```

[ ] **(164)**

```
> findtype8(24);
"n=", 10
"n=", 20
```

[ ] **(165)**

```
> FIND4F(13,10,300);
> findtype9();
```

[ ] **(166)**

==
=================

p=30

```
> phi(30);
```

8 **(167)**

```
> checkL([1,11],30);
```

1, {1, 11, 19, 29}, {7, 13, 17, 23}
7, {7, 13, 17, 23}, {1, 11, 19, 29}
11, {1, 11, 19, 29}, {7, 13, 17, 23}
13, {7, 13, 17, 23}, {1, 11, 19, 29}
17, {7, 13, 17, 23}, {1, 11, 19, 29}
19, {1, 11, 19, 29}, {7, 13, 17, 23}
23, {7, 13, 17, 23}, {1, 11, 19, 29}
29, {1, 11, 19, 29}, {7, 13, 17, 23} **(168)**

```
> G:=j->1/GetaL([1,11],30,j): H:=j->1/GetaL([7,13],30,j):
> GM:=j->1/MGetaL([1,11],30,j): HM:=j->1/MGetaL([7,13],30,j):
> GE:=j->-GetaLEXP([1,11],30,j): HE:=j->-GetaLEXP([7,13],30,j):
> GE(1),HE(1);
```

$$-\frac{31}{30}, \frac{41}{30} \qquad \textbf{(169)}$$

```
> TT1:=100:TT2:=300:
> findramtype1:=findtype1(24);
```

findramtype1 := [ ] **(170)**

```
> findramtype2:=findtype2(24);
```

findramtype2 := [ ] **(171)**

```
> findtype3(24);
```

[ ] **(172)**

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

[ ] **(173)**

```
> findtype5(24);
"n=", 10
```

```
"n=", 20
```

[ ] **(174)**

```
> TT1:=300: TT2:=600:
> findtype6(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(168)).
*** The identity below is PROVED!
[1, 1, -1]
```

$$\_G(1)\ \_HM(1) - \_GM(1)\ \_H(1) = \frac{2\,\eta(60\,\tau)^2\,\eta(6\,\tau)^2\,\eta(4\,\tau)}{\eta(30\,\tau)^3\,\eta(12\,\tau)\,\eta(2\,\tau)}$$

```
"n=", 10
"n=", 20
```

[[1, 1, -1]] **(175)**

```
> findtype7(24);
"n=", 10
"n=", 20
```

[ ] **(176)**

```
> findtype8(24);
*** There were NO errors.  Each term was modular function on
    Gamma1(60). Also -mintotord=48. To prove the identity
    we need to  check up to O(q^(50)).
    To be on the safe side we check up to O(q^(168)).
*** The identity below is PROVED!
[2, -1]
```

$$\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2) = \frac{2\,\eta(60\,\tau)^2\,\eta(6\,\tau)\,\eta(5\,\tau)\,\eta(4\,\tau)\,\eta(3\,\tau)}{\eta(30\,\tau)^2\,\eta(15\,\tau)\,\eta(12\,\tau)\,\eta(10\,\tau)\,\eta(\tau)}$$

```
"n=", 10
"n=", 20
```

[[2, -1]] **(177)**

```
> FIND4F(2,10,300);
```

$$\_G(1)^2\,\_H(2) - \_H(1)^2\,\_G(2) = \frac{2\,\eta(60\,\tau)^2\,\eta(6\,\tau)\,\eta(5\,\tau)\,\eta(4\,\tau)\,\eta(3\,\tau)}{\eta(30\,\tau)^2\,\eta(15\,\tau)\,\eta(12\,\tau)\,\eta(10\,\tau)\,\eta(\tau)} \quad \textbf{(178)}$$

```
> FIND4F(3,10,300);
> FIND4F(13,10,300);
> FIND5F(11,10,300);
> findtype9();
```

[ ] **(179)**

```
> findtype10(24);
```

[ ] **(180)**

```
==============================================================================
=================
```

p=34

```
> phi(34);
```

16 **(181)**

```
> primroot(34);
```

$$3 \quad (182)$$

```
> seq(modp(3^(2*j),34),j=0..7);
```

$$1, 9, 13, 15, 33, 25, 21, 19 \quad (183)$$

```
> seq(modp(3^(2*j+1),34),j=0..7);
```

$$3, 27, 5, 11, 31, 7, 29, 23 \quad (184)$$

```
> G:=j->1/GetaL([1,9,13,15],34,j): H:=j->1/GetaL([3,5,7,11],34,j):
> GM:=j->1/MGetaL([1,9,13,15],34,j): HM:=j->1/MGetaL([3,5,7,11],34,
  j):
> GE:=j->-GetaLEXP([1,9,13,15],34,j): HE:=j->-GetaLEXP([3,5,7,11],
  34,j):
> GE(1),HE(1);
```

$$\frac{2}{3}, -\frac{4}{3} \quad (185)$$

```
> TT1:=100: TT2:=400:
> findtype1(24);
```

$$[\,] \quad (186)$$

```
> findtype2(24);
```

$$[\,] \quad (187)$$

```
> findtype3(24);
```

$$[\,] \quad (188)$$

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

$$[\,] \quad (189)$$

```
> findtype5(24);
"n=", 10
"n=", 20
```

$$[\,] \quad (190)$$

```
> findtype6(24);
"n=", 10
"n=", 20
```

$$[\,] \quad (191)$$

```
> findtype7(24);
"n=", 10
"n=", 20
```

$$[\,] \quad (192)$$

```
> findtype8(24);
"n=", 10
"n=", 20
```

$$[\,] \quad (193)$$

```
> FIND4F(19,10,300);
> read moreprogs:
"END"
> findtype9();
*** There were NO errors.  Each term was modular function on
    Gamma1(34). Also -mintotord=16. To prove the identity
    we need to  check up to O(q^(18)).
```

```
    To be on the safe side we check up to O(q^(84)).
*** The identity below is PROVED!
[2, 1, 0]
```

$$\_G(1)^2\,\_H(1) - \_H(1)^2\,\_G(1) = -\frac{\eta(17\,\tau)\,\eta(2\,\tau)^2}{\eta(34\,\tau)^2\,\eta(\tau)}$$

$$[[2, 1, 0]] \tag{194}$$

```
> findtype10(48);
```

$$[\,] \tag{195}$$

```
> findtype11(48);
```

$$[\,] \tag{196}$$

```
================================================================================
==================
```

p=37

```
> G:=j->1/GetaL(qr(37),37,j): H:=j->1/GetaL(qnr(37),37,j):

> GM:=j->1/MGetaL(qr(37),37,j): HM:=j->1/MGetaL(qnr(37),37,j):
> GE:=j->-GetaLEXP(qr(37),37,j): HE:=j->-GetaLEXP(qnr(37),37,j):
> GE(1),HE(1);
```

$$-\frac{7}{4}, \frac{13}{4} \tag{197}$$

```
> myramtype1:=findtype1(24);
```

$$myramtype1 := [\,] \tag{198}$$

```
> findtype2(24);
```

$$[\,] \tag{199}$$

```
> findtype3(24);
```

$$[\,] \tag{200}$$

```
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
```

$$[\,] \tag{201}$$

```
> findtype5(24);
"n=", 10
"n=", 20
```

$$[\,] \tag{202}$$

```
> findtype6(24);
"n=", 10
"n=", 20
WARNING: There were 290 ebasethreshold problems.
         See the global array EBL.
```

$$[\,] \tag{203}$$

```
> findtype7(24);
"n=", 10
"n=", 20
WARNING: There were 312 ebasethreshold problems.
         See the global array EBL.
```

$$[\,] \tag{204}$$

```
> findtype8(24);
"n=", 10
"n=", 20
                                   []                                   (205)
> findtype9(24);
                                   []                                   (206)
```

==
=================

p=41

```
> G:=j->1/GetaL(qr(41),41,j): H:=j->1/GetaL(qnr(41),41,j):
> GM:=j->1/MGetaL(qr(41),41,j): HM:=j->1/MGetaL(qnr(41),41,j):
> GE:=j->-GetaLEXP(qr(41),41,j): HE:=j->-GetaLEXP(qnr(41),41,j):
> GE(1),HE(1);
```

$$-\frac{19}{6}, \frac{29}{6} \quad (207)$$

```
> myramtype1:=findtype1(24);
```

$$myramtype1 := [\,] \quad (208)$$

```
> findtype2(24);
                                   []                                   (209)
> findtype3(24);
                                   []                                   (210)
> findtype4(24);
"n=", 5
"n=", 10
"n=", 15
"n=", 20
                                   []                                   (211)
> findtype5(24);
"n=", 10
"n=", 20
                                   []                                   (212)
> findtype6(24);
"n=", 10
"n=", 20
WARNING: There were 275 ebasethreshold problems.
         See the global array EBL.
                                   []                                   (213)
> findtype7(24);
"n=", 10
"n=", 20
WARNING: There were 295 ebasethreshold problems.
         See the global array EBL.
                                   []                                   (214)
> findtype8(24);
"n=", 10
"n=", 20
                                   []                                   (215)
```

```
> findtype9(24);
```

$$[\ ]\qquad (216)$$