```
> read allprogs:
Warning, `N` is implicitly declared local to procedure `CUSPSANDWIDMAKE1`
> with(qseries):
> f:=(a,b,T)->add(a^(j*(j+1)/2)*b^(j*(j-1)/2),j=-T..T):
```

EXAMPLE 1: CHOI's IDENTITY (Theorem 2.15, p.11)

See
Youn-Seo Choi,
"Tenth order mock theta functions in Ramanujan's lost notebook. III",
Proc. Lond. Math. Soc. (3) **94** (2007), no. 1, 26-52.

```
> read "choiid1.txt";
CHOI's IDENTITY (Theorem 2.15, p.11)
```

$$\frac{\_(q^4,q^4)_{oo}^{2}\,\_(q^{10},q^{10})_{oo}^{4}\,\_(q^{20},q^{20})_{oo}}{\_(q^2,q^2)_{oo}\,\_(q^5,q^5)_{oo}^{2}\,\_(q^8,q^8)_{oo}\,\_\mathrm{f}(-q^2,-q^{18})\,\_\mathrm{f}(-q^{16},-q^{24})}=$$
$$\frac{\_(q^5,q^5)_{oo}\,\_(q^{10},q^{10})_{oo}\,\_\mathrm{f}(-q^4,-q^6)}{\_\mathrm{f}(-q^2,-q^3)\,\_\mathrm{f}(-q^2,-q^8)}-\frac{q^3\,\_(-q^{20},-q^{20})_{oo}\,\_(q^{40},q^{40})_{oo}\,\_\mathrm{f}(-q^8,-q^{32})}{\_\mathrm{f}(q^4,-q^{16})\,\_\mathrm{f}(-q^{16},-q^{24})}$$
$$+\frac{q^6\,\_(q^{40},q^{40})_{oo}\,\_(q^{80},q^{80})_{oo}\,\_\mathrm{f}(-q^8,-q^{32})}{\_\mathrm{f}(-q^{16},-q^{24})\,\_\mathrm{f}(-q^{32},-q^{48})}$$

Let CHOIID1 denote LHS-RHS:

```
> E:=n->etaq(q,n,500):
> CHOIID1:=E(4)^2*E(10)^4*E(20)/E(2)/E(5)^2/E(8)/f(-q^2,-q^18,20)/f(
  -q^16,-q^24,20)
> -( E(5)*E(10)*f(-q^4,-q^6,20)/f(-q^2,-q^3,20)/f(-q^2,-q^8,20)
> -q^3*subs(q=-q^20,E(1))*E(40)*f(-q^8,-q^32,20)/f(q^4,-q^16,20)/f(-
  q^16,-q^24,20)
> +q^6*E(40)*E(80)*f(-q^8,-q^32,20)/f(-q^16,-q^24,20)/f(-q^32,-q^48,
  20)):
> series(CHOIID1,q,500);
```

$$O(q^{500})$$

We convert each term in the identity into a quotient of Jacobi theta-functions.

```
> T1:=op(1,CHOIID1):C1:=1:
> J1:=jacprodmake(T1/C1,q,100);
```

$$J1 := \mathrm{JAC}(0,40,\infty)^9\,\mathrm{JAC}(4,40,\infty)\,\mathrm{JAC}(10,40,\infty)\,\mathrm{JAC}(12,40,\infty)\left(\frac{\mathrm{JAC}(20,40,\infty)}{\mathrm{JAC}(0,40,\infty)}\right)^{(3/2)} \Big/ \Big(\mathrm{JAC}(2,40,\infty)^2\,\mathrm{JAC}(5,40,\infty)^2\,\mathrm{JAC}(6,40,\infty)\,\mathrm{JAC}(14,40,\infty)\,\mathrm{JAC}(15,40,\infty)^2\,\mathrm{JAC}(16,40,\infty)\,\mathrm{JAC}(18,40,\infty)^2\Big)$$

```
> S1:=jac2series(J1,500):
> T2:=op(2,CHOIID1):C2:=-1:
```

```
> J2:=jacprodmake(T2/C2,q,100);
```

$$J2 := \frac{\mathrm{JAC}(0,10,\infty)^3\,\mathrm{JAC}(4,10,\infty)}{\mathrm{JAC}(2,10,\infty)^2\,\mathrm{JAC}(3,10,\infty)}$$

```
> S2:=jac2series(J2,500):
> T3:=op(3,CHOIID1):C3:=q^3:
> J3:=jacprodmake(T3/C3,q,200);
```

$$J3 := \frac{\mathrm{JAC}(4,80,\infty)\,\mathrm{JAC}(0,80,\infty)^2\,\mathrm{JAC}(32,80,\infty)\,\mathrm{JAC}(36,80,\infty)\sqrt{\frac{\mathrm{JAC}(40,80,\infty)}{\mathrm{JAC}(0,80,\infty)}}}{\mathrm{JAC}(16,80,\infty)^2\,\mathrm{JAC}(24,80,\infty)^2}$$

```
> S3:=jac2series(J3,500):
> T4:=op(4,CHOIID1):C4:=-q^6:
> J4:=jacprodmake(T4/C4,q,200);
```

$$J4 := \frac{\mathrm{JAC}(8,80,\infty)\,\mathrm{JAC}(0,80,\infty)^2\sqrt{\frac{\mathrm{JAC}(40,80,\infty)}{\mathrm{JAC}(0,80,\infty)}}}{\mathrm{JAC}(16,80,\infty)\,\mathrm{JAC}(24,80,\infty)}$$

```
> S4:=jac2series(J4,500):
> series(S1-T1,q,500);
```

$$\mathrm{O}(q^{500})$$

```
> series(T2-C2*S2,q,500);
```

$$\mathrm{O}(q^{500})$$

```
> series(T3-C3*S3,q,500);
```

$$\mathrm{O}(q^{503})$$

```
> series(T4-C4*S4,q,500);
```

$$\mathrm{O}(q^{502})$$

```
> j1:=J1*C1;
```

$$j1 := \mathrm{JAC}(0,40,\infty)^9\,\mathrm{JAC}(4,40,\infty)\,\mathrm{JAC}(10,40,\infty)\,\mathrm{JAC}(12,40,\infty)\left(\frac{\mathrm{JAC}(20,40,\infty)}{\mathrm{JAC}(0,40,\infty)}\right)^{(3/2)} \Big/ \Big(\mathrm{JAC}(2,40,\infty)^2\,\mathrm{JAC}(5,40,\infty)^2\,\mathrm{JAC}(6,40,\infty)\,\mathrm{JAC}(14,40,\infty)\,\mathrm{JAC}(15,40,\infty)^2\,\mathrm{JAC}(16,40,\infty)\,\mathrm{JAC}(18,40,\infty)^2\Big)$$

```
> j2:=J2*C2;
```

$$j2 := -\frac{\mathrm{JAC}(0,10,\infty)^3\,\mathrm{JAC}(4,10,\infty)}{\mathrm{JAC}(2,10,\infty)^2\,\mathrm{JAC}(3,10,\infty)}$$

```
> j3:=J3*C3;
```

$$j3 := \frac{\mathrm{JAC}(4,80,\infty)\,\mathrm{JAC}(0,80,\infty)^2\,\mathrm{JAC}(32,80,\infty)\,\mathrm{JAC}(36,80,\infty)\sqrt{\frac{\mathrm{JAC}(40,80,\infty)}{\mathrm{JAC}(0,80,\infty)}}\;q^3}{\mathrm{JAC}(16,80,\infty)^2\,\mathrm{JAC}(24,80,\infty)^2}$$

```
> j4:=J4*C4;
```

$$j4 := -\frac{\mathrm{JAC}(8, 80, \infty)\,\mathrm{JAC}(0, 80, \infty)^2 \sqrt{\frac{\mathrm{JAC}(40, 80, \infty)}{\mathrm{JAC}(0, 80, \infty)}}\, q^6}{\mathrm{JAC}(16, 80, \infty)\,\mathrm{JAC}(24, 80, \infty)}$$

We write the identity in symbolic form after dividing each term by j2:

```
> jid:= j1/j2+ 1 + j3/j2 + j4/j2;
```

$$jid := -\mathrm{JAC}(0, 40, \infty)^9\,\mathrm{JAC}(4, 40, \infty)\,\mathrm{JAC}(10, 40, \infty)\,\mathrm{JAC}(12, 40, \infty)\left(\frac{\mathrm{JAC}(20, 40, \infty)}{\mathrm{JAC}(0, 40, \infty)}\right)^{(3/2)}$$
$$\mathrm{JAC}(2, 10, \infty)^2\,\mathrm{JAC}(3, 10, \infty) \Big/ (\mathrm{JAC}(2, 40, \infty)^2\,\mathrm{JAC}(5, 40, \infty)^2\,\mathrm{JAC}(6, 40, \infty)$$
$$\mathrm{JAC}(14, 40, \infty)\,\mathrm{JAC}(15, 40, \infty)^2\,\mathrm{JAC}(16, 40, \infty)\,\mathrm{JAC}(18, 40, \infty)^2\,\mathrm{JAC}(0, 10, \infty)^3$$
$$\mathrm{JAC}(4, 10, \infty)) + 1 - \mathrm{JAC}(4, 80, \infty)\,\mathrm{JAC}(0, 80, \infty)^2\,\mathrm{JAC}(32, 80, \infty)\,\mathrm{JAC}(36, 80, \infty)$$
$$\sqrt{\frac{\mathrm{JAC}(40, 80, \infty)}{\mathrm{JAC}(0, 80, \infty)}}\, q^3\,\mathrm{JAC}(2, 10, \infty)^2\,\mathrm{JAC}(3, 10, \infty) \Big/ ($$
$$\mathrm{JAC}(16, 80, \infty)^2\,\mathrm{JAC}(24, 80, \infty)^2\,\mathrm{JAC}(0, 10, \infty)^3\,\mathrm{JAC}(4, 10, \infty))$$
$$+\frac{\mathrm{JAC}(8, 80, \infty)\,\mathrm{JAC}(0, 80, \infty)^2 \sqrt{\frac{\mathrm{JAC}(40, 80, \infty)}{\mathrm{JAC}(0, 80, \infty)}}\, q^6\,\mathrm{JAC}(2, 10, \infty)^2\,\mathrm{JAC}(3, 10, \infty)}{\mathrm{JAC}(16, 80, \infty)\,\mathrm{JAC}(24, 80, \infty)\,\mathrm{JAC}(0, 10, \infty)^3\,\mathrm{JAC}(4, 10, \infty)}$$

We convert each term to a uniform base:

```
> mjid:=mixedjac2jac(%,200);
"term ", 1, "of ", 4
"term ", 2, "of ", 4
"term ", 3, "of ", 4
"term ", 4, "of ", 4
```

$$mjid := -\mathrm{JAC}(3, 40, \infty)\,\mathrm{JAC}(7, 40, \infty)\,\mathrm{JAC}(8, 40, \infty)^2\,\mathrm{JAC}(12, 40, \infty)^3\,\mathrm{JAC}(13, 40, \infty)$$
$$\mathrm{JAC}(17, 40, \infty)\,\mathrm{JAC}(20, 40, \infty) \Big/ ($$
$$\mathrm{JAC}(5, 40, \infty)^2\,\mathrm{JAC}(6, 40, \infty)^2\,\mathrm{JAC}(14, 40, \infty)^2\,\mathrm{JAC}(15, 40, \infty)^2\,\mathrm{JAC}(16, 40, \infty)^2) + 1 - q^3$$
$$\mathrm{JAC}(2, 80, \infty)^2\,\mathrm{JAC}(3, 80, \infty)\,\mathrm{JAC}(7, 80, \infty)\,\mathrm{JAC}(8, 80, \infty)^2\,\mathrm{JAC}(12, 80, \infty)^2\,\mathrm{JAC}(13, 80, \infty)$$
$$\mathrm{JAC}(17, 80, \infty)\,\mathrm{JAC}(18, 80, \infty)^2\,\mathrm{JAC}(22, 80, \infty)^2\,\mathrm{JAC}(23, 80, \infty)\,\mathrm{JAC}(27, 80, \infty)$$
$$\mathrm{JAC}(28, 80, \infty)^2\,\mathrm{JAC}(32, 80, \infty)^3\,\mathrm{JAC}(33, 80, \infty)\,\mathrm{JAC}(37, 80, \infty)\,\mathrm{JAC}(38, 80, \infty)^2 \Big/ ($$
$$\mathrm{JAC}(0, 80, \infty)^{12}\,\mathrm{JAC}(6, 80, \infty)\,\mathrm{JAC}(10, 80, \infty)\,\mathrm{JAC}(14, 80, \infty)\,\mathrm{JAC}(16, 80, \infty)^3$$
$$\mathrm{JAC}(20, 80, \infty)\,\mathrm{JAC}(24, 80, \infty)^3\,\mathrm{JAC}(26, 80, \infty)\,\mathrm{JAC}(30, 80, \infty)\,\mathrm{JAC}(34, 80, \infty)) + q^6$$
$$\mathrm{JAC}(2, 80, \infty)^2\,\mathrm{JAC}(3, 80, \infty)\,\mathrm{JAC}(7, 80, \infty)\,\mathrm{JAC}(8, 80, \infty)^3\,\mathrm{JAC}(12, 80, \infty)^2\,\mathrm{JAC}(13, 80, \infty)$$
$$\mathrm{JAC}(17, 80, \infty)\,\mathrm{JAC}(18, 80, \infty)^2\,\mathrm{JAC}(22, 80, \infty)^2\,\mathrm{JAC}(23, 80, \infty)\,\mathrm{JAC}(27, 80, \infty)$$
$$\mathrm{JAC}(28, 80, \infty)^2\,\mathrm{JAC}(32, 80, \infty)^2\,\mathrm{JAC}(33, 80, \infty)\,\mathrm{JAC}(37, 80, \infty)\,\mathrm{JAC}(38, 80, \infty)^2 \Big/ ($$
$$\mathrm{JAC}(0, 80, \infty)^{12}\,\mathrm{JAC}(4, 80, \infty)\,\mathrm{JAC}(6, 80, \infty)\,\mathrm{JAC}(10, 80, \infty)\,\mathrm{JAC}(14, 80, \infty)\,\mathrm{JAC}(16, 80, \infty)^2$$
$$\mathrm{JAC}(20, 80, \infty)\,\mathrm{JAC}(24, 80, \infty)^2\,\mathrm{JAC}(26, 80, \infty)\,\mathrm{JAC}(30, 80, \infty)\,\mathrm{JAC}(34, 80, \infty)$$
$$\mathrm{JAC}(36, 80, \infty))$$

We check that the first term is a modular function on $\Gamma_1(80)$

```
> eprod1:=jac2eprod(op(1,mjid));
```

$$eprod1 := -\mathrm{GETA}(40,3)\,\mathrm{GETA}(40,7)\,\mathrm{GETA}(40,8)^2\,\mathrm{GETA}(40,12)^3\,\mathrm{GETA}(40,13)\,\mathrm{GETA}(40,17)\,\mathrm{GETA}(40,20) \Big/ \left(\mathrm{GETA}(40,5)^2\,\mathrm{GETA}(40,6)^2\,\mathrm{GETA}(40,14)^2\,\mathrm{GETA}(40,15)^2\,\mathrm{GETA}(40,16)^2\right)$$

```
> getap1:=GETAP2getalist(eprod1/(-1));
```

getap1 := [[40, 3, 1], [40, 5, -2], [40, 6, -2], [40, 7, 1], [40, 8, 2], [40, 12, 3], [40, 13, 1], [40, 14, -2], [40, 15, -2], [40, 16, -2], [40, 17, 1], [40, 20, 1]]

```
> vinf(getap1,80);
```

0

```
> Gamma1ModFunc(getap1,80);
```

"All n are divisors of ", 80

"val0=", 0

"which is even."

"valinf=", 0

"which is even."

"It IS a modfunc on Gamma1(", 80, ")"

1

We calculate a set of inequivalent cusps for $\Gamma_1(80)$

and the width of each cusp. Note: oo is the first cusp in the list.

```
> cusps80:=cuspmake1(80):
> cusp80:=cusps80 minus {[1,0]}:
> cusps80:=convert(cusp80,list):
> wids80:=map(x->cuspwid1(x[1],x[2],80),cusps80):
> wids80:=[1,op(wids80)]:
> CUSPS80:=map(x->x[1]/x[2],cusps80):
> CUSPS80:=[oo,op(CUSPS80)];
```

$$CUSPS80 := \left[oo, 0, \frac{1}{3}, \frac{1}{7}, \frac{1}{29}, \frac{1}{31}, \frac{1}{33}, \frac{1}{37}, \frac{1}{39}, \frac{1}{19}, \frac{1}{21}, \frac{1}{23}, \frac{1}{27}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{17}, \frac{1}{38}, \frac{1}{2}, \frac{1}{6}, \frac{1}{14}, \frac{1}{18}, \frac{1}{22}, \frac{1}{26}, \frac{3}{28}, \frac{1}{36}, \frac{11}{36}, \frac{1}{28}, \frac{3}{4}, \frac{1}{12}, \frac{11}{12}, \frac{1}{4}, \frac{1}{34}, \frac{1}{15}, \frac{8}{15}, \frac{7}{15}, \frac{14}{15}, \frac{4}{5}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{8}, \frac{3}{8}, \frac{7}{25}, \frac{9}{25}, \frac{1}{25}, \frac{3}{25}, \frac{1}{35}, \frac{3}{35}, \frac{17}{35}, \frac{9}{35}, \frac{1}{10}, \frac{3}{10}, \frac{7}{24}, \frac{13}{24}, \frac{11}{24}, \frac{7}{8}, \frac{5}{8}, \frac{1}{24}, \frac{1}{16}, \frac{3}{16}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{7}{10}, \frac{9}{10}, \frac{1}{30}, \frac{23}{30}, \frac{7}{30}, \frac{29}{30}, \frac{11}{20}, \frac{13}{20}, \frac{17}{20}, \frac{19}{20}, \frac{1}{20}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}, \frac{9}{32}, \frac{11}{32}, \frac{13}{32}, \frac{21}{32}, \frac{31}{32}, \frac{3}{32}, \frac{7}{32}, \frac{5}{16}, \frac{15}{16}, \frac{1}{32}, \frac{9}{40}, \frac{11}{40}, \frac{13}{40}, \frac{17}{40}, \frac{19}{40}, \frac{7}{40}, \frac{1}{40}, \frac{3}{40}, \frac{33}{80}, \frac{37}{80}, \frac{39}{80}, \frac{31}{80}, \frac{11}{80}, \frac{13}{80}, \frac{17}{80}, \frac{19}{80}, \frac{21}{80}, \frac{23}{80}, \frac{27}{80}, \frac{29}{80}, \frac{9}{80}, \frac{3}{80}, \frac{7}{80}\right]$$

```
> wids80;
```

[1, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 80, 40, 40, 40, 40, 40, 40, 40, 20, 20, 20, 20, 20, 20, 20, 20, 40, 16, 16, 16, 16, 16, 16, 16, 16, 10, 10, 16, 16, 16, 16, 16, 16, 16, 16, 8, 8, 10, 10, 10, 10, 10, 10, 5, 5, 5, 5, 5, 5, 8, 8, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

We are ready to prove the theta-function identity:

```
> provemodfuncid(mjid,CUSPS80,wids80,80);
```

```
"TERM ", 1, "of ", 4, " ************************************************************
********"
```

$$"XX=", -\mathrm{JAC}(3, 40, \infty)\,\mathrm{JAC}(7, 40, \infty)\,\mathrm{JAC}(8, 40, \infty)^2\,\mathrm{JAC}(12, 40, \infty)^3\,\mathrm{JAC}(13, 40, \infty)\,\mathrm{JAC}(17, 40, \infty)\,\mathrm{JAC}(20, 40, \infty) \Big/ \left(\mathrm{JAC}(5, 40, \infty)^2\,\mathrm{JAC}(6, 40, \infty)^2\,\mathrm{JAC}(14, 40, \infty)^2\,\mathrm{JAC}(15, 40, \infty)^2\,\mathrm{JAC}(16, 40, \infty)^2\right)$$

"Cusp ORDS: "

$$\left[[oo, 0], [0, 0], \left[\frac{1}{3}, 0\right], \left[\frac{1}{7}, 0\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, 0\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{39}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, 0\right],\right.$$

$$\left[\frac{1}{23}, 0\right], \left[\frac{1}{27}, 0\right], \left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{13}, 0\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{38}, 0\right], \left[\frac{1}{2}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{14}, 0\right], \left[\frac{1}{18}, 0\right],$$

$$\left[\frac{1}{22}, 0\right], \left[\frac{1}{26}, 0\right], \left[\frac{3}{28}, 4\right], \left[\frac{1}{36}, 4\right], \left[\frac{11}{36}, 4\right], \left[\frac{1}{28}, 4\right], \left[\frac{3}{4}, 4\right], \left[\frac{1}{12}, 4\right], \left[\frac{11}{12}, 4\right], \left[\frac{1}{4}, 4\right], \left[\frac{1}{34}, 0\right],$$

$$\left[\frac{1}{15}, -6\right], \left[\frac{8}{15}, 0\right], \left[\frac{7}{15}, 0\right], \left[\frac{14}{15}, -6\right], \left[\frac{4}{5}, -6\right], \left[\frac{1}{5}, -6\right], \left[\frac{3}{5}, 0\right], \left[\frac{2}{5}, 0\right], \left[\frac{1}{8}, -2\right], \left[\frac{3}{8}, -2\right], \left[\frac{7}{25}, 0\right],$$

$$\left[\frac{9}{25}, -6\right], \left[\frac{1}{25}, -6\right], \left[\frac{3}{25}, 0\right], \left[\frac{1}{35}, -6\right], \left[\frac{3}{35}, 0\right], \left[\frac{17}{35}, 0\right], \left[\frac{9}{35}, -6\right], \left[\frac{1}{10}, 8\right], \left[\frac{3}{10}, 4\right], \left[\frac{7}{24}, -2\right],$$

$$\left[\frac{13}{24}, -2\right], \left[\frac{11}{24}, -2\right], \left[\frac{7}{8}, -2\right], \left[\frac{5}{8}, -2\right], \left[\frac{1}{24}, -2\right], \left[\frac{1}{16}, -1\right], \left[\frac{3}{16}, -1\right], \left[\frac{7}{16}, -1\right], \left[\frac{9}{16}, -1\right], \left[\frac{11}{16}, -1\right],$$

$$\left[\frac{13}{16}, -1\right], \left[\frac{7}{10}, 4\right], \left[\frac{9}{10}, 8\right], \left[\frac{1}{30}, 8\right], \left[\frac{23}{30}, 4\right], \left[\frac{7}{30}, 4\right], \left[\frac{29}{30}, 8\right], \left[\frac{11}{20}, 0\right], \left[\frac{13}{20}, 4\right], \left[\frac{17}{20}, 4\right], \left[\frac{19}{20}, 0\right],$$

$$\left[\frac{1}{20}, 0\right], \left[\frac{3}{20}, 4\right], \left[\frac{7}{20}, 4\right], \left[\frac{9}{20}, 0\right], \left[\frac{9}{32}, -1\right], \left[\frac{11}{32}, -1\right], \left[\frac{13}{32}, -1\right], \left[\frac{21}{32}, -1\right], \left[\frac{31}{32}, -1\right], \left[\frac{3}{32}, -1\right],$$

$$\left[\frac{7}{32}, -1\right], \left[\frac{5}{16}, -1\right], \left[\frac{15}{16}, -1\right], \left[\frac{1}{32}, -1\right], \left[\frac{9}{40}, 0\right], \left[\frac{11}{40}, 0\right], \left[\frac{13}{40}, -2\right], \left[\frac{17}{40}, -2\right], \left[\frac{19}{40}, 0\right], \left[\frac{7}{40}, -2\right],$$

$$\left[\frac{1}{40}, 0\right], \left[\frac{3}{40}, -2\right], \left[\frac{33}{80}, -1\right], \left[\frac{37}{80}, -1\right], \left[\frac{39}{80}, 0\right], \left[\frac{31}{80}, 0\right], \left[\frac{11}{80}, 0\right], \left[\frac{13}{80}, -1\right], \left[\frac{17}{80}, -1\right], \left[\frac{19}{80}, 0\right],$$

$$\left.\left[\frac{21}{80}, 0\right], \left[\frac{23}{80}, -1\right], \left[\frac{27}{80}, -1\right], \left[\frac{29}{80}, 0\right], \left[\frac{9}{80}, 0\right], \left[\frac{3}{80}, -1\right], \left[\frac{7}{80}, -1\right]\right]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 80

"val0=", 0

"which is even."

"valinf=", 0

"which is even."

"It IS a modfunc on Gamma1(", 80, ")"

```
"TERM ", 2, "of ", 4, " *****************************************************************"
```

"XX=", 1

```
"TERM ", 3, "of ", 4, " *****************************************************************"
```

$$\text{"XX=", } -q^3\,\mathrm{JAC}(2,80,\infty)^2\,\mathrm{JAC}(3,80,\infty)\,\mathrm{JAC}(7,80,\infty)\,\mathrm{JAC}(8,80,\infty)^2\,\mathrm{JAC}(12,80,\infty)^2\,\mathrm{JAC}(13,80,\infty)\,\mathrm{JAC}(17,80,\infty)\,\mathrm{JAC}(18,80,\infty)^2\,\mathrm{JAC}(22,80,\infty)^2\,\mathrm{JAC}(23,80,\infty)\,\mathrm{JAC}(27,80,\infty)\,\mathrm{JAC}(28,80,\infty)^2\,\mathrm{JAC}(32,80,\infty)^3\,\mathrm{JAC}(33,80,\infty)\,\mathrm{JAC}(37,80,\infty)\,\mathrm{JAC}(38,80,\infty)^2 \Big/ \left(\mathrm{JAC}(0,80,\infty)^{12}\,\mathrm{JAC}(6,80,\infty)\,\mathrm{JAC}(10,80,\infty)\,\mathrm{JAC}(14,80,\infty)\,\mathrm{JAC}(16,80,\infty)^3\,\mathrm{JAC}(20,80,\infty)\,\mathrm{JAC}(24,80,\infty)^3\,\mathrm{JAC}(26,80,\infty)\,\mathrm{JAC}(30,80,\infty)\,\mathrm{JAC}(34,80,\infty)\right)$$

"Cusp ORDS: "

$$\Big[[oo,3],[0,1],\left[\tfrac{1}{3},1\right],\left[\tfrac{1}{7},1\right],\left[\tfrac{1}{29},1\right],\left[\tfrac{1}{31},1\right],\left[\tfrac{1}{33},1\right],\left[\tfrac{1}{37},1\right],\left[\tfrac{1}{39},1\right],\left[\tfrac{1}{19},1\right],\left[\tfrac{1}{21},1\right],$$

$$\left[\tfrac{1}{23},1\right],\left[\tfrac{1}{27},1\right],\left[\tfrac{1}{9},1\right],\left[\tfrac{1}{11},1\right],\left[\tfrac{1}{13},1\right],\left[\tfrac{1}{17},1\right],\left[\tfrac{1}{38},0\right],\left[\tfrac{1}{2},0\right],\left[\tfrac{1}{6},0\right],\left[\tfrac{1}{14},0\right],\left[\tfrac{1}{18},0\right],$$

$$\left[\tfrac{1}{22},0\right],\left[\tfrac{1}{26},0\right],\left[\tfrac{3}{28},0\right],\left[\tfrac{1}{36},0\right],\left[\tfrac{11}{36},0\right],\left[\tfrac{1}{28},0\right],\left[\tfrac{3}{4},0\right],\left[\tfrac{1}{12},0\right],\left[\tfrac{11}{12},0\right],\left[\tfrac{1}{4},0\right],\left[\tfrac{1}{34},0\right],$$

$$\left[\tfrac{1}{15},-6\right],\left[\tfrac{8}{15},1\right],\left[\tfrac{7}{15},1\right],\left[\tfrac{14}{15},-6\right],\left[\tfrac{4}{5},-6\right],\left[\tfrac{1}{5},-6\right],\left[\tfrac{3}{5},1\right],\left[\tfrac{2}{5},1\right],\left[\tfrac{1}{8},-2\right],\left[\tfrac{3}{8},-2\right],\left[\tfrac{7}{25},1\right],$$

$$\left[\tfrac{9}{25},-6\right],\left[\tfrac{1}{25},-6\right],\left[\tfrac{3}{25},1\right],\left[\tfrac{1}{35},-6\right],\left[\tfrac{3}{35},1\right],\left[\tfrac{17}{35},1\right],\left[\tfrac{9}{35},-6\right],\left[\tfrac{1}{10},0\right],\left[\tfrac{3}{10},-6\right],\left[\tfrac{7}{24},-2\right],$$

$$\left[\tfrac{13}{24},-2\right],\left[\tfrac{11}{24},-2\right],\left[\tfrac{7}{8},-2\right],\left[\tfrac{5}{8},-2\right],\left[\tfrac{1}{24},-2\right],\left[\tfrac{1}{16},0\right],\left[\tfrac{3}{16},0\right],\left[\tfrac{7}{16},0\right],\left[\tfrac{9}{16},0\right],\left[\tfrac{11}{16},0\right],\left[\tfrac{13}{16},0\right],$$

$$\left[\tfrac{7}{10},-6\right],\left[\tfrac{9}{10},0\right],\left[\tfrac{1}{30},0\right],\left[\tfrac{23}{30},-6\right],\left[\tfrac{7}{30},-6\right],\left[\tfrac{29}{30},0\right],\left[\tfrac{11}{20},0\right],\left[\tfrac{13}{20},0\right],\left[\tfrac{17}{20},0\right],\left[\tfrac{19}{20},0\right],\left[\tfrac{1}{20},0\right],$$

$$\left[\tfrac{3}{20},0\right],\left[\tfrac{7}{20},0\right],\left[\tfrac{9}{20},0\right],\left[\tfrac{9}{32},0\right],\left[\tfrac{11}{32},0\right],\left[\tfrac{13}{32},0\right],\left[\tfrac{21}{32},0\right],\left[\tfrac{31}{32},0\right],\left[\tfrac{3}{32},0\right],\left[\tfrac{7}{32},0\right],\left[\tfrac{5}{16},0\right],$$

$$\left[\tfrac{15}{16},0\right],\left[\tfrac{1}{32},0\right],\left[\tfrac{9}{40},12\right],\left[\tfrac{11}{40},12\right],\left[\tfrac{13}{40},-2\right],\left[\tfrac{17}{40},-2\right],\left[\tfrac{19}{40},12\right],\left[\tfrac{7}{40},-2\right],\left[\tfrac{1}{40},12\right],\left[\tfrac{3}{40},-2\right],$$

$\left[\frac{33}{80}, 0\right], \left[\frac{37}{80}, 0\right], \left[\frac{39}{80}, 3\right], \left[\frac{31}{80}, 3\right], \left[\frac{11}{80}, 3\right], \left[\frac{13}{80}, 0\right], \left[\frac{17}{80}, 0\right], \left[\frac{19}{80}, 3\right], \left[\frac{21}{80}, 3\right], \left[\frac{23}{80}, 0\right], \left[\frac{27}{80}, 0\right],$
$\left[\frac{29}{80}, 3\right], \left[\frac{9}{80}, 3\right], \left[\frac{3}{80}, 0\right], \left[\frac{7}{80}, 0\right]\right]$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 80

"val0=", 2

"which is even."

"valinf=", 6

"which is even."

"It IS a modfunc on Gamma1(", 80, ")"

"TERM ", 4, "of ", 4, " ***"

"XX=", q^6 JAC(2, 80, ∞)^2 JAC(3, 80, ∞) JAC(7, 80, ∞) JAC(8, 80, ∞)^3 JAC(12, 80, ∞)^2 JAC(13, 80, ∞) JAC(17, 80, ∞) JAC(18, 80, ∞)^2 JAC(22, 80, ∞)^2 JAC(23, 80, ∞) JAC(27, 80, ∞) JAC(28, 80, ∞)^2 JAC(32, 80, ∞)^2 JAC(33, 80, ∞) JAC(37, 80, ∞) JAC(38, 80, ∞)^2 ╱ (JAC(0, 80, ∞)^{12} JAC(4, 80, ∞) JAC(6, 80, ∞) JAC(10, 80, ∞) JAC(14, 80, ∞) JAC(16, 80, ∞)^2 JAC(20, 80, ∞) JAC(24, 80, ∞)^2 JAC(26, 80, ∞) JAC(30, 80, ∞) JAC(34, 80, ∞) JAC(36, 80, ∞))

"Cusp ORDS: "

$\left[[oo, 6], [0, 1], \left[\frac{1}{3}, 1\right], \left[\frac{1}{7}, 1\right], \left[\frac{1}{29}, 1\right], \left[\frac{1}{31}, 1\right], \left[\frac{1}{33}, 1\right], \left[\frac{1}{37}, 1\right], \left[\frac{1}{39}, 1\right], \left[\frac{1}{19}, 1\right], \left[\frac{1}{21}, 1\right],\right.$
$\left[\frac{1}{23}, 1\right], \left[\frac{1}{27}, 1\right], \left[\frac{1}{9}, 1\right], \left[\frac{1}{11}, 1\right], \left[\frac{1}{13}, 1\right], \left[\frac{1}{17}, 1\right], \left[\frac{1}{38}, 0\right], \left[\frac{1}{2}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{14}, 0\right], \left[\frac{1}{18}, 0\right],$
$\left[\frac{1}{22}, 0\right], \left[\frac{1}{26}, 0\right], \left[\frac{3}{28}, 0\right], \left[\frac{1}{36}, 0\right], \left[\frac{11}{36}, 0\right], \left[\frac{1}{28}, 0\right], \left[\frac{3}{4}, 0\right], \left[\frac{1}{12}, 0\right], \left[\frac{11}{12}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{34}, 0\right],$
$\left[\frac{1}{15}, -6\right], \left[\frac{8}{15}, 1\right], \left[\frac{7}{15}, 1\right], \left[\frac{14}{15}, -6\right], \left[\frac{4}{5}, -6\right], \left[\frac{1}{5}, -6\right], \left[\frac{3}{5}, 1\right], \left[\frac{2}{5}, 1\right], \left[\frac{1}{8}, 0\right], \left[\frac{3}{8}, 0\right], \left[\frac{7}{25}, 1\right],$
$\left[\frac{9}{25}, -6\right], \left[\frac{1}{25}, -6\right], \left[\frac{3}{25}, 1\right], \left[\frac{1}{35}, -6\right], \left[\frac{3}{35}, 1\right], \left[\frac{17}{35}, 1\right], \left[\frac{9}{35}, -6\right], \left[\frac{1}{10}, 0\right], \left[\frac{3}{10}, -6\right], \left[\frac{7}{24}, 0\right],$
$\left[\frac{13}{24}, 0\right], \left[\frac{11}{24}, 0\right], \left[\frac{7}{8}, 0\right], \left[\frac{5}{8}, 0\right], \left[\frac{1}{24}, 0\right], \left[\frac{1}{16}, -1\right], \left[\frac{3}{16}, -1\right], \left[\frac{7}{16}, -1\right], \left[\frac{9}{16}, -1\right], \left[\frac{11}{16}, -1\right], \left[\frac{13}{16}, -1\right],$
$\left[\frac{7}{10}, -6\right], \left[\frac{9}{10}, 0\right], \left[\frac{1}{30}, 0\right], \left[\frac{23}{30}, -6\right], \left[\frac{7}{30}, -6\right], \left[\frac{29}{30}, 0\right], \left[\frac{11}{20}, 0\right], \left[\frac{13}{20}, 0\right], \left[\frac{17}{20}, 0\right], \left[\frac{19}{20}, 0\right], \left[\frac{1}{20}, 0\right],$
$\left[\frac{3}{20}, 0\right], \left[\frac{7}{20}, 0\right], \left[\frac{9}{20}, 0\right], \left[\frac{9}{32}, -1\right], \left[\frac{11}{32}, -1\right], \left[\frac{13}{32}, -1\right], \left[\frac{21}{32}, -1\right], \left[\frac{31}{32}, -1\right], \left[\frac{3}{32}, -1\right], \left[\frac{7}{32}, -1\right],$

$$\left[\frac{5}{16}, -1\right], \left[\frac{15}{16}, -1\right], \left[\frac{1}{32}, -1\right], \left[\frac{9}{40}, 6\right], \left[\frac{11}{40}, 6\right], \left[\frac{13}{40}, 0\right], \left[\frac{17}{40}, 0\right], \left[\frac{19}{40}, 6\right], \left[\frac{7}{40}, 0\right], \left[\frac{1}{40}, 6\right], \left[\frac{3}{40}, 0\right],$$
$$\left[\frac{33}{80}, -1\right], \left[\frac{37}{80}, -1\right], \left[\frac{39}{80}, 6\right], \left[\frac{31}{80}, 6\right], \left[\frac{11}{80}, 6\right], \left[\frac{13}{80}, -1\right], \left[\frac{17}{80}, -1\right], \left[\frac{19}{80}, 6\right], \left[\frac{21}{80}, 6\right], \left[\frac{23}{80}, -1\right],$$
$$\left[\frac{27}{80}, -1\right], \left[\frac{29}{80}, 6\right], \left[\frac{9}{80}, 6\right], \left[\frac{3}{80}, -1\right], \left[\frac{7}{80}, -1\right]\right]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 80

"val0=", 2

"which is even."

"valinf=", 12

"which is even."

"It IS a modfunc on Gamma1(", 80, ")"

"min inf ord=", 0

```
"mintotord = ", -120
"TO PROVE the identity we need to show that v[oo](ID) > ", 120
"*** There were NO errors. ***"
"*** WARNING: some terms were constants. ***"
"See array CONTERMS."
To prove the identity we will need to verify if up to
q^(120).
Do you want to prove the identity? (yes/no)
```

```
> yes
```

```
You entered yes.
We verify the identity to O(q^(280)).
```

0

```
0 was returned and this proves the identity.
```

```
============================================================================
===========
```

EXAMPLE 2: Ramanujan's 40 identities for the Rogers-Ramanujan functions:

See

A.J.F. Biagioli, "A proof of some identities of Ramanujan using modular forms",

Glasgow Math.J. **31** (1989), 271-295.

```
> ramG:=r->JAC(0,5*r,infinity)/JAC(r,5*r,infinity):
  ramH:=r->JAC(0,5*r,infinity)/JAC(2*r,5*r,infinity):
  ramU:=proc(r,s)
        if modp(r+s,5)=0 then
            RETURN( ramG(r)*ramG(s)+q^((r+s)/5)*ramH(r)*ramH(s)):
        fi:
        if modp(r-s,5)=0 then
            RETURN( ramG(r)*ramH(s)-q^((r-s)/5)*ramH(r)*ramG(s)):
        fi:
```

```
  end:
```

```
> #ramP:=r->(JAC(r,2*r,infinity)/JAC(0,2*r,infinity))^(1/2):

> ramP:=r->(JAC(0,r,infinity)/JAC(0,2*r,infinity)):
> ramPS:=r->JAC(0,4*r,infinity)*ramP(2*r)/JAC(r,4*r,infinity):

  ramid1:=ramU(6,14)-ramU(42,2):
  ramid2:=2*q*ramU(6,14) - ramPS(1)*ramP(3)*ramPS(7)*ramP(21)
                         + ramP(1)*ramPS(3)*ramP(7)*ramPS(21):

  ramid3:=ramU(2,13)^2 - ramP(13)/ramP(1) +q*ramP(1)/ramP(13):
  ramid6:=ramP(11)*ramU(2,33)-ramP(3)*ramU(66,1):
```

```
> ramid1;
```

$$\frac{\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)}+\frac{q^4\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(12,30,\infty)\,\mathrm{JAC}(28,70,\infty)}-\frac{\mathrm{JAC}(0,210,\infty)\,\mathrm{JAC}(0,10,\infty)}{\mathrm{JAC}(42,210,\infty)\,\mathrm{JAC}(4,10,\infty)}+\frac{q^8\,\mathrm{JAC}(0,210,\infty)\,\mathrm{JAC}(0,10,\infty)}{\mathrm{JAC}(84,210,\infty)\,\mathrm{JAC}(2,10,\infty)}$$

```
> ramid1a:=expand(ramid1/op(1,ramid1));
```

$$ramid1a := 1+\frac{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)\,q^4}{\mathrm{JAC}(12,30,\infty)\,\mathrm{JAC}(28,70,\infty)}-\frac{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)\,\mathrm{JAC}(0,210,\infty)\,\mathrm{JAC}(0,10,\infty)}{\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)\,\mathrm{JAC}(42,210,\infty)\,\mathrm{JAC}(4,10,\infty)}+\frac{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)\,q^8\,\mathrm{JAC}(0,210,\infty)\,\mathrm{JAC}(0,10,\infty)}{\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)\,\mathrm{JAC}(84,210,\infty)\,\mathrm{JAC}(2,10,\infty)}$$

```
> ramid1b:=mixedjac2jac(ramid1a,500);
"term ", 1, "of ", 4
"term ", 2, "of ", 4
"term ", 3, "of ", 4
"term ", 4, "of ", 4
```

$$ramid1b := 1+q^4\,\mathrm{JAC}(6,210,\infty)\,\mathrm{JAC}(14,210,\infty)\,\mathrm{JAC}(24,210,\infty)\,\mathrm{JAC}(36,210,\infty)\,\mathrm{JAC}(54,210,\infty)\,\mathrm{JAC}(56,210,\infty)\,\mathrm{JAC}(66,210,\infty)\,\mathrm{JAC}(84,210,\infty)^2\,\mathrm{JAC}(96,210,\infty)\Big/\big(\mathrm{JAC}(12,210,\infty)\,\mathrm{JAC}(18,210,\infty)\,\mathrm{JAC}(28,210,\infty)\,\mathrm{JAC}(42,210,\infty)^2\,\mathrm{JAC}(48,210,\infty)\,\mathrm{JAC}(72,210,\infty)\,\mathrm{JAC}(78,210,\infty)\,\mathrm{JAC}(98,210,\infty)\,\mathrm{JAC}(102,210,\infty)\big)-\mathrm{JAC}(0,210,\infty)^{12}\,\mathrm{JAC}(84,210,\infty)\,/\,(\mathrm{JAC}(4,210,\infty)\,\mathrm{JAC}(16,210,\infty)\,\mathrm{JAC}(26,210,\infty)\,\mathrm{JAC}(34,210,\infty)\,\mathrm{JAC}(42,210,\infty)\,\mathrm{JAC}(44,210,\infty)\,\mathrm{JAC}(46,210,\infty)\,\mathrm{JAC}(64,210,\infty)\,\mathrm{JAC}(74,210,\infty)\,\mathrm{JAC}(76,210,\infty)\,\mathrm{JAC}(86,210,\infty)\,\mathrm{JAC}(94,210,\infty)\,\mathrm{JAC}(104,210,\infty))+q^8\,\mathrm{JAC}(6,210,\infty)\,\mathrm{JAC}(0,210,\infty)^{12}\,\mathrm{JAC}(14,210,\infty)\,\mathrm{JAC}(24,210,\infty)\,\mathrm{JAC}(36,210,\infty)$$

$\mathrm{JAC}(54, 210, \infty)\,\mathrm{JAC}(56, 210, \infty)\,\mathrm{JAC}(66, 210, \infty)\,\mathrm{JAC}(84, 210, \infty)\,\mathrm{JAC}(96, 210, \infty) \,/\, ($
$\mathrm{JAC}(2, 210, \infty)\,\mathrm{JAC}(8, 210, \infty)\,\mathrm{JAC}(12, 210, \infty)\,\mathrm{JAC}(18, 210, \infty)\,\mathrm{JAC}(22, 210, \infty)$
$\mathrm{JAC}(28, 210, \infty)\,\mathrm{JAC}(32, 210, \infty)\,\mathrm{JAC}(38, 210, \infty)\,\mathrm{JAC}(42, 210, \infty)\,\mathrm{JAC}(48, 210, \infty)$
$\mathrm{JAC}(52, 210, \infty)\,\mathrm{JAC}(58, 210, \infty)\,\mathrm{JAC}(62, 210, \infty)\,\mathrm{JAC}(68, 210, \infty)\,\mathrm{JAC}(72, 210, \infty)$
$\mathrm{JAC}(78, 210, \infty)\,\mathrm{JAC}(82, 210, \infty)\,\mathrm{JAC}(88, 210, \infty)\,\mathrm{JAC}(92, 210, \infty)\,\mathrm{JAC}(98, 210, \infty)$
$\mathrm{JAC}(102, 210, \infty))$

We calculate a set of inequivalent cusps for $\Gamma_1(210)$
and the width of each cusp. Note: oo is the first cusp in the list.

```
> cusps210:=cuspmake1(210):
> cusp210:=cusps210 minus {[1,0]}:
> cusps210:=convert(cusp210,list):
> wids210:=map(x->cuspwid1(x[1],x[2],210),cusps210):
> wids210:=[1,op(wids210)]:
> CUSPS210:=map(x->x[1]/x[2],cusps210):
> CUSPS210:=[oo,op(CUSPS210)];
```

$$CUSPS210 := \left[oo, \frac{17}{55}, \frac{13}{55}, \frac{28}{65}, \frac{11}{21}, \frac{13}{21}, \frac{1}{65}, \frac{1}{21}, \frac{19}{55}, \frac{13}{63}, \frac{19}{85}, \frac{57}{85}, \frac{3}{10}, \frac{13}{85}, \frac{1}{10}, \frac{4}{21}, \frac{1}{63}, \frac{5}{21}, \frac{20}{21}, \frac{1}{85}, \frac{16}{21}, \frac{10}{21}, \right.$$
$$\frac{2}{21}, \frac{8}{21}, \frac{19}{65}, \frac{19}{21}, \frac{17}{21}, \frac{17}{65}, \frac{9}{10}, \frac{19}{25}, \frac{17}{25}, \frac{13}{25}, \frac{47}{63}, \frac{4}{63}, \frac{41}{63}, \frac{1}{25}, \frac{39}{95}, \frac{37}{63}, \frac{29}{63}, \frac{17}{95}, \frac{13}{95}, \frac{7}{10}, \frac{1}{95}, \frac{19}{63}, \frac{17}{63}, \frac{1}{56}, \frac{19}{20}, \frac{11}{42},$$
$$\frac{17}{20}, \frac{11}{18}, \frac{13}{20}, \frac{1}{20}, \frac{19}{90}, \frac{17}{90}, \frac{13}{90}, \frac{19}{80}, \frac{17}{80}, \frac{11}{90}, \frac{1}{90}, \frac{29}{30}, \frac{7}{30}, \frac{19}{30}, \frac{13}{30}, \frac{17}{30}, \frac{1}{30}, \frac{11}{30}, \frac{13}{80}, \frac{1}{80}, \frac{17}{50}, \frac{13}{50}, \frac{1}{50}, \frac{19}{60}, \frac{23}{60}, \frac{17}{60},$$
$$\frac{19}{100}, \frac{37}{90}, \frac{17}{100}, \frac{29}{90}, \frac{13}{100}, \frac{1}{100}, \frac{23}{90}, \frac{19}{40}, \frac{17}{40}, \frac{13}{40}, \frac{19}{50}, \frac{29}{35}, \frac{23}{35}, \frac{19}{35}, \frac{17}{35}, \frac{13}{35}, \frac{11}{35}, \frac{1}{35}, \frac{1}{6}, \frac{23}{63}, \frac{27}{35}, \frac{11}{63}, \frac{9}{35}, \frac{3}{35}, \frac{32}{35},$$
$$\frac{1}{66}, \frac{26}{35}, \frac{24}{35}, \frac{18}{35}, \frac{12}{35}, \frac{8}{35}, \frac{6}{35}, \frac{1}{14}, \frac{31}{35}, \frac{2}{35}, \frac{5}{6}, \frac{11}{102}, \frac{1}{102}, \frac{3}{14}, \frac{11}{78}, \frac{1}{78}, \frac{13}{14}, \frac{11}{14}, \frac{34}{35}, \frac{23}{66}, \frac{22}{35}, \frac{16}{35}, \frac{23}{84}, \frac{29}{84}, \frac{17}{84}, \frac{19}{84},$$
$$\frac{13}{84}, \frac{1}{84}, \frac{5}{42}, \frac{1}{28}, \frac{1}{12}, \frac{31}{42}, \frac{19}{42}, \frac{23}{42}, \frac{17}{42}, \frac{13}{42}, \frac{1}{42}, \frac{5}{14}, \frac{17}{28}, \frac{11}{36}, \frac{13}{28}, \frac{1}{36}, \frac{11}{28}, \frac{67}{84}, \frac{23}{30}, \frac{11}{12}, \frac{1}{64}, \frac{1}{49}, \frac{23}{75}, \frac{23}{91}, \frac{19}{75}, \frac{2}{5},$$
$$\frac{19}{91}, \frac{17}{75}, \frac{17}{70}, \frac{19}{70}, \frac{11}{56}, \frac{13}{70}, \frac{11}{70}, \frac{1}{72}, \frac{1}{70}, \frac{11}{48}, \frac{23}{28}, \frac{1}{48}, \frac{19}{28}, \frac{1}{60}, \frac{11}{60}, \frac{39}{70}, \frac{51}{70}, \frac{13}{56}, \frac{27}{70}, \frac{33}{70}, \frac{3}{70}, \frac{9}{70}, \frac{67}{70}, \frac{61}{70}, \frac{59}{70}, \frac{47}{70},$$
$$\frac{43}{70}, \frac{41}{70}, \frac{37}{70}, \frac{31}{70}, \frac{23}{70}, \frac{29}{70}, \frac{29}{60}, \frac{37}{60}, \frac{11}{72}, \frac{13}{60}, \frac{19}{56}, \frac{1}{18}, \frac{17}{56}, \frac{69}{70}, \frac{57}{70}, \frac{41}{105}, \frac{31}{105}, \frac{37}{105}, \frac{19}{105}, \frac{23}{105}, \frac{17}{105}, \frac{13}{105}, \frac{11}{105},$$
$$\frac{1}{24}, \frac{1}{105}, \frac{11}{54}, \frac{1}{98}, \frac{23}{56}, \frac{1}{54}, \frac{23}{98}, \frac{11}{96}, \frac{1}{96}, \frac{19}{98}, \frac{17}{98}, \frac{13}{98}, \frac{52}{105}, \frac{32}{105}, \frac{34}{105}, \frac{38}{105}, \frac{11}{98}, \frac{11}{24}, \frac{22}{105}, \frac{16}{105}, \frac{8}{105}, \frac{2}{105}, \frac{29}{210},$$
$$\frac{23}{210}, \frac{17}{210}, \frac{19}{210}, \frac{13}{210}, \frac{11}{210}, \frac{33}{35}, \frac{4}{35}, \frac{53}{210}, \frac{59}{210}, \frac{43}{210}, \frac{47}{210}, \frac{37}{210}, \frac{41}{210}, \frac{31}{210}, \frac{101}{210}, \frac{103}{210}, \frac{1}{67}, \frac{1}{71}, \frac{1}{73}, \frac{1}{79}, \frac{1}{83},$$
$$\frac{97}{210}, \frac{1}{89}, \frac{1}{61}, \frac{1}{47}, \frac{89}{210}, \frac{1}{53}, \frac{83}{210}, \frac{1}{59}, \frac{29}{42}, \frac{1}{97}, \frac{1}{101}, \frac{1}{103}, \frac{1}{29}, \frac{1}{31}, \frac{1}{37}, \frac{1}{41}, \frac{73}{210}, \frac{1}{43}, \frac{1}{13}, \frac{1}{17}, \frac{1}{19}, \frac{1}{23}, \frac{1}{11}, 0,$$

$$\frac{67}{210}, \frac{61}{210}, \frac{1}{15}, \frac{13}{15}, \frac{1}{2}, \frac{1}{22}, \frac{1}{26}, \frac{1}{34}, \frac{1}{38}, \frac{1}{46}, \frac{1}{58}, \frac{25}{42}, \frac{11}{15}, \frac{29}{105}, \frac{37}{42}, \frac{41}{42}, \frac{1}{76}, \frac{1}{88}, \frac{1}{92}, \frac{1}{104}, \frac{1}{68}, \frac{1}{44}, \frac{1}{52}, \frac{1}{16}, \frac{1}{32},$$
$$\frac{4}{7}, \frac{4}{15}, \frac{1}{7}, \frac{1}{86}, \frac{1}{94}, \frac{1}{4}, \frac{1}{8}, \frac{2}{15}, \frac{1}{62}, \frac{1}{74}, \frac{1}{82}, \frac{1}{3}, \frac{2}{3}, \frac{1}{33}, \frac{23}{33}, \frac{3}{7}, \frac{11}{84}, \frac{14}{15}, \frac{6}{7}, \frac{8}{15}, \frac{11}{45}, \frac{1}{45}, \frac{5}{7}, \frac{1}{39}, \frac{11}{39}, \frac{1}{51}, \frac{11}{51}, \frac{7}{15}, \frac{41}{84},$$
$$\frac{1}{77}, \frac{1}{99}, \frac{23}{99}, \frac{9}{14}, \frac{4}{105}, \frac{11}{81}, \frac{19}{45}, \frac{17}{45}, \frac{11}{93}, \frac{1}{9}, \frac{2}{9}, \frac{1}{27}, \frac{11}{27}, \frac{1}{81}, \frac{2}{7}, \frac{1}{69}, \frac{11}{69}, \frac{1}{87}, \frac{11}{87}, \frac{1}{93}, \frac{13}{45}, \frac{29}{45}, \frac{67}{77}, \frac{47}{84}, \frac{31}{84}, \frac{37}{84}, \frac{23}{45},$$
$$\frac{13}{77}, \frac{1}{40}, \frac{31}{63}, \frac{19}{77}, \frac{17}{77}, \frac{37}{45}, \frac{26}{105}, \frac{43}{105}, \frac{47}{105}, \frac{1}{91}, \frac{23}{77}, \frac{1}{75}, \frac{44}{105}, \frac{46}{105}, \frac{79}{210}, \frac{11}{91}, \frac{11}{75}, \frac{71}{210}, \frac{53}{70}, \frac{1}{5}, \frac{13}{75}, \frac{17}{91}, \frac{3}{5}, \frac{41}{91}, \frac{4}{5},$$
$$\left.\frac{11}{49}, \frac{37}{75}, \frac{29}{75}, \frac{13}{49}, \frac{19}{49}, \frac{1}{55}, \frac{17}{49}, \frac{23}{49}, \frac{1}{57}, \frac{11}{57}\right]$$

```
> nops(CUSPS210);
```

384

```
> provemodfuncid(ramid1b,CUSPS210,wids210,210);
"TERM ", 1, "of ", 4, " ****************************************************************************"
```

"XX=", 1

```
"TERM ", 2, "of ", 4, " ****************************************************************************"
```

"XX=", $q^4\,\mathrm{JAC}(6,210,\infty)\,\mathrm{JAC}(14,210,\infty)\,\mathrm{JAC}(24,210,\infty)\,\mathrm{JAC}(36,210,\infty)\,\mathrm{JAC}(54,210,\infty)$
$\mathrm{JAC}(56,210,\infty)\,\mathrm{JAC}(66,210,\infty)\,\mathrm{JAC}(84,210,\infty)^2\,\mathrm{JAC}(96,210,\infty) \Big/ (\mathrm{JAC}(12,210,\infty)$
$\mathrm{JAC}(18,210,\infty)\,\mathrm{JAC}(28,210,\infty)\,\mathrm{JAC}(42,210,\infty)^2\,\mathrm{JAC}(48,210,\infty)\,\mathrm{JAC}(72,210,\infty)$
$\mathrm{JAC}(78,210,\infty)\,\mathrm{JAC}(98,210,\infty)\,\mathrm{JAC}(102,210,\infty))$

"Cusp ORDS: "

$$\Big[[oo, 4], \left[\frac{17}{55}, -2\right], \left[\frac{13}{55}, -2\right], \left[\frac{28}{65}, -2\right], \left[\frac{11}{21}, 0\right], \left[\frac{13}{21}, 0\right], \left[\frac{1}{65}, 2\right], \left[\frac{1}{21}, 0\right], \left[\frac{19}{55}, 2\right], \left[\frac{13}{63}, 0\right],$$
$$\left[\frac{19}{85}, 2\right], \left[\frac{57}{85}, -2\right], \left[\frac{3}{10}, 4\right], \left[\frac{13}{85}, -2\right], \left[\frac{1}{10}, -4\right], \left[\frac{4}{21}, 0\right], \left[\frac{1}{63}, 0\right], \left[\frac{5}{21}, 0\right], \left[\frac{20}{21}, 0\right], \left[\frac{1}{85}, 2\right], \left[\frac{16}{21}, 0\right],$$
$$\left[\frac{10}{21}, 0\right], \left[\frac{2}{21}, 0\right], \left[\frac{8}{21}, 0\right], \left[\frac{19}{65}, 2\right], \left[\frac{19}{21}, 0\right], \left[\frac{17}{21}, 0\right], \left[\frac{17}{65}, -2\right], \left[\frac{9}{10}, -4\right], \left[\frac{19}{25}, 2\right], \left[\frac{17}{25}, -2\right], \left[\frac{13}{25}, -2\right],$$
$$\left[\frac{47}{63}, 0\right], \left[\frac{4}{63}, 0\right], \left[\frac{41}{63}, 0\right], \left[\frac{1}{25}, 2\right], \left[\frac{39}{95}, 2\right], \left[\frac{37}{63}, 0\right], \left[\frac{29}{63}, 0\right], \left[\frac{17}{95}, -2\right], \left[\frac{13}{95}, -2\right], \left[\frac{7}{10}, 4\right], \left[\frac{1}{95}, 2\right],$$
$$\left[\frac{19}{63}, 0\right], \left[\frac{17}{63}, 0\right], \left[\frac{1}{56}, 0\right], \left[\frac{19}{20}, -4\right], \left[\frac{11}{42}, 0\right], \left[\frac{17}{20}, 4\right], \left[\frac{11}{18}, 0\right], \left[\frac{13}{20}, 4\right], \left[\frac{1}{20}, -4\right], \left[\frac{19}{90}, 8\right], \left[\frac{17}{90}, -8\right],$$
$$\left[\frac{13}{90}, -8\right], \left[\frac{19}{80}, -4\right], \left[\frac{17}{80}, 4\right], \left[\frac{11}{90}, 8\right], \left[\frac{1}{90}, 8\right], \left[\frac{29}{30}, 8\right], \left[\frac{7}{30}, -8\right], \left[\frac{19}{30}, 8\right], \left[\frac{13}{30}, -8\right], \left[\frac{17}{30}, -8\right],$$
$$\left[\frac{1}{30}, 8\right], \left[\frac{11}{30}, 8\right], \left[\frac{13}{80}, 4\right], \left[\frac{1}{80}, -4\right], \left[\frac{17}{50}, 4\right], \left[\frac{13}{50}, 4\right], \left[\frac{1}{50}, -4\right], \left[\frac{19}{60}, 8\right], \left[\frac{23}{60}, -8\right], \left[\frac{17}{60}, -8\right],$$

$$\left[\frac{19}{100}, -4\right], \left[\frac{37}{90}, -8\right], \left[\frac{17}{100}, 4\right], \left[\frac{29}{90}, 8\right], \left[\frac{13}{100}, 4\right], \left[\frac{1}{100}, -4\right], \left[\frac{23}{90}, -8\right], \left[\frac{19}{40}, -4\right], \left[\frac{17}{40}, 4\right], \left[\frac{13}{40}, 4\right],$$

$$\left[\frac{19}{50}, -4\right], \left[\frac{29}{35}, -4\right], \left[\frac{23}{35}, 4\right], \left[\frac{19}{35}, -4\right], \left[\frac{17}{35}, 4\right], \left[\frac{13}{35}, 4\right], \left[\frac{11}{35}, -4\right], \left[\frac{1}{35}, -4\right], \left[\frac{1}{6}, 0\right], \left[\frac{23}{63}, 0\right], \left[\frac{27}{35}, 4\right],$$

$$\left[\frac{11}{63}, 0\right], \left[\frac{9}{35}, -4\right], \left[\frac{3}{35}, 4\right], \left[\frac{32}{35}, 4\right], \left[\frac{1}{66}, 0\right], \left[\frac{26}{35}, -4\right], \left[\frac{24}{35}, -4\right], \left[\frac{18}{35}, 4\right], \left[\frac{12}{35}, 4\right], \left[\frac{8}{35}, 4\right], \left[\frac{6}{35}, -4\right],$$

$$\left[\frac{1}{14}, 0\right], \left[\frac{31}{35}, -4\right], \left[\frac{2}{35}, 4\right], \left[\frac{5}{6}, 0\right], \left[\frac{11}{102}, 0\right], \left[\frac{1}{102}, 0\right], \left[\frac{3}{14}, 0\right], \left[\frac{11}{78}, 0\right], \left[\frac{1}{78}, 0\right], \left[\frac{13}{14}, 0\right], \left[\frac{11}{14}, 0\right],$$

$$\left[\frac{34}{35}, -4\right], \left[\frac{23}{66}, 0\right], \left[\frac{22}{35}, 4\right], \left[\frac{16}{35}, -4\right], \left[\frac{23}{84}, 0\right], \left[\frac{29}{84}, 0\right], \left[\frac{17}{84}, 0\right], \left[\frac{19}{84}, 0\right], \left[\frac{13}{84}, 0\right], \left[\frac{1}{84}, 0\right], \left[\frac{5}{42}, 0\right],$$

$$\left[\frac{1}{28}, 0\right], \left[\frac{1}{12}, 0\right], \left[\frac{31}{42}, 0\right], \left[\frac{19}{42}, 0\right], \left[\frac{23}{42}, 0\right], \left[\frac{17}{42}, 0\right], \left[\frac{13}{42}, 0\right], \left[\frac{1}{42}, 0\right], \left[\frac{5}{14}, 0\right], \left[\frac{17}{28}, 0\right], \left[\frac{11}{36}, 0\right],$$

$$\left[\frac{13}{28}, 0\right], \left[\frac{1}{36}, 0\right], \left[\frac{11}{28}, 0\right], \left[\frac{67}{84}, 0\right], \left[\frac{23}{30}, -8\right], \left[\frac{11}{12}, 0\right], \left[\frac{1}{64}, 0\right], \left[\frac{1}{49}, 0\right], \left[\frac{23}{75}, 4\right], \left[\frac{23}{91}, 0\right], \left[\frac{19}{75}, -4\right],$$

$$\left[\frac{2}{5}, -2\right], \left[\frac{19}{91}, 0\right], \left[\frac{17}{75}, 4\right], \left[\frac{17}{70}, -8\right], \left[\frac{19}{70}, 8\right], \left[\frac{11}{56}, 0\right], \left[\frac{13}{70}, -8\right], \left[\frac{11}{70}, 8\right], \left[\frac{1}{72}, 0\right], \left[\frac{1}{70}, 8\right], \left[\frac{11}{48}, 0\right],$$

$$\left[\frac{23}{28}, 0\right], \left[\frac{1}{48}, 0\right], \left[\frac{19}{28}, 0\right], \left[\frac{1}{60}, 8\right], \left[\frac{11}{60}, 8\right], \left[\frac{39}{70}, 8\right], \left[\frac{51}{70}, 8\right], \left[\frac{13}{56}, 0\right], \left[\frac{27}{70}, -8\right], \left[\frac{33}{70}, -8\right], \left[\frac{3}{70}, -8\right],$$

$$\left[\frac{9}{70}, 8\right], \left[\frac{67}{70}, -8\right], \left[\frac{61}{70}, 8\right], \left[\frac{59}{70}, 8\right], \left[\frac{47}{70}, -8\right], \left[\frac{43}{70}, -8\right], \left[\frac{41}{70}, 8\right], \left[\frac{37}{70}, -8\right], \left[\frac{31}{70}, 8\right], \left[\frac{23}{70}, -8\right],$$

$$\left[\frac{29}{70}, 8\right], \left[\frac{29}{60}, 8\right], \left[\frac{37}{60}, -8\right], \left[\frac{11}{72}, 0\right], \left[\frac{13}{60}, -8\right], \left[\frac{19}{56}, 0\right], \left[\frac{1}{18}, 0\right], \left[\frac{17}{56}, 0\right], \left[\frac{69}{70}, 8\right], \left[\frac{57}{70}, -8\right],$$

$$\left[\frac{41}{105}, -2\right], \left[\frac{31}{105}, -2\right], \left[\frac{37}{105}, 2\right], \left[\frac{19}{105}, -2\right], \left[\frac{23}{105}, 2\right], \left[\frac{17}{105}, 2\right], \left[\frac{13}{105}, 2\right], \left[\frac{11}{105}, -2\right], \left[\frac{1}{24}, 0\right],$$

$$\left[\frac{1}{105}, -2\right], \left[\frac{11}{54}, 0\right], \left[\frac{1}{98}, 0\right], \left[\frac{23}{56}, 0\right], \left[\frac{1}{54}, 0\right], \left[\frac{23}{98}, 0\right], \left[\frac{11}{96}, 0\right], \left[\frac{1}{96}, 0\right], \left[\frac{19}{98}, 0\right], \left[\frac{17}{98}, 0\right], \left[\frac{13}{98}, 0\right],$$

$$\left[\frac{52}{105}, 2\right], \left[\frac{32}{105}, 2\right], \left[\frac{34}{105}, -2\right], \left[\frac{38}{105}, 2\right], \left[\frac{11}{98}, 0\right], \left[\frac{11}{24}, 0\right], \left[\frac{22}{105}, 2\right], \left[\frac{16}{105}, -2\right], \left[\frac{8}{105}, 2\right], \left[\frac{2}{105}, 2\right],$$

$$\left[\frac{29}{210}, 4\right], \left[\frac{23}{210}, -4\right], \left[\frac{17}{210}, -4\right], \left[\frac{19}{210}, 4\right], \left[\frac{13}{210}, -4\right], \left[\frac{11}{210}, 4\right], \left[\frac{33}{35}, 4\right], \left[\frac{4}{35}, -4\right], \left[\frac{53}{210}, -4\right],$$

$$\left[\frac{59}{210}, 4\right], \left[\frac{43}{210}, -4\right], \left[\frac{47}{210}, -4\right], \left[\frac{37}{210}, -4\right], \left[\frac{41}{210}, 4\right], \left[\frac{31}{210}, 4\right], \left[\frac{101}{210}, 4\right], \left[\frac{103}{210}, -4\right], \left[\frac{1}{67}, 0\right],$$

$$\left[\frac{1}{71}, 0\right], \left[\frac{1}{73}, 0\right], \left[\frac{1}{79}, 0\right], \left[\frac{1}{83}, 0\right], \left[\frac{97}{210}, -4\right], \left[\frac{1}{89}, 0\right], \left[\frac{1}{61}, 0\right], \left[\frac{1}{47}, 0\right], \left[\frac{89}{210}, 4\right], \left[\frac{1}{53}, 0\right],$$

$$\left[\frac{83}{210}, -4\right], \left[\frac{1}{59}, 0\right], \left[\frac{29}{42}, 0\right], \left[\frac{1}{97}, 0\right], \left[\frac{1}{101}, 0\right], \left[\frac{1}{103}, 0\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right],$$

$$\left[\frac{73}{210}, -4\right], \left[\frac{1}{43}, 0\right], \left[\frac{1}{13}, 0\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{23}, 0\right], \left[\frac{1}{11}, 0\right], [0, 0], \left[\frac{67}{210}, -4\right], \left[\frac{61}{210}, 4\right],$$

$$\left[\frac{1}{15}, -4\right], \left[\frac{13}{15}, 4\right], \left[\frac{1}{2}, 0\right], \left[\frac{1}{22}, 0\right], \left[\frac{1}{26}, 0\right], \left[\frac{1}{34}, 0\right], \left[\frac{1}{38}, 0\right], \left[\frac{1}{46}, 0\right], \left[\frac{1}{58}, 0\right], \left[\frac{25}{42}, 0\right], \left[\frac{11}{15}, -4\right],$$

$$\left[\frac{29}{105}, -2\right], \left[\frac{37}{42}, 0\right], \left[\frac{41}{42}, 0\right], \left[\frac{1}{76}, 0\right], \left[\frac{1}{88}, 0\right], \left[\frac{1}{92}, 0\right], \left[\frac{1}{104}, 0\right], \left[\frac{1}{68}, 0\right], \left[\frac{1}{44}, 0\right], \left[\frac{1}{52}, 0\right], \left[\frac{1}{16}, 0\right],$$

$$\left[\frac{1}{32}, 0\right], \left[\frac{4}{7}, 0\right], \left[\frac{4}{15}, -4\right], \left[\frac{1}{7}, 0\right], \left[\frac{1}{86}, 0\right], \left[\frac{1}{94}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{8}, 0\right], \left[\frac{2}{15}, 4\right], \left[\frac{1}{62}, 0\right], \left[\frac{1}{74}, 0\right],$$

$$\left[\frac{1}{82}, 0\right], \left[\frac{1}{3}, 0\right], \left[\frac{2}{3}, 0\right], \left[\frac{1}{33}, 0\right], \left[\frac{23}{33}, 0\right], \left[\frac{3}{7}, 0\right], \left[\frac{11}{84}, 0\right], \left[\frac{14}{15}, -4\right], \left[\frac{6}{7}, 0\right], \left[\frac{8}{15}, 4\right], \left[\frac{11}{45}, -4\right],$$

$$\left[\frac{1}{45}, -4\right], \left[\frac{5}{7}, 0\right], \left[\frac{1}{39}, 0\right], \left[\frac{11}{39}, 0\right], \left[\frac{1}{51}, 0\right], \left[\frac{11}{51}, 0\right], \left[\frac{7}{15}, 4\right], \left[\frac{41}{84}, 0\right], \left[\frac{1}{77}, 0\right], \left[\frac{1}{99}, 0\right], \left[\frac{23}{99}, 0\right],$$

$$\left[\frac{9}{14}, 0\right], \left[\frac{4}{105}, -2\right], \left[\frac{11}{81}, 0\right], \left[\frac{19}{45}, -4\right], \left[\frac{17}{45}, 4\right], \left[\frac{11}{93}, 0\right], \left[\frac{1}{9}, 0\right], \left[\frac{2}{9}, 0\right], \left[\frac{1}{27}, 0\right], \left[\frac{11}{27}, 0\right], \left[\frac{1}{81}, 0\right],$$

$$\left[\frac{2}{7}, 0\right], \left[\frac{1}{69}, 0\right], \left[\frac{11}{69}, 0\right], \left[\frac{1}{87}, 0\right], \left[\frac{11}{87}, 0\right], \left[\frac{1}{93}, 0\right], \left[\frac{13}{45}, 4\right], \left[\frac{29}{45}, -4\right], \left[\frac{67}{77}, 0\right], \left[\frac{47}{84}, 0\right], \left[\frac{31}{84}, 0\right],$$

$$\left[\frac{37}{84}, 0\right], \left[\frac{23}{45}, 4\right], \left[\frac{13}{77}, 0\right], \left[\frac{1}{40}, -4\right], \left[\frac{31}{63}, 0\right], \left[\frac{19}{77}, 0\right], \left[\frac{17}{77}, 0\right], \left[\frac{37}{45}, 4\right], \left[\frac{26}{105}, -2\right], \left[\frac{43}{105}, 2\right],$$

$$\left[\frac{47}{105}, 2\right], \left[\frac{1}{91}, 0\right], \left[\frac{23}{77}, 0\right], \left[\frac{1}{75}, -4\right], \left[\frac{44}{105}, -2\right], \left[\frac{46}{105}, -2\right], \left[\frac{79}{210}, 4\right], \left[\frac{11}{91}, 0\right], \left[\frac{11}{75}, -4\right], \left[\frac{71}{210}, 4\right],$$

$$\left[\frac{53}{70}, -8\right], \left[\frac{1}{5}, 2\right], \left[\frac{13}{75}, 4\right], \left[\frac{17}{91}, 0\right], \left[\frac{3}{5}, -2\right], \left[\frac{41}{91}, 0\right], \left[\frac{4}{5}, 2\right], \left[\frac{11}{49}, 0\right], \left[\frac{37}{75}, 4\right], \left[\frac{29}{75}, -4\right], \left[\frac{13}{49}, 0\right],$$

$$\left[\frac{19}{49}, 0\right], \left[\frac{1}{55}, 2\right], \left[\frac{17}{49}, 0\right], \left[\frac{23}{49}, 0\right], \left[\frac{1}{57}, 0\right], \left[\frac{11}{57}, 0\right]\right]$$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 210

"val0=", 0

"which is even."

"valinf=", 8

"which is even."

"It IS a modfunc on Gamma1(", 210, ")"

"TERM ", 3, "of ", 4, " ***"

$$\text{"XX=", } -\mathrm{JAC}(0, 210, \infty)^{12}\, \mathrm{JAC}(84, 210, \infty) / (\mathrm{JAC}(4, 210, \infty)\, \mathrm{JAC}(16, 210, \infty)$$
$$\mathrm{JAC}(26, 210, \infty)\, \mathrm{JAC}(34, 210, \infty)\, \mathrm{JAC}(42, 210, \infty)\, \mathrm{JAC}(44, 210, \infty)\, \mathrm{JAC}(46, 210, \infty)$$
$$\mathrm{JAC}(64, 210, \infty)\, \mathrm{JAC}(74, 210, \infty)\, \mathrm{JAC}(76, 210, \infty)\, \mathrm{JAC}(86, 210, \infty)\, \mathrm{JAC}(94, 210, \infty)$$

JAC(104, 210, ∞))

"Cusp ORDS: "

$$\left[[oo, 0], \left[\frac{17}{55}, 2\right], \left[\frac{13}{55}, 2\right], \left[\frac{28}{65}, 2\right], \left[\frac{11}{21}, -1\right], \left[\frac{13}{21}, -1\right], \left[\frac{1}{65}, 0\right], \left[\frac{1}{21}, -1\right], \left[\frac{19}{55}, 0\right], \left[\frac{13}{63}, -1\right], \right.$$

$$\left[\frac{19}{85}, 0\right], \left[\frac{57}{85}, 2\right], \left[\frac{3}{10}, 0\right], \left[\frac{13}{85}, 2\right], \left[\frac{1}{10}, 4\right], \left[\frac{4}{21}, -1\right], \left[\frac{1}{63}, -1\right], \left[\frac{5}{21}, -1\right], \left[\frac{20}{21}, -1\right], \left[\frac{1}{85}, 0\right],$$

$$\left[\frac{16}{21}, -1\right], \left[\frac{10}{21}, -1\right], \left[\frac{2}{21}, -1\right], \left[\frac{8}{21}, -1\right], \left[\frac{19}{65}, 0\right], \left[\frac{19}{21}, -1\right], \left[\frac{17}{21}, -1\right], \left[\frac{17}{65}, 2\right], \left[\frac{9}{10}, 4\right], \left[\frac{19}{25}, 0\right],$$

$$\left[\frac{17}{25}, 2\right], \left[\frac{13}{25}, 2\right], \left[\frac{47}{63}, -1\right], \left[\frac{4}{63}, -1\right], \left[\frac{41}{63}, -1\right], \left[\frac{1}{25}, 0\right], \left[\frac{39}{95}, 0\right], \left[\frac{37}{63}, -1\right], \left[\frac{29}{63}, -1\right], \left[\frac{17}{95}, 2\right],$$

$$\left[\frac{13}{95}, 2\right], \left[\frac{7}{10}, 0\right], \left[\frac{1}{95}, 0\right], \left[\frac{19}{63}, -1\right], \left[\frac{17}{63}, -1\right], \left[\frac{1}{56}, 2\right], \left[\frac{19}{20}, 4\right], \left[\frac{11}{42}, -2\right], \left[\frac{17}{20}, 0\right], \left[\frac{11}{18}, 2\right], \left[\frac{13}{20}, 0\right],$$

$$\left[\frac{1}{20}, 4\right], \left[\frac{19}{90}, 4\right], \left[\frac{17}{90}, -8\right], \left[\frac{13}{90}, -8\right], \left[\frac{19}{80}, 4\right], \left[\frac{17}{80}, 0\right], \left[\frac{11}{90}, 4\right], \left[\frac{1}{90}, 4\right], \left[\frac{29}{30}, 4\right], \left[\frac{7}{30}, -8\right], \left[\frac{19}{30}, 4\right],$$

$$\left[\frac{13}{30}, -8\right], \left[\frac{17}{30}, -8\right], \left[\frac{1}{30}, 4\right], \left[\frac{11}{30}, 4\right], \left[\frac{13}{80}, 0\right], \left[\frac{1}{80}, 4\right], \left[\frac{17}{50}, 0\right], \left[\frac{13}{50}, 0\right], \left[\frac{1}{50}, 4\right], \left[\frac{19}{60}, 4\right], \left[\frac{23}{60}, -8\right],$$

$$\left[\frac{17}{60}, -8\right], \left[\frac{19}{100}, 4\right], \left[\frac{37}{90}, -8\right], \left[\frac{17}{100}, 0\right], \left[\frac{29}{90}, 4\right], \left[\frac{13}{100}, 0\right], \left[\frac{1}{100}, 4\right], \left[\frac{23}{90}, -8\right], \left[\frac{19}{40}, 4\right], \left[\frac{17}{40}, 0\right],$$

$$\left[\frac{13}{40}, 0\right], \left[\frac{19}{50}, 4\right], \left[\frac{29}{35}, -4\right], \left[\frac{23}{35}, 2\right], \left[\frac{19}{35}, -4\right], \left[\frac{17}{35}, 2\right], \left[\frac{13}{35}, 2\right], \left[\frac{11}{35}, -4\right], \left[\frac{1}{35}, -4\right], \left[\frac{1}{6}, 2\right], \left[\frac{23}{63}, -1\right],$$

$$\left[\frac{27}{35}, 2\right], \left[\frac{11}{63}, -1\right], \left[\frac{9}{35}, -4\right], \left[\frac{3}{35}, 2\right], \left[\frac{32}{35}, 2\right], \left[\frac{1}{66}, 2\right], \left[\frac{26}{35}, -4\right], \left[\frac{24}{35}, -4\right], \left[\frac{18}{35}, 2\right], \left[\frac{12}{35}, 2\right], \left[\frac{8}{35}, 2\right],$$

$$\left[\frac{6}{35}, -4\right], \left[\frac{1}{14}, 2\right], \left[\frac{31}{35}, -4\right], \left[\frac{2}{35}, 2\right], \left[\frac{5}{6}, 2\right], \left[\frac{11}{102}, 2\right], \left[\frac{1}{102}, 2\right], \left[\frac{3}{14}, 2\right], \left[\frac{11}{78}, 2\right], \left[\frac{1}{78}, 2\right], \left[\frac{13}{14}, 2\right],$$

$$\left[\frac{11}{14}, 2\right], \left[\frac{34}{35}, -4\right], \left[\frac{23}{66}, 2\right], \left[\frac{22}{35}, 2\right], \left[\frac{16}{35}, -4\right], \left[\frac{23}{84}, -2\right], \left[\frac{29}{84}, -2\right], \left[\frac{17}{84}, -2\right], \left[\frac{19}{84}, -2\right], \left[\frac{13}{84}, -2\right],$$

$$\left[\frac{1}{84}, -2\right], \left[\frac{5}{42}, -2\right], \left[\frac{1}{28}, 2\right], \left[\frac{1}{12}, 2\right], \left[\frac{31}{42}, -2\right], \left[\frac{19}{42}, -2\right], \left[\frac{23}{42}, -2\right], \left[\frac{17}{42}, -2\right], \left[\frac{13}{42}, -2\right], \left[\frac{1}{42}, -2\right],$$

$$\left[\frac{5}{14}, 2\right], \left[\frac{17}{28}, 2\right], \left[\frac{11}{36}, 2\right], \left[\frac{13}{28}, 2\right], \left[\frac{1}{36}, 2\right], \left[\frac{11}{28}, 2\right], \left[\frac{67}{84}, -2\right], \left[\frac{23}{30}, -8\right], \left[\frac{11}{12}, 2\right], \left[\frac{1}{64}, -2\right], \left[\frac{1}{49}, 1\right],$$

$$\left[\frac{23}{75}, 2\right], \left[\frac{23}{91}, 1\right], \left[\frac{19}{75}, -4\right], \left[\frac{2}{5}, 2\right], \left[\frac{19}{91}, 1\right], \left[\frac{17}{75}, 2\right], \left[\frac{17}{70}, -8\right], \left[\frac{19}{70}, 4\right], \left[\frac{11}{56}, 2\right], \left[\frac{13}{70}, -8\right], \left[\frac{11}{70}, 4\right],$$

$$\left[\frac{1}{72}, 2\right], \left[\frac{1}{70}, 4\right], \left[\frac{11}{48}, 2\right], \left[\frac{23}{28}, 2\right], \left[\frac{1}{48}, 2\right], \left[\frac{19}{28}, 2\right], \left[\frac{1}{60}, 4\right], \left[\frac{11}{60}, 4\right], \left[\frac{39}{70}, 4\right], \left[\frac{51}{70}, 4\right], \left[\frac{13}{56}, 2\right],$$

$$\left[\frac{27}{70}, -8\right], \left[\frac{33}{70}, -8\right], \left[\frac{3}{70}, -8\right], \left[\frac{9}{70}, 4\right], \left[\frac{67}{70}, -8\right], \left[\frac{61}{70}, 4\right], \left[\frac{59}{70}, 4\right], \left[\frac{47}{70}, -8\right], \left[\frac{43}{70}, -8\right], \left[\frac{41}{70}, 4\right],$$

$\left[\frac{37}{70}, -8\right], \left[\frac{31}{70}, 4\right], \left[\frac{23}{70}, -8\right], \left[\frac{29}{70}, 4\right], \left[\frac{29}{60}, 4\right], \left[\frac{37}{60}, -8\right], \left[\frac{11}{72}, 2\right], \left[\frac{13}{60}, -8\right], \left[\frac{19}{56}, 2\right], \left[\frac{1}{18}, 2\right], \left[\frac{17}{56}, 2\right],$

$\left[\frac{69}{70}, 4\right], \left[\frac{57}{70}, -8\right], \left[\frac{41}{105}, 2\right], \left[\frac{31}{105}, 2\right], \left[\frac{37}{105}, 0\right], \left[\frac{19}{105}, 2\right], \left[\frac{23}{105}, 0\right], \left[\frac{17}{105}, 0\right], \left[\frac{13}{105}, 0\right], \left[\frac{11}{105}, 2\right],$

$\left[\frac{1}{24}, 2\right], \left[\frac{1}{105}, 2\right], \left[\frac{11}{54}, 2\right], \left[\frac{1}{98}, 2\right], \left[\frac{23}{56}, 2\right], \left[\frac{1}{54}, 2\right], \left[\frac{23}{98}, 2\right], \left[\frac{11}{96}, 2\right], \left[\frac{1}{96}, 2\right], \left[\frac{19}{98}, 2\right], \left[\frac{17}{98}, 2\right],$

$\left[\frac{13}{98}, 2\right], \left[\frac{52}{105}, 0\right], \left[\frac{32}{105}, 0\right], \left[\frac{34}{105}, 2\right], \left[\frac{38}{105}, 0\right], \left[\frac{11}{98}, 2\right], \left[\frac{11}{24}, 2\right], \left[\frac{22}{105}, 0\right], \left[\frac{16}{105}, 2\right], \left[\frac{8}{105}, 0\right],$

$\left[\frac{2}{105}, 0\right], \left[\frac{29}{210}, 0\right], \left[\frac{23}{210}, 4\right], \left[\frac{17}{210}, 4\right], \left[\frac{19}{210}, 0\right], \left[\frac{13}{210}, 4\right], \left[\frac{11}{210}, 0\right], \left[\frac{33}{35}, 2\right], \left[\frac{4}{35}, -4\right], \left[\frac{53}{210}, 4\right],$

$\left[\frac{59}{210}, 0\right], \left[\frac{43}{210}, 4\right], \left[\frac{47}{210}, 4\right], \left[\frac{37}{210}, 4\right], \left[\frac{41}{210}, 0\right], \left[\frac{31}{210}, 0\right], \left[\frac{101}{210}, 0\right], \left[\frac{103}{210}, 4\right], \left[\frac{1}{67}, -1\right], \left[\frac{1}{71}, -1\right],$

$\left[\frac{1}{73}, -1\right], \left[\frac{1}{79}, -1\right], \left[\frac{1}{83}, -1\right], \left[\frac{97}{210}, 4\right], \left[\frac{1}{89}, -1\right], \left[\frac{1}{61}, -1\right], \left[\frac{1}{47}, -1\right], \left[\frac{89}{210}, 0\right], \left[\frac{1}{53}, -1\right], \left[\frac{83}{210}, 4\right],$

$\left[\frac{1}{59}, -1\right], \left[\frac{29}{42}, -2\right], \left[\frac{1}{97}, -1\right], \left[\frac{1}{101}, -1\right], \left[\frac{1}{103}, -1\right], \left[\frac{1}{29}, -1\right], \left[\frac{1}{31}, -1\right], \left[\frac{1}{37}, -1\right], \left[\frac{1}{41}, -1\right], \left[\frac{73}{210}, 4\right],$

$\left[\frac{1}{43}, -1\right], \left[\frac{1}{13}, -1\right], \left[\frac{1}{17}, -1\right], \left[\frac{1}{19}, -1\right], \left[\frac{1}{23}, -1\right], \left[\frac{1}{11}, -1\right], [0, -1], \left[\frac{67}{210}, 4\right], \left[\frac{61}{210}, 0\right], \left[\frac{1}{15}, -4\right],$

$\left[\frac{13}{15}, 2\right], \left[\frac{1}{2}, -2\right], \left[\frac{1}{22}, -2\right], \left[\frac{1}{26}, -2\right], \left[\frac{1}{34}, -2\right], \left[\frac{1}{38}, -2\right], \left[\frac{1}{46}, -2\right], \left[\frac{1}{58}, -2\right], \left[\frac{25}{42}, -2\right], \left[\frac{11}{15}, -4\right],$

$\left[\frac{29}{105}, 2\right], \left[\frac{37}{42}, -2\right], \left[\frac{41}{42}, -2\right], \left[\frac{1}{76}, -2\right], \left[\frac{1}{88}, -2\right], \left[\frac{1}{92}, -2\right], \left[\frac{1}{104}, -2\right], \left[\frac{1}{68}, -2\right], \left[\frac{1}{44}, -2\right], \left[\frac{1}{52}, -2\right],$

$\left[\frac{1}{16}, -2\right], \left[\frac{1}{32}, -2\right], \left[\frac{4}{7}, 1\right], \left[\frac{4}{15}, -4\right], \left[\frac{1}{7}, 1\right], \left[\frac{1}{86}, -2\right], \left[\frac{1}{94}, -2\right], \left[\frac{1}{4}, -2\right], \left[\frac{1}{8}, -2\right], \left[\frac{2}{15}, 2\right], \left[\frac{1}{62}, -2\right],$

$\left[\frac{1}{74}, -2\right], \left[\frac{1}{82}, -2\right], \left[\frac{1}{3}, 1\right], \left[\frac{2}{3}, 1\right], \left[\frac{1}{33}, 1\right], \left[\frac{23}{33}, 1\right], \left[\frac{3}{7}, 1\right], \left[\frac{11}{84}, -2\right], \left[\frac{14}{15}, -4\right], \left[\frac{6}{7}, 1\right], \left[\frac{8}{15}, 2\right],$

$\left[\frac{11}{45}, -4\right], \left[\frac{1}{45}, -4\right], \left[\frac{5}{7}, 1\right], \left[\frac{1}{39}, 1\right], \left[\frac{11}{39}, 1\right], \left[\frac{1}{51}, 1\right], \left[\frac{11}{51}, 1\right], \left[\frac{7}{15}, 2\right], \left[\frac{41}{84}, -2\right], \left[\frac{1}{77}, 1\right], \left[\frac{1}{99}, 1\right],$

$\left[\frac{23}{99}, 1\right], \left[\frac{9}{14}, 2\right], \left[\frac{4}{105}, 2\right], \left[\frac{11}{81}, 1\right], \left[\frac{19}{45}, -4\right], \left[\frac{17}{45}, 2\right], \left[\frac{11}{93}, 1\right], \left[\frac{1}{9}, 1\right], \left[\frac{2}{9}, 1\right], \left[\frac{1}{27}, 1\right], \left[\frac{11}{27}, 1\right],$

$\left[\frac{1}{81}, 1\right], \left[\frac{2}{7}, 1\right], \left[\frac{1}{69}, 1\right], \left[\frac{11}{69}, 1\right], \left[\frac{1}{87}, 1\right], \left[\frac{11}{87}, 1\right], \left[\frac{1}{93}, 1\right], \left[\frac{13}{45}, 2\right], \left[\frac{29}{45}, -4\right], \left[\frac{67}{77}, 1\right], \left[\frac{47}{84}, -2\right],$

$\left[\frac{31}{84}, -2\right], \left[\frac{37}{84}, -2\right], \left[\frac{23}{45}, 2\right], \left[\frac{13}{77}, 1\right], \left[\frac{1}{40}, 4\right], \left[\frac{31}{63}, -1\right], \left[\frac{19}{77}, 1\right], \left[\frac{17}{77}, 1\right], \left[\frac{37}{45}, 2\right], \left[\frac{26}{105}, 2\right],$

$\left[\frac{43}{105}, 0\right], \left[\frac{47}{105}, 0\right], \left[\frac{1}{91}, 1\right], \left[\frac{23}{77}, 1\right], \left[\frac{1}{75}, -4\right], \left[\frac{44}{105}, 2\right], \left[\frac{46}{105}, 2\right], \left[\frac{79}{210}, 0\right], \left[\frac{11}{91}, 1\right], \left[\frac{11}{75}, -4\right],$

$\left[\frac{71}{210}, 0\right], \left[\frac{53}{70}, -8\right], \left[\frac{1}{5}, 0\right], \left[\frac{13}{75}, 2\right], \left[\frac{17}{91}, 1\right], \left[\frac{3}{5}, 2\right], \left[\frac{41}{91}, 1\right], \left[\frac{4}{5}, 0\right], \left[\frac{11}{49}, 1\right], \left[\frac{37}{75}, 2\right], \left[\frac{29}{75}, -4\right],$
$\left[\frac{13}{49}, 1\right], \left[\frac{19}{49}, 1\right], \left[\frac{1}{55}, 0\right], \left[\frac{17}{49}, 1\right], \left[\frac{23}{49}, 1\right], \left[\frac{1}{57}, 1\right], \left[\frac{11}{57}, 1\right]\right]$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 210

"val0=", -2

"which is even."

"valinf=", 0

"which is even."

"It IS a modfunc on Gamma1(", 210, ")"

"TERM ", 4, "of ", 4, " ***"

"XX=", q^8 JAC(6, 210, ∞) JAC(0, 210, ∞)^{12} JAC(14, 210, ∞) JAC(24, 210, ∞) JAC(36, 210, ∞) JAC(54, 210, ∞) JAC(56, 210, ∞) JAC(66, 210, ∞) JAC(84, 210, ∞) JAC(96, 210, ∞) / (JAC(2, 210, ∞) JAC(8, 210, ∞) JAC(12, 210, ∞) JAC(18, 210, ∞) JAC(22, 210, ∞) JAC(28, 210, ∞) JAC(32, 210, ∞) JAC(38, 210, ∞) JAC(42, 210, ∞) JAC(48, 210, ∞) JAC(52, 210, ∞) JAC(58, 210, ∞) JAC(62, 210, ∞) JAC(68, 210, ∞) JAC(72, 210, ∞) JAC(78, 210, ∞) JAC(82, 210, ∞) JAC(88, 210, ∞) JAC(92, 210, ∞) JAC(98, 210, ∞) JAC(102, 210, ∞))

"Cusp ORDS: "

$\left[[oo, 8], \left[\frac{17}{55}, -2\right], \left[\frac{13}{55}, -2\right], \left[\frac{28}{65}, -2\right], \left[\frac{11}{21}, -1\right], \left[\frac{13}{21}, -1\right], \left[\frac{1}{65}, 4\right], \left[\frac{1}{21}, -1\right], \left[\frac{19}{55}, 4\right], \left[\frac{13}{63}, -1\right],\right.$
$\left[\frac{19}{85}, 4\right], \left[\frac{57}{85}, -2\right], \left[\frac{3}{10}, 8\right], \left[\frac{13}{85}, -2\right], \left[\frac{1}{10}, -4\right], \left[\frac{4}{21}, -1\right], \left[\frac{1}{63}, -1\right], \left[\frac{5}{21}, -1\right], \left[\frac{20}{21}, -1\right], \left[\frac{1}{85}, 4\right],$
$\left[\frac{16}{21}, -1\right], \left[\frac{10}{21}, -1\right], \left[\frac{2}{21}, -1\right], \left[\frac{8}{21}, -1\right], \left[\frac{19}{65}, 4\right], \left[\frac{19}{21}, -1\right], \left[\frac{17}{21}, -1\right], \left[\frac{17}{65}, -2\right], \left[\frac{9}{10}, -4\right], \left[\frac{19}{25}, 4\right],$
$\left[\frac{17}{25}, -2\right], \left[\frac{13}{25}, -2\right], \left[\frac{47}{63}, -1\right], \left[\frac{4}{63}, -1\right], \left[\frac{41}{63}, -1\right], \left[\frac{1}{25}, 4\right], \left[\frac{39}{95}, 4\right], \left[\frac{37}{63}, -1\right], \left[\frac{29}{63}, -1\right], \left[\frac{17}{95}, -2\right],$
$\left[\frac{13}{95}, -2\right], \left[\frac{7}{10}, 8\right], \left[\frac{1}{95}, 4\right], \left[\frac{19}{63}, -1\right], \left[\frac{17}{63}, -1\right], \left[\frac{1}{56}, 2\right], \left[\frac{19}{20}, -4\right], \left[\frac{11}{42}, -2\right], \left[\frac{17}{20}, 8\right], \left[\frac{11}{18}, 2\right],$
$\left[\frac{13}{20}, 8\right], \left[\frac{1}{20}, -4\right], \left[\frac{19}{90}, 0\right], \left[\frac{17}{90}, -4\right], \left[\frac{13}{90}, -4\right], \left[\frac{19}{80}, -4\right], \left[\frac{17}{80}, 8\right], \left[\frac{11}{90}, 0\right], \left[\frac{1}{90}, 0\right], \left[\frac{29}{30}, 0\right],$
$\left[\frac{7}{30}, -4\right], \left[\frac{19}{30}, 0\right], \left[\frac{13}{30}, -4\right], \left[\frac{17}{30}, -4\right], \left[\frac{1}{30}, 0\right], \left[\frac{11}{30}, 0\right], \left[\frac{13}{80}, 8\right], \left[\frac{1}{80}, -4\right], \left[\frac{17}{50}, 8\right], \left[\frac{13}{50}, 8\right],$

$$\left[\frac{1}{50}, -4\right], \left[\frac{19}{60}, 0\right], \left[\frac{23}{60}, -4\right], \left[\frac{17}{60}, -4\right], \left[\frac{19}{100}, -4\right], \left[\frac{37}{90}, -4\right], \left[\frac{17}{100}, 8\right], \left[\frac{29}{90}, 0\right], \left[\frac{13}{100}, 8\right], \left[\frac{1}{100}, -4\right],$$

$$\left[\frac{23}{90}, -4\right], \left[\frac{19}{40}, -4\right], \left[\frac{17}{40}, 8\right], \left[\frac{13}{40}, 8\right], \left[\frac{19}{50}, -4\right], \left[\frac{29}{35}, -2\right], \left[\frac{23}{35}, 0\right], \left[\frac{19}{35}, -2\right], \left[\frac{17}{35}, 0\right], \left[\frac{13}{35}, 0\right],$$

$$\left[\frac{11}{35}, -2\right], \left[\frac{1}{35}, -2\right], \left[\frac{1}{6}, 2\right], \left[\frac{23}{63}, -1\right], \left[\frac{27}{35}, 0\right], \left[\frac{11}{63}, -1\right], \left[\frac{9}{35}, -2\right], \left[\frac{3}{35}, 0\right], \left[\frac{32}{35}, 0\right], \left[\frac{1}{66}, 2\right],$$

$$\left[\frac{26}{35}, -2\right], \left[\frac{24}{35}, -2\right], \left[\frac{18}{35}, 0\right], \left[\frac{12}{35}, 0\right], \left[\frac{8}{35}, 0\right], \left[\frac{6}{35}, -2\right], \left[\frac{1}{14}, 2\right], \left[\frac{31}{35}, -2\right], \left[\frac{2}{35}, 0\right], \left[\frac{5}{6}, 2\right], \left[\frac{11}{102}, 2\right],$$

$$\left[\frac{1}{102}, 2\right], \left[\frac{3}{14}, 2\right], \left[\frac{11}{78}, 2\right], \left[\frac{1}{78}, 2\right], \left[\frac{13}{14}, 2\right], \left[\frac{11}{14}, 2\right], \left[\frac{34}{35}, -2\right], \left[\frac{23}{66}, 2\right], \left[\frac{22}{35}, 0\right], \left[\frac{16}{35}, -2\right],$$

$$\left[\frac{23}{84}, -2\right], \left[\frac{29}{84}, -2\right], \left[\frac{17}{84}, -2\right], \left[\frac{19}{84}, -2\right], \left[\frac{13}{84}, -2\right], \left[\frac{1}{84}, -2\right], \left[\frac{5}{42}, -2\right], \left[\frac{1}{28}, 2\right], \left[\frac{1}{12}, 2\right], \left[\frac{31}{42}, -2\right],$$

$$\left[\frac{19}{42}, -2\right], \left[\frac{23}{42}, -2\right], \left[\frac{17}{42}, -2\right], \left[\frac{13}{42}, -2\right], \left[\frac{1}{42}, -2\right], \left[\frac{5}{14}, 2\right], \left[\frac{17}{28}, 2\right], \left[\frac{11}{36}, 2\right], \left[\frac{13}{28}, 2\right], \left[\frac{1}{36}, 2\right],$$

$$\left[\frac{11}{28}, 2\right], \left[\frac{67}{84}, -2\right], \left[\frac{23}{30}, -4\right], \left[\frac{11}{12}, 2\right], \left[\frac{1}{64}, -2\right], \left[\frac{1}{49}, 1\right], \left[\frac{23}{75}, 0\right], \left[\frac{23}{91}, 1\right], \left[\frac{19}{75}, -2\right], \left[\frac{2}{5}, -2\right], \left[\frac{19}{91}, 1\right],$$

$$\left[\frac{17}{75}, 0\right], \left[\frac{17}{70}, -4\right], \left[\frac{19}{70}, 0\right], \left[\frac{11}{56}, 2\right], \left[\frac{13}{70}, -4\right], \left[\frac{11}{70}, 0\right], \left[\frac{1}{72}, 2\right], \left[\frac{1}{70}, 0\right], \left[\frac{11}{48}, 2\right], \left[\frac{23}{28}, 2\right], \left[\frac{1}{48}, 2\right],$$

$$\left[\frac{19}{28}, 2\right], \left[\frac{1}{60}, 0\right], \left[\frac{11}{60}, 0\right], \left[\frac{39}{70}, 0\right], \left[\frac{51}{70}, 0\right], \left[\frac{13}{56}, 2\right], \left[\frac{27}{70}, -4\right], \left[\frac{33}{70}, -4\right], \left[\frac{3}{70}, -4\right], \left[\frac{9}{70}, 0\right], \left[\frac{67}{70}, -4\right],$$

$$\left[\frac{61}{70}, 0\right], \left[\frac{59}{70}, 0\right], \left[\frac{47}{70}, -4\right], \left[\frac{43}{70}, -4\right], \left[\frac{41}{70}, 0\right], \left[\frac{37}{70}, -4\right], \left[\frac{31}{70}, 0\right], \left[\frac{23}{70}, -4\right], \left[\frac{29}{70}, 0\right], \left[\frac{29}{60}, 0\right],$$

$$\left[\frac{37}{60}, -4\right], \left[\frac{11}{72}, 2\right], \left[\frac{13}{60}, -4\right], \left[\frac{19}{56}, 2\right], \left[\frac{1}{18}, 2\right], \left[\frac{17}{56}, 2\right], \left[\frac{69}{70}, 0\right], \left[\frac{57}{70}, -4\right], \left[\frac{41}{105}, -2\right], \left[\frac{31}{105}, -2\right],$$

$$\left[\frac{37}{105}, 4\right], \left[\frac{19}{105}, -2\right], \left[\frac{23}{105}, 4\right], \left[\frac{17}{105}, 4\right], \left[\frac{13}{105}, 4\right], \left[\frac{11}{105}, -2\right], \left[\frac{1}{24}, 2\right], \left[\frac{1}{105}, -2\right], \left[\frac{11}{54}, 2\right], \left[\frac{1}{98}, 2\right],$$

$$\left[\frac{23}{56}, 2\right], \left[\frac{1}{54}, 2\right], \left[\frac{23}{98}, 2\right], \left[\frac{11}{96}, 2\right], \left[\frac{1}{96}, 2\right], \left[\frac{19}{98}, 2\right], \left[\frac{17}{98}, 2\right], \left[\frac{13}{98}, 2\right], \left[\frac{52}{105}, 4\right], \left[\frac{32}{105}, 4\right],$$

$$\left[\frac{34}{105}, -2\right], \left[\frac{38}{105}, 4\right], \left[\frac{11}{98}, 2\right], \left[\frac{11}{24}, 2\right], \left[\frac{22}{105}, 4\right], \left[\frac{16}{105}, -2\right], \left[\frac{8}{105}, 4\right], \left[\frac{2}{105}, 4\right], \left[\frac{29}{210}, 8\right],$$

$$\left[\frac{23}{210}, -4\right], \left[\frac{17}{210}, -4\right], \left[\frac{19}{210}, 8\right], \left[\frac{13}{210}, -4\right], \left[\frac{11}{210}, 8\right], \left[\frac{33}{35}, 0\right], \left[\frac{4}{35}, -2\right], \left[\frac{53}{210}, -4\right], \left[\frac{59}{210}, 8\right],$$

$$\left[\frac{43}{210}, -4\right], \left[\frac{47}{210}, -4\right], \left[\frac{37}{210}, -4\right], \left[\frac{41}{210}, 8\right], \left[\frac{31}{210}, 8\right], \left[\frac{101}{210}, 8\right], \left[\frac{103}{210}, -4\right], \left[\frac{1}{67}, -1\right], \left[\frac{1}{71}, -1\right],$$

$$\left[\frac{1}{73}, -1\right], \left[\frac{1}{79}, -1\right], \left[\frac{1}{83}, -1\right], \left[\frac{97}{210}, -4\right], \left[\frac{1}{89}, -1\right], \left[\frac{1}{61}, -1\right], \left[\frac{1}{47}, -1\right], \left[\frac{89}{210}, 8\right], \left[\frac{1}{53}, -1\right], \left[\frac{83}{210}, -4\right],$$

$$\left[\frac{1}{59}, -1\right], \left[\frac{29}{42}, -2\right], \left[\frac{1}{97}, -1\right], \left[\frac{1}{101}, -1\right], \left[\frac{1}{103}, -1\right], \left[\frac{1}{29}, -1\right], \left[\frac{1}{31}, -1\right], \left[\frac{1}{37}, -1\right], \left[\frac{1}{41}, -1\right], \left[\frac{73}{210}, -4\right],$$

$$\left[\frac{1}{43}, -1\right], \left[\frac{1}{13}, -1\right], \left[\frac{1}{17}, -1\right], \left[\frac{1}{19}, -1\right], \left[\frac{1}{23}, -1\right], \left[\frac{1}{11}, -1\right], [0, -1], \left[\frac{67}{210}, -4\right], \left[\frac{61}{210}, 8\right], \left[\frac{1}{15}, -2\right],$$

$$\left[\frac{13}{15}, 0\right], \left[\frac{1}{2}, -2\right], \left[\frac{1}{22}, -2\right], \left[\frac{1}{26}, -2\right], \left[\frac{1}{34}, -2\right], \left[\frac{1}{38}, -2\right], \left[\frac{1}{46}, -2\right], \left[\frac{1}{58}, -2\right], \left[\frac{25}{42}, -2\right], \left[\frac{11}{15}, -2\right],$$

$$\left[\frac{29}{105}, -2\right], \left[\frac{37}{42}, -2\right], \left[\frac{41}{42}, -2\right], \left[\frac{1}{76}, -2\right], \left[\frac{1}{88}, -2\right], \left[\frac{1}{92}, -2\right], \left[\frac{1}{104}, -2\right], \left[\frac{1}{68}, -2\right], \left[\frac{1}{44}, -2\right], \left[\frac{1}{52}, -2\right],$$

$$\left[\frac{1}{16}, -2\right], \left[\frac{1}{32}, -2\right], \left[\frac{4}{7}, 1\right], \left[\frac{4}{15}, -2\right], \left[\frac{1}{7}, 1\right], \left[\frac{1}{86}, -2\right], \left[\frac{1}{94}, -2\right], \left[\frac{1}{4}, -2\right], \left[\frac{1}{8}, -2\right], \left[\frac{2}{15}, 0\right], \left[\frac{1}{62}, -2\right],$$

$$\left[\frac{1}{74}, -2\right], \left[\frac{1}{82}, -2\right], \left[\frac{1}{3}, 1\right], \left[\frac{2}{3}, 1\right], \left[\frac{1}{33}, 1\right], \left[\frac{23}{33}, 1\right], \left[\frac{3}{7}, 1\right], \left[\frac{11}{84}, -2\right], \left[\frac{14}{15}, -2\right], \left[\frac{6}{7}, 1\right], \left[\frac{8}{15}, 0\right],$$

$$\left[\frac{11}{45}, -2\right], \left[\frac{1}{45}, -2\right], \left[\frac{5}{7}, 1\right], \left[\frac{1}{39}, 1\right], \left[\frac{11}{39}, 1\right], \left[\frac{1}{51}, 1\right], \left[\frac{11}{51}, 1\right], \left[\frac{7}{15}, 0\right], \left[\frac{41}{84}, -2\right], \left[\frac{1}{77}, 1\right], \left[\frac{1}{99}, 1\right],$$

$$\left[\frac{23}{99}, 1\right], \left[\frac{9}{14}, 2\right], \left[\frac{4}{105}, -2\right], \left[\frac{11}{81}, 1\right], \left[\frac{19}{45}, -2\right], \left[\frac{17}{45}, 0\right], \left[\frac{11}{93}, 1\right], \left[\frac{1}{9}, 1\right], \left[\frac{2}{9}, 1\right], \left[\frac{1}{27}, 1\right], \left[\frac{11}{27}, 1\right],$$

$$\left[\frac{1}{81}, 1\right], \left[\frac{2}{7}, 1\right], \left[\frac{1}{69}, 1\right], \left[\frac{11}{69}, 1\right], \left[\frac{1}{87}, 1\right], \left[\frac{11}{87}, 1\right], \left[\frac{1}{93}, 1\right], \left[\frac{13}{45}, 0\right], \left[\frac{29}{45}, -2\right], \left[\frac{67}{77}, 1\right], \left[\frac{47}{84}, -2\right],$$

$$\left[\frac{31}{84}, -2\right], \left[\frac{37}{84}, -2\right], \left[\frac{23}{45}, 0\right], \left[\frac{13}{77}, 1\right], \left[\frac{1}{40}, -4\right], \left[\frac{31}{63}, -1\right], \left[\frac{19}{77}, 1\right], \left[\frac{17}{77}, 1\right], \left[\frac{37}{45}, 0\right], \left[\frac{26}{105}, -2\right],$$

$$\left[\frac{43}{105}, 4\right], \left[\frac{47}{105}, 4\right], \left[\frac{1}{91}, 1\right], \left[\frac{23}{77}, 1\right], \left[\frac{1}{75}, -2\right], \left[\frac{44}{105}, -2\right], \left[\frac{46}{105}, -2\right], \left[\frac{79}{210}, 8\right], \left[\frac{11}{91}, 1\right], \left[\frac{11}{75}, -2\right],$$

$$\left[\frac{71}{210}, 8\right], \left[\frac{53}{70}, -4\right], \left[\frac{1}{5}, 4\right], \left[\frac{13}{75}, 0\right], \left[\frac{17}{91}, 1\right], \left[\frac{3}{5}, -2\right], \left[\frac{41}{91}, 1\right], \left[\frac{4}{5}, 4\right], \left[\frac{11}{49}, 1\right], \left[\frac{37}{75}, 0\right], \left[\frac{29}{75}, -2\right],$$

$$\left[\frac{13}{49}, 1\right], \left[\frac{19}{49}, 1\right], \left[\frac{1}{55}, 4\right], \left[\frac{17}{49}, 1\right], \left[\frac{23}{49}, 1\right], \left[\frac{1}{57}, 1\right], \left[\frac{11}{57}, 1\right]\Big]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 210

"val0=", -2

"which is even."

"valinf=", 16

"which is even."

"It IS a modfunc on Gamma1(", 210, ")"

"min inf ord=", 0

```
"mintotord = ", -576
"TO PROVE the identity we need to show that v[oo](ID) > ", 576
```

```
"*** There were NO errors. ***"
"*** WARNING: some terms were constants. ***"
"See array CONTERMS."
To prove the identity we will need to verify if up to
q^(576).
Do you want to prove the identity? (yes/no)
```

```
> yes
```

```
You entered yes.
We verify the identity to O(q^(996)).
```

$$0$$

```
0 was returned and this proves the identity.
```

```
> ramid2;
```

$$2\,q\left(\frac{\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)}+\frac{q^4\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(12,30,\infty)\,\mathrm{JAC}(28,70,\infty)}\right)-\frac{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(0,3,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(0,21,\infty)}{\mathrm{JAC}(1,4,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(7,28,\infty)\,\mathrm{JAC}(0,42,\infty)}+\frac{\mathrm{JAC}(0,1,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(0,7,\infty)\,\mathrm{JAC}(0,42,\infty)}{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(3,12,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(21,84,\infty)}$$

```
> ramid2:=expand(ramid2);
```

$$ramid2 := 2\,\frac{q\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)}+\frac{2\,q^5\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(12,30,\infty)\,\mathrm{JAC}(28,70,\infty)}-\frac{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(0,3,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(0,21,\infty)}{\mathrm{JAC}(1,4,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(7,28,\infty)\,\mathrm{JAC}(0,42,\infty)}+\frac{\mathrm{JAC}(0,1,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(0,7,\infty)\,\mathrm{JAC}(0,42,\infty)}{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(3,12,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(21,84,\infty)}$$

```
> ramid2a:=expand(ramid2/op(3,ramid2));
```

$$ramid2a := -2\,\frac{\mathrm{JAC}(1,4,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(7,28,\infty)\,\mathrm{JAC}(0,42,\infty)\,q\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(0,3,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(0,21,\infty)\,\mathrm{JAC}(6,30,\infty)\,\mathrm{JAC}(14,70,\infty)}-\frac{2\,\mathrm{JAC}(1,4,\infty)\,\mathrm{JAC}(0,6,\infty)\,\mathrm{JAC}(7,28,\infty)\,\mathrm{JAC}(0,42,\infty)\,q^5\,\mathrm{JAC}(0,30,\infty)\,\mathrm{JAC}(0,70,\infty)}{\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(0,3,\infty)\,\mathrm{JAC}(0,14,\infty)\,\mathrm{JAC}(0,21,\infty)\,\mathrm{JAC}(12,30,\infty)\,\mathrm{JAC}(28,70,\infty)}+1-\frac{\mathrm{JAC}(1,4,\infty)\,\mathrm{JAC}(0,6,\infty)^2\,\mathrm{JAC}(7,28,\infty)\,\mathrm{JAC}(0,42,\infty)^2\,\mathrm{JAC}(0,1,\infty)\,\mathrm{JAC}(0,7,\infty)}{\mathrm{JAC}(0,2,\infty)^2\,\mathrm{JAC}(0,3,\infty)\,\mathrm{JAC}(0,14,\infty)^2\,\mathrm{JAC}(0,21,\infty)\,\mathrm{JAC}(3,12,\infty)\,\mathrm{JAC}(21,84,\infty)}$$

```
> ilcm(4,6,28,30,70,84);
```

$$420$$

```
> ramid2b:=mixedjac2jac(ramid2a,900);
```

```
"term ", 1, "of ", 4
"term ", 2, "of ", 4
"term ", 3, "of ", 4
"term ", 4, "of ", 4
```

$\mathit{ramid2b} := -2\, q\, \mathrm{JAC}(1, 420, \infty)\, \mathrm{JAC}(5, 420, \infty)\, \mathrm{JAC}(7, 420, \infty)^2\, \mathrm{JAC}(11, 420, \infty)$

$\mathrm{JAC}(13, 420, \infty)\, \mathrm{JAC}(17, 420, \infty)\, \mathrm{JAC}(19, 420, \infty)\, \mathrm{JAC}(23, 420, \infty)\, \mathrm{JAC}(25, 420, \infty)$

$\mathrm{JAC}(29, 420, \infty)\, \mathrm{JAC}(31, 420, \infty)\, \mathrm{JAC}(35, 420, \infty)^2\, \mathrm{JAC}(37, 420, \infty)\, \mathrm{JAC}(41, 420, \infty)$

$\mathrm{JAC}(43, 420, \infty)\, \mathrm{JAC}(47, 420, \infty)\, \mathrm{JAC}(49, 420, \infty)^2\, \mathrm{JAC}(53, 420, \infty)\, \mathrm{JAC}(55, 420, \infty)$

$\mathrm{JAC}(59, 420, \infty)\, \mathrm{JAC}(61, 420, \infty)\, \mathrm{JAC}(65, 420, \infty)\, \mathrm{JAC}(67, 420, \infty)\, \mathrm{JAC}(71, 420, \infty)$

$\mathrm{JAC}(73, 420, \infty)\, \mathrm{JAC}(77, 420, \infty)^2\, \mathrm{JAC}(79, 420, \infty)\, \mathrm{JAC}(83, 420, \infty)\, \mathrm{JAC}(85, 420, \infty)$

$\mathrm{JAC}(89, 420, \infty)\, \mathrm{JAC}(91, 420, \infty)^2\, \mathrm{JAC}(95, 420, \infty)\, \mathrm{JAC}(97, 420, \infty)\, \mathrm{JAC}(101, 420, \infty)$

$\mathrm{JAC}(103, 420, \infty)\, \mathrm{JAC}(107, 420, \infty)\, \mathrm{JAC}(109, 420, \infty)\, \mathrm{JAC}(113, 420, \infty)\, \mathrm{JAC}(115, 420, \infty)$

$\mathrm{JAC}(119, 420, \infty)^2\, \mathrm{JAC}(121, 420, \infty)\, \mathrm{JAC}(125, 420, \infty)\, \mathrm{JAC}(127, 420, \infty)\, \mathrm{JAC}(131, 420, \infty)$

$\mathrm{JAC}(133, 420, \infty)^2\, \mathrm{JAC}(137, 420, \infty)\, \mathrm{JAC}(139, 420, \infty)\, \mathrm{JAC}(143, 420, \infty)\, \mathrm{JAC}(145, 420, \infty)$

$\mathrm{JAC}(149, 420, \infty)\, \mathrm{JAC}(151, 420, \infty)\, \mathrm{JAC}(155, 420, \infty)\, \mathrm{JAC}(157, 420, \infty)\, \mathrm{JAC}(161, 420, \infty)^2$

$\mathrm{JAC}(163, 420, \infty)\, \mathrm{JAC}(167, 420, \infty)\, \mathrm{JAC}(169, 420, \infty)\, \mathrm{JAC}(173, 420, \infty)\, \mathrm{JAC}(175, 420, \infty)^2$

$\mathrm{JAC}(179, 420, \infty)\, \mathrm{JAC}(181, 420, \infty)\, \mathrm{JAC}(185, 420, \infty)\, \mathrm{JAC}(187, 420, \infty)\, \mathrm{JAC}(191, 420, \infty)$

$\mathrm{JAC}(193, 420, \infty)\, \mathrm{JAC}(197, 420, \infty)\, \mathrm{JAC}(199, 420, \infty)\, \mathrm{JAC}(203, 420, \infty)^2\, \mathrm{JAC}(205, 420, \infty)$

$\mathrm{JAC}(209, 420, \infty) \,/\, (\mathrm{JAC}(2, 420, \infty)\, \mathrm{JAC}(6, 420, \infty)^2\, \mathrm{JAC}(10, 420, \infty)\, \mathrm{JAC}(14, 420, \infty)^3$

$\mathrm{JAC}(18, 420, \infty)\, \mathrm{JAC}(22, 420, \infty)\, \mathrm{JAC}(24, 420, \infty)\, \mathrm{JAC}(26, 420, \infty)\, \mathrm{JAC}(30, 420, \infty)$

$\mathrm{JAC}(34, 420, \infty)\, \mathrm{JAC}(36, 420, \infty)\, \mathrm{JAC}(38, 420, \infty)\, \mathrm{JAC}(42, 420, \infty)^2\, \mathrm{JAC}(46, 420, \infty)$

$\mathrm{JAC}(50, 420, \infty)\, \mathrm{JAC}(54, 420, \infty)^2\, \mathrm{JAC}(56, 420, \infty)\, \mathrm{JAC}(58, 420, \infty)\, \mathrm{JAC}(62, 420, \infty)$

$\mathrm{JAC}(66, 420, \infty)^2\, \mathrm{JAC}(70, 420, \infty)^2\, \mathrm{JAC}(74, 420, \infty)\, \mathrm{JAC}(78, 420, \infty)\, \mathrm{JAC}(82, 420, \infty)$

$\mathrm{JAC}(84, 420, \infty)^2\, \mathrm{JAC}(86, 420, \infty)\, \mathrm{JAC}(90, 420, \infty)\, \mathrm{JAC}(94, 420, \infty)\, \mathrm{JAC}(96, 420, \infty)$

$\mathrm{JAC}(98, 420, \infty)^2\, \mathrm{JAC}(102, 420, \infty)\, \mathrm{JAC}(106, 420, \infty)\, \mathrm{JAC}(110, 420, \infty)\, \mathrm{JAC}(114, 420, \infty)^2$

$\mathrm{JAC}(118, 420, \infty)\, \mathrm{JAC}(122, 420, \infty)\, \mathrm{JAC}(126, 420, \infty)^4\, \mathrm{JAC}(130, 420, \infty)\, \mathrm{JAC}(134, 420, \infty)$

$\mathrm{JAC}(138, 420, \infty)\, \mathrm{JAC}(142, 420, \infty)\, \mathrm{JAC}(144, 420, \infty)\, \mathrm{JAC}(146, 420, \infty)\, \mathrm{JAC}(150, 420, \infty)$

$\mathrm{JAC}(154, 420, \infty)^3\, \mathrm{JAC}(156, 420, \infty)\, \mathrm{JAC}(158, 420, \infty)\, \mathrm{JAC}(162, 420, \infty)\, \mathrm{JAC}(166, 420, \infty)$

$\mathrm{JAC}(170, 420, \infty)\, \mathrm{JAC}(174, 420, \infty)^2\, \mathrm{JAC}(178, 420, \infty)\, \mathrm{JAC}(182, 420, \infty)^2\, \mathrm{JAC}(186, 420, \infty)^2$

$\mathrm{JAC}(190, 420, \infty)\, \mathrm{JAC}(194, 420, \infty)\, \mathrm{JAC}(196, 420, \infty)\, \mathrm{JAC}(198, 420, \infty)\, \mathrm{JAC}(202, 420, \infty)$

$\mathrm{JAC}(204, 420, \infty)\, \mathrm{JAC}(206, 420, \infty)\, \mathrm{JAC}(210, 420, \infty)) - 2\, q^5\, \mathrm{JAC}(1, 420, \infty)\, \mathrm{JAC}(5, 420, \infty)$

$\mathrm{JAC}(7, 420, \infty)^2\, \mathrm{JAC}(11, 420, \infty)\, \mathrm{JAC}(13, 420, \infty)\, \mathrm{JAC}(17, 420, \infty)\, \mathrm{JAC}(19, 420, \infty)$

$\mathrm{JAC}(23, 420, \infty)\, \mathrm{JAC}(25, 420, \infty)\, \mathrm{JAC}(29, 420, \infty)\, \mathrm{JAC}(31, 420, \infty)\, \mathrm{JAC}(35, 420, \infty)^2$

$\mathrm{JAC}(37, 420, \infty)\, \mathrm{JAC}(41, 420, \infty)\, \mathrm{JAC}(43, 420, \infty)\, \mathrm{JAC}(47, 420, \infty)\, \mathrm{JAC}(49, 420, \infty)^2$

$\mathrm{JAC}(53, 420, \infty)\, \mathrm{JAC}(55, 420, \infty)\, \mathrm{JAC}(59, 420, \infty)\, \mathrm{JAC}(61, 420, \infty)\, \mathrm{JAC}(65, 420, \infty)$

$\mathrm{JAC}(67, 420, \infty)\, \mathrm{JAC}(71, 420, \infty)\, \mathrm{JAC}(73, 420, \infty)\, \mathrm{JAC}(77, 420, \infty)^2\, \mathrm{JAC}(79, 420, \infty)$

$\mathrm{JAC}(83, 420, \infty)\, \mathrm{JAC}(85, 420, \infty)\, \mathrm{JAC}(89, 420, \infty)\, \mathrm{JAC}(91, 420, \infty)^2\, \mathrm{JAC}(95, 420, \infty)$

$$\mathrm{JAC}(97, 420, \infty)\, \mathrm{JAC}(101, 420, \infty)\, \mathrm{JAC}(103, 420, \infty)\, \mathrm{JAC}(107, 420, \infty)\, \mathrm{JAC}(109, 420, \infty)$$
$$\mathrm{JAC}(113, 420, \infty)\, \mathrm{JAC}(115, 420, \infty)\, \mathrm{JAC}(119, 420, \infty)^2\, \mathrm{JAC}(121, 420, \infty)\, \mathrm{JAC}(125, 420, \infty)$$
$$\mathrm{JAC}(127, 420, \infty)\, \mathrm{JAC}(131, 420, \infty)\, \mathrm{JAC}(133, 420, \infty)^2\, \mathrm{JAC}(137, 420, \infty)\, \mathrm{JAC}(139, 420, \infty)$$
$$\mathrm{JAC}(143, 420, \infty)\, \mathrm{JAC}(145, 420, \infty)\, \mathrm{JAC}(149, 420, \infty)\, \mathrm{JAC}(151, 420, \infty)\, \mathrm{JAC}(155, 420, \infty)$$
$$\mathrm{JAC}(157, 420, \infty)\, \mathrm{JAC}(161, 420, \infty)^2\, \mathrm{JAC}(163, 420, \infty)\, \mathrm{JAC}(167, 420, \infty)\, \mathrm{JAC}(169, 420, \infty)$$
$$\mathrm{JAC}(173, 420, \infty)\, \mathrm{JAC}(175, 420, \infty)^2\, \mathrm{JAC}(179, 420, \infty)\, \mathrm{JAC}(181, 420, \infty)\, \mathrm{JAC}(185, 420, \infty)$$
$$\mathrm{JAC}(187, 420, \infty)\, \mathrm{JAC}(191, 420, \infty)\, \mathrm{JAC}(193, 420, \infty)\, \mathrm{JAC}(197, 420, \infty)\, \mathrm{JAC}(199, 420, \infty)$$
$$\mathrm{JAC}(203, 420, \infty)^2\, \mathrm{JAC}(205, 420, \infty)\, \mathrm{JAC}(209, 420, \infty) \Big/ (\mathrm{JAC}(2, 420, \infty)\, \mathrm{JAC}(6, 420, \infty)$$
$$\mathrm{JAC}(10, 420, \infty)\, \mathrm{JAC}(12, 420, \infty)\, \mathrm{JAC}(14, 420, \infty)^2\, \mathrm{JAC}(18, 420, \infty)^2\, \mathrm{JAC}(22, 420, \infty)$$
$$\mathrm{JAC}(26, 420, \infty)\, \mathrm{JAC}(28, 420, \infty)\, \mathrm{JAC}(30, 420, \infty)\, \mathrm{JAC}(34, 420, \infty)\, \mathrm{JAC}(38, 420, \infty)$$
$$\mathrm{JAC}(42, 420, \infty)^4\, \mathrm{JAC}(46, 420, \infty)\, \mathrm{JAC}(48, 420, \infty)\, \mathrm{JAC}(50, 420, \infty)\, \mathrm{JAC}(54, 420, \infty)$$
$$\mathrm{JAC}(58, 420, \infty)\, \mathrm{JAC}(62, 420, \infty)\, \mathrm{JAC}(66, 420, \infty)\, \mathrm{JAC}(70, 420, \infty)^2\, \mathrm{JAC}(72, 420, \infty)$$
$$\mathrm{JAC}(74, 420, \infty)\, \mathrm{JAC}(78, 420, \infty)^2\, \mathrm{JAC}(82, 420, \infty)\, \mathrm{JAC}(86, 420, \infty)\, \mathrm{JAC}(90, 420, \infty)$$
$$\mathrm{JAC}(94, 420, \infty)\, \mathrm{JAC}(98, 420, \infty)^3\, \mathrm{JAC}(102, 420, \infty)^2\, \mathrm{JAC}(106, 420, \infty)\, \mathrm{JAC}(108, 420, \infty)$$
$$\mathrm{JAC}(110, 420, \infty)\, \mathrm{JAC}(112, 420, \infty)\, \mathrm{JAC}(114, 420, \infty)\, \mathrm{JAC}(118, 420, \infty)\, \mathrm{JAC}(122, 420, \infty)$$
$$\mathrm{JAC}(126, 420, \infty)^2\, \mathrm{JAC}(130, 420, \infty)\, \mathrm{JAC}(132, 420, \infty)\, \mathrm{JAC}(134, 420, \infty)\, \mathrm{JAC}(138, 420, \infty)^2$$
$$\mathrm{JAC}(142, 420, \infty)\, \mathrm{JAC}(146, 420, \infty)\, \mathrm{JAC}(150, 420, \infty)\, \mathrm{JAC}(154, 420, \infty)^2\, \mathrm{JAC}(158, 420, \infty)$$
$$\mathrm{JAC}(162, 420, \infty)^2\, \mathrm{JAC}(166, 420, \infty)\, \mathrm{JAC}(168, 420, \infty)^2\, \mathrm{JAC}(170, 420, \infty)\, \mathrm{JAC}(174, 420, \infty)$$
$$\mathrm{JAC}(178, 420, \infty)\, \mathrm{JAC}(182, 420, \infty)^3\, \mathrm{JAC}(186, 420, \infty)\, \mathrm{JAC}(190, 420, \infty)\, \mathrm{JAC}(192, 420, \infty)$$
$$\mathrm{JAC}(194, 420, \infty)\, \mathrm{JAC}(198, 420, \infty)^2\, \mathrm{JAC}(202, 420, \infty)\, \mathrm{JAC}(206, 420, \infty)\, \mathrm{JAC}(210, 420, \infty))$$
$$+ 1 - \mathrm{JAC}(1, 84, \infty)^2\, \mathrm{JAC}(5, 84, \infty)^2\, \mathrm{JAC}(7, 84, \infty)^4\, \mathrm{JAC}(11, 84, \infty)^2\, \mathrm{JAC}(13, 84, \infty)^2$$
$$\mathrm{JAC}(17, 84, \infty)^2\, \mathrm{JAC}(19, 84, \infty)^2\, \mathrm{JAC}(23, 84, \infty)^2\, \mathrm{JAC}(25, 84, \infty)^2\, \mathrm{JAC}(29, 84, \infty)^2$$
$$\mathrm{JAC}(31, 84, \infty)^2\, \mathrm{JAC}(35, 84, \infty)^4\, \mathrm{JAC}(37, 84, \infty)^2\, \mathrm{JAC}(41, 84, \infty)^2 \Big/ (\mathrm{JAC}(0, 84, \infty)^{24}$$
$$\mathrm{JAC}(2, 84, \infty)\, \mathrm{JAC}(10, 84, \infty)\, \mathrm{JAC}(14, 84, \infty)^2\, \mathrm{JAC}(22, 84, \infty)\, \mathrm{JAC}(26, 84, \infty)$$
$$\mathrm{JAC}(34, 84, \infty)\, \mathrm{JAC}(38, 84, \infty))$$

We calculate a set of inequivalent cusps for $\Gamma_1(420)$

and the width of each cusp. Note: oo is the first cusp in the list.

```
> cusps420:=cuspmake1(420):
> cusp420:=cusps420 minus {[1,0]}:
> cusps420:=convert(cusp420,list):
> wids420:=map(x->cuspwid1(x[1],x[2],420),cusps420):
> wids420:=[1,op(wids420)]:
> CUSPS420:=map(x->x[1]/x[2],cusps420):
> CUSPS420:=[oo,op(CUSPS420)];
```

$$CUSPS420 := \Big[oo, \frac{17}{108}, \frac{71}{140}, \frac{31}{70}, \frac{37}{70}, \frac{1}{125}, \frac{34}{35}, \frac{19}{144}, \frac{19}{25}, \frac{13}{25}, \frac{17}{144}, \frac{1}{108}, \frac{61}{70}, 0, \frac{59}{70}, \frac{1}{175}, \frac{19}{125}, \frac{53}{70}, \frac{32}{105},$$

$$\frac{11}{108}, \frac{47}{70}, \frac{11}{35}, \frac{43}{70}, \frac{41}{70}, \frac{17}{175}, \frac{13}{125}, \frac{73}{140}, \frac{11}{175}, \frac{13}{45}, \frac{1}{17}, \frac{31}{175}, \frac{11}{45}, \frac{1}{13}, \frac{97}{140}, \frac{47}{200}, \frac{13}{50}, \frac{1}{45}, \frac{1}{50}, \frac{89}{140}, \frac{11}{128}, \frac{29}{175},$$

$$\frac{1}{11}, \frac{34}{105}, \frac{29}{40}, \frac{83}{140}, \frac{69}{70}, \frac{1}{128}, \frac{1}{7}, \frac{19}{175}, \frac{23}{40}, \frac{33}{70}, \frac{39}{70}, \frac{51}{70}, \frac{27}{70}, \frac{9}{70}, \frac{3}{70}, \frac{79}{140}, \frac{23}{175}, \frac{13}{175}, \frac{67}{70}, \frac{113}{140}, \frac{1}{49}, \frac{11}{126}, \frac{53}{175},$$

$$\frac{1}{47}, \frac{1}{126}, \frac{11}{135}, \frac{44}{105}, \frac{11}{54}, \frac{17}{110}, \frac{29}{140}, \frac{109}{140}, \frac{1}{43}, \frac{1}{135}, \frac{47}{175}, \frac{107}{140}, \frac{47}{100}, \frac{1}{54}, \frac{23}{140}, \frac{29}{45}, \frac{19}{140}, \frac{1}{41}, \frac{43}{175}, \frac{17}{140}, \frac{1}{37},$$

$$\frac{11}{18}, \frac{13}{140}, \frac{1}{31}, \frac{1}{29}, \frac{41}{175}, \frac{103}{140}, \frac{23}{45}, \frac{11}{140}, \frac{127}{140}, \frac{19}{45}, \frac{1}{23}, \frac{38}{105}, \frac{1}{19}, \frac{1}{140}, \frac{17}{45}, \frac{19}{50}, \frac{101}{140}, \frac{37}{175}, \frac{79}{84}, \frac{131}{140}, \frac{5}{21}, \frac{4}{21}, \frac{20}{21},$$

$$\frac{1}{2}, \frac{16}{21}, \frac{67}{84}, \frac{61}{84}, \frac{8}{21}, \frac{73}{175}, \frac{2}{21}, \frac{53}{84}, \frac{59}{84}, \frac{1}{21}, \frac{1}{27}, \frac{17}{21}, \frac{19}{21}, \frac{47}{84}, \frac{13}{21}, \frac{1}{207}, \frac{11}{207}, \frac{41}{84}, \frac{11}{189}, \frac{1}{59}, \frac{46}{105}, \frac{37}{84}, \frac{23}{35}, \frac{31}{84}, \frac{29}{84},$$

$$\frac{13}{135}, \frac{1}{153}, \frac{13}{84}, \frac{29}{135}, \frac{67}{175}, \frac{1}{53}, \frac{1}{99}, \frac{11}{81}, \frac{121}{140}, \frac{23}{135}, \frac{1}{3}, \frac{1}{9}, \frac{1}{63}, \frac{11}{27}, \frac{61}{175}, \frac{2}{9}, \frac{1}{198}, \frac{19}{135}, \frac{11}{162}, \frac{1}{80}, \frac{1}{162}, \frac{17}{135}, \frac{59}{175},$$

$$\frac{23}{147}, \frac{1}{132}, \frac{29}{35}, \frac{61}{168}, \frac{51}{140}, \frac{17}{84}, \frac{1}{72}, \frac{59}{168}, \frac{11}{84}, \frac{13}{147}, \frac{17}{147}, \frac{53}{168}, \frac{19}{147}, \frac{39}{140}, \frac{47}{168}, \frac{43}{168}, \frac{43}{84}, \frac{41}{189}, \frac{33}{140}, \frac{41}{168}, \frac{47}{189},$$

$$\frac{103}{175}, \frac{37}{189}, \frac{27}{140}, \frac{1}{84}, \frac{37}{168}, \frac{31}{189}, \frac{5}{12}, \frac{29}{189}, \frac{9}{140}, \frac{7}{12}, \frac{31}{168}, \frac{19}{189}, \frac{23}{189}, \frac{17}{189}, \frac{13}{189}, \frac{3}{140}, \frac{1}{6}, \frac{1}{18}, \frac{52}{105}, \frac{139}{140}, \frac{47}{63},$$

$$\frac{4}{63}, \frac{1}{71}, \frac{97}{175}, \frac{1}{67}, \frac{1}{61}, \frac{37}{63}, \frac{41}{63}, \frac{1}{33}, \frac{1}{39}, \frac{31}{63}, \frac{29}{63}, \frac{137}{140}, \frac{23}{63}, \frac{17}{63}, \frac{19}{63}, \frac{65}{84}, \frac{13}{63}, \frac{55}{84}, \frac{79}{175}, \frac{83}{84}, \frac{5}{84}, \frac{25}{84}, \frac{123}{140}, \frac{13}{28},$$

$$\frac{113}{420}, \frac{17}{28}, \frac{11}{36}, \frac{11}{14}, \frac{43}{130}, \frac{109}{420}, \frac{31}{35}, \frac{1}{96}, \frac{11}{28}, \frac{103}{420}, \frac{107}{420}, \frac{101}{420}, \frac{139}{175}, \frac{58}{105}, \frac{97}{420}, \frac{1}{12}, \frac{89}{420}, \frac{11}{12}, \frac{117}{140}, \frac{1}{28}, \frac{3}{28}, \frac{1}{4},$$

$$\frac{111}{140}, \frac{79}{420}, \frac{83}{420}, \frac{99}{140}, \frac{11}{204}, \frac{71}{420}, \frac{73}{420}, \frac{19}{72}, \frac{127}{175}, \frac{1}{204}, \frac{149}{168}, \frac{67}{420}, \frac{93}{140}, \frac{139}{168}, \frac{61}{420}, \frac{17}{156}, \frac{87}{140}, \frac{109}{168}, \frac{1}{77}, \frac{17}{72},$$

$$\frac{83}{168}, \frac{89}{168}, \frac{11}{156}, \frac{67}{147}, \frac{81}{140}, \frac{79}{168}, \frac{121}{175}, \frac{47}{147}, \frac{17}{132}, \frac{1}{156}, \frac{41}{147}, \frac{69}{140}, \frac{73}{168}, \frac{37}{147}, \frac{71}{168}, \frac{11}{72}, \frac{31}{147}, \frac{109}{175}, \frac{29}{147}, \frac{57}{140},$$

$$\frac{67}{168}, \frac{173}{420}, \frac{17}{112}, \frac{1}{92}, \frac{1}{60}, \frac{1}{68}, \frac{9}{14}, \frac{167}{420}, \frac{17}{130}, \frac{1}{52}, \frac{19}{96}, \frac{2}{35}, \frac{27}{56}, \frac{19}{132}, \frac{1}{83}, \frac{29}{75}, \frac{5}{56}, \frac{17}{96}, \frac{5}{14}, \frac{1}{73}, \frac{53}{56}, \frac{15}{28}, \frac{43}{56}, \frac{23}{75},$$

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$$\frac{13}{196}, \frac{11}{60}, \frac{191}{420}, \frac{14}{15}, \frac{1}{35}, \frac{1}{188}, \frac{11}{196}, \frac{3}{35}, \frac{17}{35}, \frac{9}{35}, \frac{1}{25}, \frac{187}{420}, \frac{83}{112}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{172}, \frac{4}{5}, \frac{13}{154}, \frac{1}{15}, \frac{8}{15}, \frac{7}{15}, \frac{61}{112}, \frac{1}{164},$$

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"DSET", 174, "nops=", 2
"DSET", 175, "nops=", 24
"DSET", 176, "nops=", 2
"DSET", 177, "nops=", 2
"DSET", 178, "nops=", 1
"DSET", 179, "nops=", 1
"DSET", 180, "nops=", 16
"DSET", 181, "nops=", 1
"DSET", 182, "nops=", 6
"DSET", 183, "nops=", 2
"DSET", 184, "nops=", 2
"DSET", 185, "nops=", 4
"DSET", 186, "nops=", 2
"DSET", 187, "nops=", 1
"DSET", 188, "nops=", 2
"DSET", 189, "nops=", 12
"DSET", 190, "nops=", 4
"DSET", 191, "nops=", 1
"DSET", 192, "nops=", 4
"DSET", 193, "nops=", 1
"DSET", 194, "nops=", 1
"DSET", 195, "nops=", 8
"DSET", 196, "nops=", 12
"DSET", 197, "nops=", 1
"DSET", 198, "nops=", 2
"DSET", 199, "nops=", 1
"DSET", 200, "nops=", 8
"DSET", 201, "nops=", 2
"DSET", 202, "nops=", 1
"DSET", 203, "nops=", 6
```

```
"DSET", 204, "nops=", 4
"DSET", 205, "nops=", 4
"DSET", 206, "nops=", 1
"DSET", 207, "nops=", 2
"DSET", 208, "nops=", 2
"DSET", 209, "nops=", 1
"DSET", 210, "nops=", 24
All cusps in the set are inequivalent.
```

true

```
> numcuspequiv1(3);
```

2

```
> op(numcuspequiv1);
```

```
proc()
local N, dd, xx, d;
    if nargs = 0 then
        printf("-------------------------------------------------------\n");
        printf("numcuspequiv1(N)                                  \n");
        printf("   Returns the number of inequivalent cusps            \n");
        printf("   of Gamma[1](N).                                 \n");
        printf("-------------------------------------------------------\n")
    elif nargs = 1 and type(args[1], posint) then
        N := args[1];
        dd := numtheory[divisors](N);
        xx := 0;
        for d in dd do xx := numtheory[phi](d)*numtheory[phi](N/d) + xx end do;
        RETURN(1/2*xx)
    else ERROR(`invalid input type`)
    end if
end proc
```

```
> nops(CUSPS420);
```

960

```
> provemodfuncid(ramid2b,CUSPS420,wids420,420);
"TERM ", 1, "of ", 4, " ***********************************************************
********"
```

"XX=", $-2\,q\,\mathrm{JAC}(1,420,\infty)\,\mathrm{JAC}(5,420,\infty)\,\mathrm{JAC}(7,420,\infty)^2\,\mathrm{JAC}(11,420,\infty)$
$\mathrm{JAC}(13,420,\infty)\,\mathrm{JAC}(17,420,\infty)\,\mathrm{JAC}(19,420,\infty)\,\mathrm{JAC}(23,420,\infty)\,\mathrm{JAC}(25,420,\infty)$
$\mathrm{JAC}(29,420,\infty)\,\mathrm{JAC}(31,420,\infty)\,\mathrm{JAC}(35,420,\infty)^2\,\mathrm{JAC}(37,420,\infty)\,\mathrm{JAC}(41,420,\infty)$
$\mathrm{JAC}(43,420,\infty)\,\mathrm{JAC}(47,420,\infty)\,\mathrm{JAC}(49,420,\infty)^2\,\mathrm{JAC}(53,420,\infty)\,\mathrm{JAC}(55,420,\infty)$
$\mathrm{JAC}(59,420,\infty)\,\mathrm{JAC}(61,420,\infty)\,\mathrm{JAC}(65,420,\infty)\,\mathrm{JAC}(67,420,\infty)\,\mathrm{JAC}(71,420,\infty)$
$\mathrm{JAC}(73,420,\infty)\,\mathrm{JAC}(77,420,\infty)^2\,\mathrm{JAC}(79,420,\infty)\,\mathrm{JAC}(83,420,\infty)\,\mathrm{JAC}(85,420,\infty)$

$\mathrm{JAC}(89, 420, \infty)\ \mathrm{JAC}(91, 420, \infty)^2\ \mathrm{JAC}(95, 420, \infty)\ \mathrm{JAC}(97, 420, \infty)\ \mathrm{JAC}(101, 420, \infty)$
$\mathrm{JAC}(103, 420, \infty)\ \mathrm{JAC}(107, 420, \infty)\ \mathrm{JAC}(109, 420, \infty)\ \mathrm{JAC}(113, 420, \infty)\ \mathrm{JAC}(115, 420, \infty)$
$\mathrm{JAC}(119, 420, \infty)^2\ \mathrm{JAC}(121, 420, \infty)\ \mathrm{JAC}(125, 420, \infty)\ \mathrm{JAC}(127, 420, \infty)\ \mathrm{JAC}(131, 420, \infty)$
$\mathrm{JAC}(133, 420, \infty)^2\ \mathrm{JAC}(137, 420, \infty)\ \mathrm{JAC}(139, 420, \infty)\ \mathrm{JAC}(143, 420, \infty)\ \mathrm{JAC}(145, 420, \infty)$
$\mathrm{JAC}(149, 420, \infty)\ \mathrm{JAC}(151, 420, \infty)\ \mathrm{JAC}(155, 420, \infty)\ \mathrm{JAC}(157, 420, \infty)\ \mathrm{JAC}(161, 420, \infty)^2$
$\mathrm{JAC}(163, 420, \infty)\ \mathrm{JAC}(167, 420, \infty)\ \mathrm{JAC}(169, 420, \infty)\ \mathrm{JAC}(173, 420, \infty)\ \mathrm{JAC}(175, 420, \infty)^2$
$\mathrm{JAC}(179, 420, \infty)\ \mathrm{JAC}(181, 420, \infty)\ \mathrm{JAC}(185, 420, \infty)\ \mathrm{JAC}(187, 420, \infty)\ \mathrm{JAC}(191, 420, \infty)$
$\mathrm{JAC}(193, 420, \infty)\ \mathrm{JAC}(197, 420, \infty)\ \mathrm{JAC}(199, 420, \infty)\ \mathrm{JAC}(203, 420, \infty)^2\ \mathrm{JAC}(205, 420, \infty)$
$\mathrm{JAC}(209, 420, \infty) \Big/ (\mathrm{JAC}(2, 420, \infty)\ \mathrm{JAC}(6, 420, \infty)^2\ \mathrm{JAC}(10, 420, \infty)\ \mathrm{JAC}(14, 420, \infty)^3$
$\mathrm{JAC}(18, 420, \infty)\ \mathrm{JAC}(22, 420, \infty)\ \mathrm{JAC}(24, 420, \infty)\ \mathrm{JAC}(26, 420, \infty)\ \mathrm{JAC}(30, 420, \infty)$
$\mathrm{JAC}(34, 420, \infty)\ \mathrm{JAC}(36, 420, \infty)\ \mathrm{JAC}(38, 420, \infty)\ \mathrm{JAC}(42, 420, \infty)^2\ \mathrm{JAC}(46, 420, \infty)$
$\mathrm{JAC}(50, 420, \infty)\ \mathrm{JAC}(54, 420, \infty)^2\ \mathrm{JAC}(56, 420, \infty)\ \mathrm{JAC}(58, 420, \infty)\ \mathrm{JAC}(62, 420, \infty)$
$\mathrm{JAC}(66, 420, \infty)^2\ \mathrm{JAC}(70, 420, \infty)^2\ \mathrm{JAC}(74, 420, \infty)\ \mathrm{JAC}(78, 420, \infty)\ \mathrm{JAC}(82, 420, \infty)$
$\mathrm{JAC}(84, 420, \infty)^2\ \mathrm{JAC}(86, 420, \infty)\ \mathrm{JAC}(90, 420, \infty)\ \mathrm{JAC}(94, 420, \infty)\ \mathrm{JAC}(96, 420, \infty)$
$\mathrm{JAC}(98, 420, \infty)^2\ \mathrm{JAC}(102, 420, \infty)\ \mathrm{JAC}(106, 420, \infty)\ \mathrm{JAC}(110, 420, \infty)\ \mathrm{JAC}(114, 420, \infty)^2$
$\mathrm{JAC}(118, 420, \infty)\ \mathrm{JAC}(122, 420, \infty)\ \mathrm{JAC}(126, 420, \infty)^4\ \mathrm{JAC}(130, 420, \infty)\ \mathrm{JAC}(134, 420, \infty)$
$\mathrm{JAC}(138, 420, \infty)\ \mathrm{JAC}(142, 420, \infty)\ \mathrm{JAC}(144, 420, \infty)\ \mathrm{JAC}(146, 420, \infty)\ \mathrm{JAC}(150, 420, \infty)$
$\mathrm{JAC}(154, 420, \infty)^3\ \mathrm{JAC}(156, 420, \infty)\ \mathrm{JAC}(158, 420, \infty)\ \mathrm{JAC}(162, 420, \infty)\ \mathrm{JAC}(166, 420, \infty)$
$\mathrm{JAC}(170, 420, \infty)\ \mathrm{JAC}(174, 420, \infty)^2\ \mathrm{JAC}(178, 420, \infty)\ \mathrm{JAC}(182, 420, \infty)^2\ \mathrm{JAC}(186, 420, \infty)^2$
$\mathrm{JAC}(190, 420, \infty)\ \mathrm{JAC}(194, 420, \infty)\ \mathrm{JAC}(196, 420, \infty)\ \mathrm{JAC}(198, 420, \infty)\ \mathrm{JAC}(202, 420, \infty)$
$\mathrm{JAC}(204, 420, \infty)\ \mathrm{JAC}(206, 420, \infty)\ \mathrm{JAC}(210, 420, \infty))$

"Cusp ORDS: "

$\left[[oo, 1], \left[\frac{1}{147}, -10\right], \left[\frac{19}{203}, -2\right], \left[\frac{59}{168}, 5\right], \left[\frac{17}{203}, -2\right], \left[\frac{32}{105}, -2\right], \left[\frac{67}{189}, -10\right], \left[\frac{13}{203}, -2\right], \left[\frac{13}{49}, -2\right], \right.$

$\left[\frac{52}{105}, -2\right], \left[\frac{11}{49}, -2\right], \left[\frac{13}{147}, -10\right], \left[\frac{46}{105}, 2\right], \left[\frac{37}{90}, 16\right], \left[\frac{44}{105}, 2\right], \left[\frac{23}{203}, -2\right], \left[\frac{11}{147}, -10\right], \left[\frac{38}{105}, -2\right],$

$\left[\frac{34}{105}, 2\right], \left[\frac{74}{105}, 2\right], \left[\frac{13}{14}, -24\right], \left[\frac{11}{14}, -24\right], \left[\frac{47}{147}, -10\right], \left[\frac{41}{147}, -10\right], \left[\frac{68}{105}, -2\right], \left[\frac{83}{168}, 5\right], \left[\frac{37}{147}, -10\right],$

$\left[\frac{23}{49}, -2\right], \left[\frac{31}{147}, -10\right], \left[\frac{64}{105}, 2\right], \left[\frac{19}{49}, -2\right], \left[\frac{29}{147}, -10\right], \left[\frac{61}{105}, 2\right], \left[\frac{23}{147}, -10\right], \left[\frac{62}{105}, -2\right], \left[\frac{67}{168}, 5\right],$

$\left[\frac{17}{147}, -10\right], \left[\frac{58}{105}, -2\right], \left[\frac{17}{49}, -2\right], \left[\frac{61}{168}, 5\right], \left[\frac{17}{60}, 9\right], \left[\frac{92}{105}, -2\right], \left[\frac{17}{154}, -24\right], \left[\frac{13}{154}, -24\right], \left[\frac{11}{60}, 1\right],$

$\left[\frac{88}{105}, -2\right], \left[\frac{123}{154}, -24\right], \left[\frac{1}{150}, 0\right], \left[\frac{86}{105}, 2\right], \left[\frac{169}{420}, 1\right], \left[\frac{1}{154}, -24\right], \left[\frac{149}{168}, 5\right], \left[\frac{9}{14}, -24\right], \left[\frac{82}{105}, -2\right],$

$$\left[\frac{139}{168}, 5\right], \left[\frac{43}{130}, -4\right], \left[\frac{5}{14}, -24\right], \left[\frac{76}{105}, 2\right], \left[\frac{109}{168}, 5\right], \left[\frac{3}{14}, -24\right], \left[\frac{17}{133}, -2\right], \left[\frac{67}{147}, -10\right], \left[\frac{89}{168}, 5\right],$$

$$\left[\frac{5}{28}, 3\right], \left[\frac{41}{60}, 1\right], \left[\frac{25}{28}, 3\right], \left[\frac{197}{420}, 5\right], \left[\frac{15}{28}, 3\right], \left[\frac{9}{28}, 3\right], \left[\frac{3}{28}, 3\right], \left[\frac{37}{60}, 9\right], \left[\frac{17}{182}, -24\right], \left[\frac{23}{28}, 3\right],$$

$$\left[\frac{193}{420}, 5\right], \left[\frac{69}{182}, -24\right], \left[\frac{17}{28}, 3\right], \left[\frac{19}{28}, 3\right], \left[\frac{13}{28}, 3\right], \left[\frac{67}{105}, -2\right], \left[\frac{31}{60}, 1\right], \left[\frac{191}{420}, 1\right], \left[\frac{187}{420}, 5\right], \left[\frac{11}{28}, 3\right],$$

$$\left[\frac{181}{420}, 1\right], \left[\frac{173}{420}, 5\right], \left[\frac{179}{420}, 1\right], \left[\frac{104}{105}, 2\right], \left[\frac{167}{420}, 5\right], \left[\frac{1}{182}, -24\right], \left[\frac{23}{60}, 9\right], \left[\frac{94}{105}, 2\right], \left[\frac{23}{154}, -24\right],$$

$$\left[\frac{19}{154}, -24\right], \left[\frac{17}{180}, 9\right], \left[\frac{29}{60}, 1\right], \left[\frac{83}{112}, 3\right], \left[\frac{19}{98}, -24\right], \left[\frac{11}{150}, 0\right], \left[\frac{13}{180}, 9\right], \left[\frac{61}{112}, 3\right], \left[\frac{53}{112}, 3\right],$$

$$\left[\frac{11}{210}, 0\right], \left[\frac{43}{112}, 3\right], \left[\frac{11}{180}, 1\right], \left[\frac{31}{112}, 3\right], \left[\frac{37}{112}, 3\right], \left[\frac{17}{98}, -24\right], \left[\frac{1}{180}, 1\right], \left[\frac{1}{210}, 0\right], \left[\frac{13}{98}, -24\right],$$

$$\left[\frac{11}{182}, -24\right], \left[\frac{17}{112}, 3\right], \left[\frac{19}{112}, 3\right], \left[\frac{23}{112}, 3\right], \left[\frac{11}{98}, -24\right], \left[\frac{49}{60}, 1\right], \left[\frac{13}{112}, 3\right], \left[\frac{11}{112}, 3\right], \left[\frac{1}{112}, 3\right],$$

$$\left[\frac{27}{56}, 3\right], \left[\frac{5}{56}, 3\right], \left[\frac{59}{60}, 1\right], \left[\frac{53}{56}, 3\right], \left[\frac{40}{133}, -2\right], \left[\frac{1}{98}, -24\right], \left[\frac{37}{56}, 3\right], \left[\frac{43}{56}, 3\right], \left[\frac{53}{60}, 9\right], \left[\frac{23}{56}, 3\right],$$

$$\left[\frac{31}{56}, 3\right], \left[\frac{19}{56}, 3\right], \left[\frac{17}{56}, 3\right], \left[\frac{13}{56}, 3\right], \left[\frac{47}{60}, 9\right], \left[\frac{23}{182}, -24\right], \left[\frac{11}{56}, 3\right], \left[\frac{19}{182}, -24\right], \left[\frac{43}{60}, 9\right], \left[\frac{199}{420}, 1\right],$$

$$\left[\frac{209}{420}, 1\right], \left[\frac{27}{28}, 3\right], \left[\frac{17}{30}, 16\right], \left[\frac{59}{180}, 1\right], \left[\frac{11}{30}, 0\right], \left[\frac{43}{210}, 8\right], \left[\frac{17}{35}, 0\right], \left[\frac{53}{180}, 9\right], \left[\frac{41}{210}, 0\right], \left[\frac{47}{180}, 9\right],$$

$$\left[\frac{13}{35}, 0\right], \left[\frac{37}{210}, 8\right], \left[\frac{43}{180}, 9\right], \left[\frac{83}{196}, 3\right], \left[\frac{11}{35}, 8\right], \left[\frac{31}{210}, 0\right], \left[\frac{61}{196}, 3\right], \left[\frac{41}{180}, 1\right], \left[\frac{37}{196}, 3\right], \left[\frac{43}{196}, 3\right],$$

$$\left[\frac{53}{196}, 3\right], \left[\frac{37}{180}, 9\right], \left[\frac{23}{196}, 3\right], \left[\frac{31}{196}, 3\right], \left[\frac{29}{210}, 0\right], \left[\frac{31}{180}, 1\right], \left[\frac{23}{210}, 8\right], \left[\frac{29}{180}, 1\right], \left[\frac{17}{196}, 3\right],$$

$$\left[\frac{19}{196}, 3\right], \left[\frac{19}{210}, 0\right], \left[\frac{23}{180}, 9\right], \left[\frac{13}{196}, 3\right], \left[\frac{11}{196}, 3\right], \left[\frac{17}{210}, 8\right], \left[\frac{19}{180}, 1\right], \left[\frac{13}{210}, 8\right], \left[\frac{23}{98}, -24\right],$$

$$\left[\frac{103}{210}, 8\right], \left[\frac{53}{120}, 9\right], \left[\frac{47}{120}, 9\right], \left[\frac{2}{15}, -2\right], \left[\frac{8}{35}, 0\right], \left[\frac{101}{210}, 0\right], \left[\frac{43}{120}, 9\right], \left[\frac{97}{210}, 8\right], \left[\frac{1}{130}, 4\right], \left[\frac{11}{15}, 6\right],$$

$$\left[\frac{41}{120}, 1\right], \left[\frac{89}{210}, 0\right], \left[\frac{37}{120}, 9\right], \left[\frac{2}{35}, 0\right], \left[\frac{31}{120}, 1\right], \left[\frac{6}{35}, 8\right], \left[\frac{53}{105}, -2\right], \left[\frac{83}{210}, 8\right], \left[\frac{29}{120}, 1\right], \left[\frac{43}{105}, -2\right],$$

$$\left[\frac{47}{105}, -2\right], \left[\frac{41}{105}, 2\right], \left[\frac{79}{210}, 0\right], \left[\frac{19}{110}, 4\right], \left[\frac{37}{105}, -2\right], \left[\frac{73}{210}, 8\right], \left[\frac{23}{120}, 9\right], \left[\frac{13}{90}, 16\right], \left[\frac{19}{105}, 2\right],$$

$$\left[\frac{23}{105}, -2\right], \left[\frac{29}{105}, 2\right], \left[\frac{71}{210}, 0\right], \left[\frac{17}{105}, -2\right], \left[\frac{13}{105}, -2\right], \left[\frac{19}{120}, 1\right], \left[\frac{17}{110}, -4\right], \left[\frac{17}{120}, 9\right], \left[\frac{67}{210}, 8\right],$$

$$\left[\frac{31}{35}, 8\right], \left[\frac{11}{105}, 2\right], \left[\frac{29}{90}, 0\right], \left[\frac{61}{210}, 0\right], \left[\frac{1}{105}, 2\right], \left[\frac{23}{90}, 16\right], \left[\frac{13}{120}, 9\right], \left[\frac{59}{210}, 0\right], \left[\frac{19}{90}, 0\right], \left[\frac{11}{120}, 1\right],$$

$\left[\frac{17}{90}, 16\right], \left[\frac{13}{110}, -4\right], \left[\frac{1}{120}, 1\right], \left[\frac{29}{35}, 8\right], \left[\frac{53}{210}, 8\right], \left[\frac{23}{35}, 0\right], \left[\frac{1}{110}, 4\right], \left[\frac{109}{180}, 1\right], \left[\frac{1}{90}, 0\right], \left[\frac{71}{105}, 2\right],$

$\left[\frac{11}{90}, 0\right], \left[\frac{47}{210}, 8\right], \left[\frac{19}{35}, 8\right], \left[\frac{67}{180}, 9\right], \left[\frac{29}{30}, 0\right], \left[\frac{23}{30}, 16\right], \left[\frac{19}{165}, 6\right], \left[\frac{17}{165}, -2\right], \left[\frac{13}{165}, -2\right], \left[\frac{1}{47}, 0\right],$

$\left[\frac{1}{49}, -2\right], \left[\frac{1}{51}, -12\right], \left[\frac{1}{53}, 0\right], \left[\frac{1}{57}, -12\right], \left[\frac{1}{59}, 0\right], \left[\frac{1}{170}, 4\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right], \left[\frac{1}{43}, 0\right], \left[\frac{26}{35}, 8\right],$

$\left[\frac{101}{165}, 6\right], \left[\frac{13}{150}, 16\right], \left[\frac{1}{165}, 6\right], \left[\frac{109}{120}, 1\right], \left[\frac{1}{27}, -12\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, -12\right], \left[\frac{24}{35}, 8\right], \left[\frac{67}{120}, 9\right],$

$\left[\frac{18}{35}, 0\right], \left[\frac{14}{15}, 6\right], \left[\frac{59}{120}, 1\right], \left[\frac{1}{7}, -2\right], \left[\frac{1}{9}, -12\right], \left[\frac{1}{11}, 0\right], \left[\frac{8}{15}, -2\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, -10\right],$

$\left[\frac{1}{23}, 0\right], \left[\frac{12}{35}, 0\right], \left[\frac{73}{105}, -2\right], [0, 0], \left[\frac{1}{3}, -12\right], \left[\frac{4}{35}, 8\right], \left[\frac{43}{195}, -2\right], \left[\frac{23}{133}, -2\right], \left[\frac{33}{35}, 0\right], \left[\frac{11}{195}, 6\right],$

$\left[\frac{1}{195}, 6\right], \left[\frac{19}{170}, 4\right], \left[\frac{37}{165}, -2\right], \left[\frac{27}{35}, 0\right], \left[\frac{29}{165}, 6\right], \left[\frac{97}{170}, -4\right], \left[\frac{23}{165}, -2\right], \left[\frac{13}{170}, -4\right], \left[\frac{1}{61}, 0\right],$

$\left[\frac{1}{63}, -10\right], \left[\frac{13}{45}, -2\right], \left[\frac{1}{58}, -20\right], \left[\frac{1}{62}, -20\right], \left[\frac{1}{4}, 5\right], \left[\frac{59}{190}, 4\right], \left[\frac{1}{8}, 5\right], \left[\frac{1}{54}, -4\right], \left[\frac{1}{42}, 0\right], \left[\frac{22}{35}, 0\right],$

$\left[\frac{11}{45}, 6\right], \left[\frac{17}{190}, -4\right], \left[\frac{1}{46}, -20\right], \left[\frac{1}{38}, -20\right], \left[\frac{1}{34}, -20\right], \left[\frac{37}{195}, -2\right], \left[\frac{1}{6}, -4\right], \left[\frac{1}{14}, -24\right], \left[\frac{1}{18}, -4\right],$

$\left[\frac{1}{22}, -20\right], \left[\frac{16}{35}, 8\right], \left[\frac{29}{195}, 6\right], \left[\frac{79}{105}, 2\right], \left[\frac{13}{190}, -4\right], \left[\frac{23}{195}, -2\right], \left[\frac{19}{195}, 6\right], \left[\frac{1}{2}, -20\right], \left[\frac{1}{190}, 4\right],$

$\left[\frac{17}{195}, -2\right], \left[\frac{13}{135}, -2\right], \left[\frac{1}{48}, 3\right], \left[\frac{1}{56}, 3\right], \left[\frac{1}{64}, 5\right], \left[\frac{11}{135}, 6\right], \left[\frac{1}{44}, 5\right], \left[\frac{1}{28}, 3\right], \left[\frac{1}{32}, 5\right], \left[\frac{1}{36}, 3\right],$

$\left[\frac{13}{175}, 0\right], \left[\frac{17}{50}, -4\right], \left[\frac{1}{135}, 6\right], \left[\frac{37}{45}, -2\right], \left[\frac{1}{24}, 3\right], \left[\frac{29}{45}, 6\right], \left[\frac{13}{50}, -4\right], \left[\frac{11}{175}, 8\right], \left[\frac{17}{150}, 16\right], \left[\frac{23}{45}, -2\right],$

$\left[\frac{1}{12}, 3\right], \left[\frac{1}{16}, 5\right], \left[\frac{1}{175}, 8\right], \left[\frac{19}{45}, 6\right], \left[\frac{34}{35}, 8\right], \left[\frac{17}{45}, -2\right], \left[\frac{23}{175}, 0\right], \left[\frac{23}{135}, -2\right], \left[\frac{7}{15}, -2\right], \left[\frac{4}{15}, 6\right],$

$\left[\frac{1}{35}, 8\right], \left[\frac{3}{35}, 0\right], \left[\frac{19}{135}, 6\right], \left[\frac{19}{175}, 8\right], \left[\frac{83}{105}, -2\right], \left[\frac{1}{15}, 6\right], \left[\frac{13}{15}, -2\right], \left[\frac{19}{50}, 4\right], \left[\frac{1}{5}, 0\right], \left[\frac{3}{5}, 4\right], \left[\frac{2}{5}, 4\right],$

$\left[\frac{4}{5}, 0\right], \left[\frac{17}{175}, 0\right], \left[\frac{19}{100}, 5\right], \left[\frac{8}{105}, -2\right], \left[\frac{41}{420}, 1\right], \left[\frac{11}{144}, 3\right], \left[\frac{3}{140}, 9\right], \left[\frac{11}{126}, 0\right], \left[\frac{17}{100}, 1\right], \left[\frac{121}{161}, -2\right],$

$\left[\frac{1}{144}, 3\right], \left[\frac{1}{126}, 0\right], \left[\frac{4}{105}, 2\right], \left[\frac{139}{140}, 1\right], \left[\frac{41}{175}, 8\right], \left[\frac{13}{75}, -2\right], \left[\frac{11}{75}, 6\right], \left[\frac{1}{55}, 0\right], \left[\frac{1}{25}, 0\right], \left[\frac{37}{175}, 0\right],$

$\left[\frac{17}{135}, -2\right], \left[\frac{37}{189}, -10\right], \left[\frac{41}{189}, -10\right], \left[\frac{31}{175}, 8\right], \left[\frac{32}{35}, 0\right], \left[\frac{9}{35}, 8\right], \left[\frac{1}{45}, 6\right], \left[\frac{37}{135}, -2\right], \left[\frac{29}{175}, 8\right],$

$\left[\frac{29}{135}, 6\right], \left[\frac{17}{75}, -2\right], \left[\frac{59}{175}, 8\right], \left[\frac{1}{30}, 0\right], \left[\frac{13}{30}, 16\right], \left[\frac{7}{30}, 16\right], \left[\frac{19}{30}, 0\right], \left[\frac{1}{40}, 5\right], \left[\frac{23}{75}, -2\right], \left[\frac{53}{175}, 0\right],$

$\left[\frac{1}{10}, 4\right], \left[\frac{3}{10}, -4\right], \left[\frac{7}{10}, -4\right], \left[\frac{9}{10}, 4\right], \left[\frac{47}{175}, 0\right], \left[\frac{19}{75}, 6\right], \left[\frac{1}{75}, 6\right], \left[\frac{43}{175}, 0\right], \left[\frac{73}{175}, 0\right], \left[\frac{67}{175}, 0\right],$

$\left[\frac{3}{20}, 1\right], \left[\frac{7}{20}, 1\right], \left[\frac{9}{20}, 5\right], \left[\frac{1}{20}, 5\right], \left[\frac{37}{75}, -2\right], \left[\frac{7}{60}, 9\right], \left[\frac{19}{60}, 1\right], \left[\frac{1}{50}, 4\right], \left[\frac{7}{40}, 1\right], \left[\frac{1}{60}, 1\right], \left[\frac{13}{60}, 9\right],$

$\left[\frac{1}{161}, -2\right], \left[\frac{61}{175}, 8\right], \left[\frac{29}{75}, 6\right], \left[\frac{109}{175}, 8\right], \left[\frac{19}{70}, -4\right], \left[\frac{17}{70}, 12\right], \left[\frac{1}{13}, 0\right], \left[\frac{103}{175}, 0\right], \left[\frac{13}{70}, 12\right], \left[\frac{11}{70}, -4\right],$

$\left[\frac{97}{175}, 0\right], \left[\frac{19}{150}, 0\right], \left[\frac{1}{70}, -4\right], \left[\frac{79}{175}, 8\right], \left[\frac{41}{70}, -4\right], \left[\frac{1}{140}, 1\right], \left[\frac{37}{70}, 12\right], \left[\frac{139}{175}, 8\right], \left[\frac{31}{70}, -4\right], \left[\frac{127}{175}, 0\right],$

$\left[\frac{29}{70}, -4\right], \left[\frac{121}{175}, 8\right], \left[\frac{23}{70}, 12\right], \left[\frac{89}{105}, 2\right], \left[\frac{53}{70}, 12\right], \left[\frac{97}{105}, -2\right], \left[\frac{47}{70}, 12\right], \left[\frac{13}{140}, 9\right], \left[\frac{23}{150}, 16\right],$

$\left[\frac{11}{161}, -2\right], \left[\frac{43}{70}, 12\right], \left[\frac{11}{140}, 1\right], \left[\frac{1}{39}, -12\right], \left[\frac{11}{39}, -12\right], \left[\frac{1}{52}, 5\right], \left[\frac{11}{20}, 5\right], \left[\frac{3}{70}, 12\right], \left[\frac{1}{26}, -20\right],$

$\left[\frac{67}{70}, 12\right], \left[\frac{19}{140}, 1\right], \left[\frac{5}{12}, 3\right], \left[\frac{11}{12}, 3\right], \left[\frac{61}{70}, -4\right], \left[\frac{17}{140}, 9\right], \left[\frac{59}{70}, -4\right], \left[\frac{31}{140}, 1\right], \left[\frac{29}{140}, 1\right], \left[\frac{1}{132}, 3\right],$

$\left[\frac{39}{70}, -4\right], \left[\frac{33}{70}, 12\right], \left[\frac{7}{12}, 3\right], \left[\frac{19}{20}, 5\right], \left[\frac{27}{70}, 12\right], \left[\frac{23}{140}, 9\right], \left[\frac{17}{20}, 1\right], \left[\frac{9}{70}, -4\right], \left[\frac{11}{52}, 5\right], \left[\frac{13}{20}, 1\right],$

$\left[\frac{1}{156}, 3\right], \left[\frac{101}{105}, 2\right], \left[\frac{41}{140}, 1\right], \left[\frac{11}{40}, 5\right], \left[\frac{69}{70}, -4\right], \left[\frac{19}{132}, 3\right], \left[\frac{37}{140}, 9\right], \left[\frac{57}{70}, 12\right], \left[\frac{13}{161}, -2\right], \left[\frac{17}{65}, 4\right],$

$\left[\frac{19}{65}, 0\right], \left[\frac{29}{150}, 0\right], \left[\frac{51}{70}, -4\right], \left[\frac{1}{65}, 0\right], \left[\frac{17}{132}, 3\right], \left[\frac{35}{132}, 3\right], \left[\frac{31}{189}, -10\right], \left[\frac{19}{40}, 5\right], \left[\frac{11}{156}, 3\right], \left[\frac{17}{40}, 1\right],$

$\left[\frac{13}{40}, 1\right], \left[\frac{43}{140}, 9\right], \left[\frac{19}{130}, 4\right], \left[\frac{19}{156}, 3\right], \left[\frac{29}{40}, 5\right], \left[\frac{17}{130}, -4\right], \left[\frac{17}{156}, 3\right], \left[\frac{53}{140}, 9\right], \left[\frac{23}{40}, 1\right], \left[\frac{47}{140}, 9\right],$

$\left[\frac{67}{140}, 9\right], \left[\frac{103}{105}, -2\right], \left[\frac{11}{204}, 3\right], \left[\frac{37}{150}, 16\right], \left[\frac{61}{140}, 1\right], \left[\frac{1}{160}, 5\right], \left[\frac{1}{204}, 3\right], \left[\frac{23}{189}, -10\right], \left[\frac{29}{189}, -10\right],$

$\left[\frac{59}{140}, 1\right], \left[\frac{73}{140}, 9\right], \left[\frac{29}{204}, 3\right], \left[\frac{17}{160}, 1\right], \left[\frac{71}{140}, 1\right], \left[\frac{17}{161}, -2\right], \left[\frac{13}{160}, 1\right], \left[\frac{11}{160}, 5\right], \left[\frac{29}{160}, 5\right],$

$\left[\frac{83}{140}, 9\right], \left[\frac{23}{160}, 1\right], \left[\frac{79}{140}, 1\right], \left[\frac{19}{204}, 3\right], \left[\frac{19}{160}, 5\right], \left[\frac{1}{200}, 5\right], \left[\frac{89}{140}, 1\right], \left[\frac{11}{42}, 0\right], \left[\frac{47}{160}, 1\right], \left[\frac{101}{140}, 1\right],$

$\left[\frac{17}{420}, 5\right], \left[\frac{19}{200}, 5\right], \left[\frac{13}{189}, -10\right], \left[\frac{17}{189}, -10\right], \left[\frac{19}{189}, -10\right], \left[\frac{17}{200}, 1\right], \left[\frac{17}{42}, 0\right], \left[\frac{13}{420}, 5\right], \left[\frac{13}{200}, 1\right],$

$\left[\frac{17}{24}, 3\right], \left[\frac{97}{140}, 9\right], \left[\frac{11}{24}, 3\right], \left[\frac{11}{200}, 5\right], \left[\frac{13}{42}, 0\right], \left[\frac{11}{420}, 1\right], \left[\frac{11}{72}, 3\right], \left[\frac{107}{140}, 9\right], \left[\frac{23}{42}, 0\right], \left[\frac{23}{200}, 1\right],$

$\left[\frac{1}{72}, 3\right], \left[\frac{103}{140}, 9\right], \left[\frac{19}{42}, 0\right], \left[\frac{19}{24}, 3\right], \left[\frac{29}{63}, -10\right], \left[\frac{31}{63}, -10\right], \left[\frac{37}{63}, -10\right], \left[\frac{41}{63}, -10\right], \left[\frac{29}{200}, 5\right], \left[\frac{29}{42}, 0\right],$

$\left[\frac{109}{140}, 1\right], \left[\frac{19}{72}, 3\right], \left[\frac{121}{140}, 1\right], \left[\frac{37}{42}, 0\right], \left[\frac{1}{100}, 5\right], \left[\frac{19}{63}, -10\right], \left[\frac{23}{63}, -10\right], \left[\frac{29}{420}, 1\right], \left[\frac{47}{200}, 1\right], \left[\frac{31}{42}, 0\right],$

$$\left[\frac{113}{140}, 9\right], \left[\frac{23}{420}, 5\right], \left[\frac{17}{72}, 3\right], \left[\frac{13}{63}, -10\right], \left[\frac{31}{420}, 1\right], \left[\frac{127}{140}, 9\right], \left[\frac{1}{96}, 3\right], \left[\frac{47}{168}, 5\right], \left[\frac{53}{168}, 5\right], \left[\frac{17}{63}, -10\right],$$

$$\left[\frac{11}{63}, -10\right], \left[\frac{19}{420}, 1\right], \left[\frac{11}{100}, 5\right], \left[\frac{43}{168}, 5\right], \left[\frac{2}{105}, -2\right], \left[\frac{41}{42}, 0\right], \left[\frac{19}{96}, 3\right], \left[\frac{137}{140}, 9\right], \left[\frac{25}{42}, 0\right], \left[\frac{17}{96}, 3\right],$$

$$\left[\frac{5}{42}, 0\right], \left[\frac{37}{420}, 5\right], \left[\frac{19}{161}, -2\right], \left[\frac{131}{140}, 1\right], \left[\frac{11}{96}, 3\right], \left[\frac{1}{203}, -2\right], \left[\frac{17}{126}, 0\right], \left[\frac{23}{100}, 1\right], \left[\frac{17}{144}, 3\right], \left[\frac{13}{126}, 0\right],$$

$$\left[\frac{43}{420}, 5\right], \left[\frac{9}{140}, 1\right], \left[\frac{53}{420}, 5\right], \left[\frac{13}{100}, 1\right], \left[\frac{23}{126}, 0\right], \left[\frac{33}{140}, 9\right], \left[\frac{5}{21}, -10\right], \left[\frac{4}{21}, -10\right], \left[\frac{16}{105}, 2\right],$$

$$\left[\frac{47}{100}, 1\right], \left[\frac{29}{100}, 5\right], \left[\frac{47}{420}, 5\right], \left[\frac{19}{126}, 0\right], \left[\frac{27}{140}, 9\right], \left[\frac{19}{144}, 3\right], \left[\frac{13}{80}, 1\right], \left[\frac{31}{126}, 0\right], \left[\frac{17}{36}, 3\right], \left[\frac{51}{140}, 1\right],$$

$$\left[\frac{29}{126}, 0\right], \left[\frac{11}{80}, 5\right], \left[\frac{11}{36}, 3\right], \left[\frac{39}{140}, 1\right], \left[\frac{1}{80}, 5\right], \left[\frac{67}{420}, 5\right], \left[\frac{19}{36}, 3\right], \left[\frac{57}{140}, 9\right], \left[\frac{19}{80}, 5\right], \left[\frac{37}{126}, 0\right],$$

$$\left[\frac{17}{80}, 1\right], \left[\frac{61}{420}, 1\right], \left[\frac{20}{21}, -10\right], \left[\frac{59}{420}, 1\right], \left[\frac{47}{80}, 1\right], \left[\frac{69}{140}, 1\right], \left[\frac{29}{80}, 5\right], \left[\frac{71}{420}, 1\right], \left[\frac{1}{108}, 3\right], \left[\frac{41}{126}, 0\right],$$

$$\left[\frac{16}{21}, -10\right], \left[\frac{23}{80}, 1\right], \left[\frac{8}{21}, -10\right], \left[\frac{41}{168}, 5\right], \left[\frac{10}{21}, -10\right], \left[\frac{73}{420}, 5\right], \left[\frac{31}{168}, 5\right], \left[\frac{37}{168}, 5\right], \left[\frac{79}{420}, 1\right],$$

$$\left[\frac{19}{21}, -10\right], \left[\frac{2}{21}, -10\right], \left[\frac{11}{108}, 3\right], \left[\frac{11}{203}, -2\right], \left[\frac{13}{21}, -10\right], \left[\frac{17}{21}, -10\right], \left[\frac{47}{126}, 0\right], \left[\frac{81}{140}, 1\right], \left[\frac{29}{168}, 5\right],$$

$$\left[\frac{83}{420}, 5\right], \left[\frac{11}{21}, -10\right], \left[\frac{11}{84}, 5\right], \left[\frac{97}{420}, 5\right], \left[\frac{1}{84}, 5\right], \left[\frac{17}{108}, 3\right], \left[\frac{23}{168}, 5\right], \left[\frac{89}{420}, 1\right], \left[\frac{19}{147}, -10\right], \left[\frac{19}{84}, 5\right],$$

$$\left[\frac{19}{108}, 3\right], \left[\frac{103}{420}, 5\right], \left[\frac{47}{189}, -10\right], \left[\frac{87}{140}, 9\right], \left[\frac{17}{84}, 5\right], \left[\frac{17}{168}, 5\right], \left[\frac{19}{168}, 5\right], \left[\frac{1}{189}, -10\right], \left[\frac{11}{189}, -10\right],$$

$$\left[\frac{22}{105}, -2\right], \left[\frac{101}{420}, 1\right], \left[\frac{47}{63}, -10\right], \left[\frac{4}{63}, -10\right], \left[\frac{67}{126}, 0\right], \left[\frac{29}{84}, 5\right], \left[\frac{109}{420}, 1\right], \left[\frac{1}{196}, 3\right], \left[\frac{13}{84}, 5\right], \left[\frac{25}{84}, 5\right],$$

$$\left[\frac{55}{84}, 5\right], \left[\frac{65}{84}, 5\right], \left[\frac{1}{168}, 5\right], \left[\frac{11}{168}, 5\right], \left[\frac{13}{168}, 5\right], \left[\frac{23}{84}, 5\right], \left[\frac{93}{140}, 9\right], \left[\frac{107}{420}, 5\right], \left[\frac{79}{84}, 5\right], \left[\frac{83}{84}, 5\right],$$

$$\left[\frac{5}{84}, 5\right], \left[\frac{31}{84}, 5\right], \left[\frac{157}{420}, 5\right], \left[\frac{163}{420}, 5\right], \left[\frac{113}{420}, 5\right], \left[\frac{99}{140}, 1\right], \left[\frac{59}{105}, 2\right], \left[\frac{31}{105}, 2\right], \left[\frac{73}{168}, 5\right], \left[\frac{79}{168}, 5\right],$$

$$\left[\frac{71}{168}, 5\right], \left[\frac{137}{420}, 5\right], \left[\frac{53}{84}, 5\right], \left[\frac{59}{84}, 5\right], \left[\frac{1}{192}, 3\right], \left[\frac{129}{140}, 1\right], \left[\frac{61}{84}, 5\right], \left[\frac{11}{192}, 3\right], \left[\frac{139}{420}, 1\right], \left[\frac{17}{192}, 3\right],$$

$$\left[\frac{73}{84}, 5\right], \left[\frac{26}{105}, 2\right], \left[\frac{71}{84}, 5\right], \left[\frac{143}{420}, 5\right], \left[\frac{19}{192}, 3\right], \left[\frac{67}{84}, 5\right], \left[\frac{111}{140}, 1\right], \left[\frac{11}{48}, 3\right], \left[\frac{41}{84}, 5\right], \left[\frac{121}{420}, 1\right],$$

$$\left[\frac{37}{84}, 5\right], \left[\frac{17}{48}, 3\right], \left[\frac{117}{140}, 9\right], \left[\frac{43}{84}, 5\right], \left[\frac{127}{420}, 5\right], \left[\frac{47}{84}, 5\right], \left[\frac{131}{420}, 1\right], \left[\frac{13}{133}, -2\right], \left[\frac{23}{77}, -2\right], \left[\frac{1}{91}, -2\right],$$

$$\left[\frac{11}{91}, -2\right], \left[\frac{69}{91}, -2\right], \left[\frac{17}{91}, -2\right], \left[\frac{19}{91}, -2\right], \left[\frac{23}{91}, -2\right], \left[\frac{46}{77}, -2\right], \left[\frac{13}{77}, -2\right], \left[\frac{17}{77}, -2\right], \left[\frac{19}{77}, -2\right], \left[\frac{1}{119}, -2\right],$$

$$\left[\frac{11}{119}, -2\right], \left[\frac{13}{119}, -2\right], \left[\frac{80}{119}, -2\right], \left[\frac{19}{119}, -2\right], \left[\frac{23}{119}, -2\right], \left[\frac{1}{133}, -2\right], \left[\frac{11}{133}, -2\right], \left[\frac{19}{48}, 3\right], \left[\frac{4}{7}, -2\right],$$

$$\left[\frac{6}{7}, -2\right], \left[\frac{3}{7}, -2\right], \left[\frac{5}{7}, -2\right], \left[\frac{2}{7}, -2\right], \left[\frac{1}{77}, -2\right], \left[\frac{123}{140}, 9\right], \left[\frac{35}{198}, -4\right], \left[\frac{1}{198}, -4\right], \left[\frac{11}{102}, -4\right], \left[\frac{1}{114}, -4\right],$$

$$\left[\frac{11}{114}, -4\right], \left[\frac{1}{138}, -4\right], \left[\frac{11}{138}, -4\right], \left[\frac{1}{174}, -4\right], \left[\frac{11}{174}, -4\right], \left[\frac{1}{186}, -4\right], \left[\frac{11}{186}, -4\right], \left[\frac{11}{18}, -4\right], \left[\frac{11}{54}, -4\right],$$

$$\left[\frac{1}{162}, -4\right], \left[\frac{11}{162}, -4\right], \left[\frac{5}{6}, -4\right], \left[\frac{1}{66}, -4\right], \left[\frac{35}{66}, -4\right], \left[\frac{1}{78}, -4\right], \left[\frac{11}{78}, -4\right], \left[\frac{1}{102}, -4\right], \left[\frac{1}{125}, 0\right], \left[\frac{13}{125}, 4\right],$$

$$\left[\frac{17}{125}, 4\right], \left[\frac{19}{125}, 0\right], \left[\frac{13}{205}, 4\right], \left[\frac{17}{205}, 4\right], \left[\frac{19}{205}, 0\right], \left[\frac{13}{25}, 4\right], \left[\frac{17}{25}, 4\right], \left[\frac{19}{25}, 0\right], \left[\frac{17}{155}, 4\right], \left[\frac{19}{155}, 0\right],$$

$$\left[\frac{1}{185}, 0\right], \left[\frac{13}{185}, 4\right], \left[\frac{17}{185}, 4\right], \left[\frac{19}{185}, 0\right], \left[\frac{1}{205}, 0\right], \left[\frac{1}{85}, 0\right], \left[\frac{13}{85}, 4\right], \left[\frac{12}{85}, 4\right], \left[\frac{19}{85}, 0\right], \left[\frac{1}{95}, 0\right],$$

$$\left[\frac{13}{95}, 4\right], \left[\frac{17}{95}, 4\right], \left[\frac{59}{95}, 0\right], \left[\frac{43}{65}, 4\right], \left[\frac{13}{55}, 4\right], \left[\frac{17}{55}, 4\right], \left[\frac{19}{55}, 0\right], \left[\frac{11}{184}, 5\right], \left[\frac{1}{208}, 5\right], \left[\frac{11}{208}, 5\right],$$

$$\left[\frac{1}{176}, 5\right], \left[\frac{79}{176}, 5\right], \left[\frac{1}{184}, 5\right], \left[\frac{11}{104}, 5\right], \left[\frac{1}{128}, 5\right], \left[\frac{11}{128}, 5\right], \left[\frac{1}{136}, 5\right], \left[\frac{11}{136}, 5\right], \left[\frac{1}{152}, 5\right],$$

$$\left[\frac{11}{152}, 5\right], \left[\frac{1}{115}, 0\right], \left[\frac{13}{115}, 4\right], \left[\frac{17}{115}, 4\right], \left[\frac{19}{115}, 0\right], \left[\frac{1}{145}, 0\right], \left[\frac{13}{145}, 4\right], \left[\frac{17}{145}, 4\right], \left[\frac{19}{145}, 0\right],$$

$$\left[\frac{1}{155}, 0\right], \left[\frac{13}{155}, 4\right], \left[\frac{11}{164}, 5\right], \left[\frac{1}{172}, 5\right], \left[\frac{1}{116}, 5\right], \left[\frac{11}{116}, 5\right], \left[\frac{1}{124}, 5\right], \left[\frac{11}{124}, 5\right], \left[\frac{1}{148}, 5\right],$$

$$\left[\frac{11}{148}, 5\right], \left[\frac{1}{164}, 5\right], \left[\frac{35}{44}, 5\right], \left[\frac{1}{68}, 5\right], \left[\frac{11}{68}, 5\right], \left[\frac{1}{76}, 5\right], \left[\frac{11}{76}, 5\right], \left[\frac{1}{92}, 5\right], \left[\frac{11}{92}, 5\right], \left[\frac{3}{4}, 5\right],$$

$$\left[\frac{11}{207}, -12\right], \left[\frac{1}{117}, -12\right], \left[\frac{11}{117}, -12\right], \left[\frac{1}{153}, -12\right], \left[\frac{11}{153}, -12\right], \left[\frac{1}{171}, -12\right], \left[\frac{11}{171}, -12\right], \left[\frac{1}{207}, -12\right],$$

$$\left[\frac{1}{99}, -12\right], \left[\frac{35}{99}, -12\right], \left[\frac{11}{201}, -12\right], \left[\frac{2}{9}, -12\right], \left[\frac{11}{27}, -12\right], \left[\frac{1}{81}, -12\right], \left[\frac{11}{81}, -12\right], \left[\frac{11}{123}, -12\right],$$

$$\left[\frac{1}{129}, -12\right], \left[\frac{11}{129}, -12\right], \left[\frac{1}{141}, -12\right], \left[\frac{11}{141}, -12\right], \left[\frac{1}{87}, -12\right], \left[\frac{11}{87}, -12\right], \left[\frac{1}{93}, -12\right], \left[\frac{11}{93}, -12\right],$$

$$\left[\frac{1}{111}, -12\right], \left[\frac{11}{111}, -12\right], \left[\frac{1}{123}, -12\right], \left[\frac{11}{51}, -12\right], \left[\frac{11}{57}, -12\right], \left[\frac{1}{69}, -12\right], \left[\frac{11}{69}, -12\right], \left[\frac{1}{159}, -12\right],$$

$$\left[\frac{11}{159}, -12\right], \left[\frac{1}{177}, -12\right], \left[\frac{11}{177}, -12\right], \left[\frac{1}{183}, -12\right], \left[\frac{11}{183}, -12\right], \left[\frac{1}{201}, -12\right], \left[\frac{11}{172}, 5\right], \left[\frac{1}{188}, 5\right],$$

$$\left[\frac{11}{188}, 5\right], \left[\frac{3}{8}, 5\right], \left[\frac{11}{16}, 5\right], \left[\frac{11}{32}, 5\right], \left[\frac{11}{64}, 5\right], \left[\frac{1}{88}, 5\right], \left[\frac{79}{88}, 5\right], \left[\frac{1}{104}, 5\right], \left[\frac{2}{3}, -12\right], \left[\frac{2}{33}, -12\right],$$

$$\left[\frac{1}{202}, -20\right], \left[\frac{1}{206}, -20\right], \left[\frac{1}{122}, -20\right], \left[\frac{1}{134}, -20\right], \left[\frac{1}{142}, -20\right], \left[\frac{1}{146}, -20\right], \left[\frac{1}{158}, -20\right], \left[\frac{1}{166}, -20\right],$$

$\left[\frac{1}{178}, -20\right], \left[\frac{1}{194}, -20\right], \left[\frac{1}{74}, -20\right], \left[\frac{1}{82}, -20\right], \left[\frac{1}{86}, -20\right], \left[\frac{1}{94}, -20\right], \left[\frac{1}{106}, -20\right], \left[\frac{1}{118}, -20\right],$
$\left[\frac{1}{181}, 0\right], \left[\frac{1}{187}, 0\right], \left[\frac{1}{191}, 0\right], \left[\frac{1}{193}, 0\right], \left[\frac{1}{197}, 0\right], \left[\frac{1}{199}, 0\right], \left[\frac{1}{209}, 0\right], \left[\frac{1}{169}, 0\right], \left[\frac{1}{173}, 0\right],$
$\left[\frac{1}{179}, 0\right], \left[\frac{1}{139}, 0\right], \left[\frac{1}{143}, 0\right], \left[\frac{1}{149}, 0\right], \left[\frac{1}{151}, 0\right], \left[\frac{1}{157}, 0\right], \left[\frac{1}{163}, 0\right], \left[\frac{1}{167}, 0\right], \left[\frac{1}{121}, 0\right],$
$\left[\frac{1}{127}, 0\right], \left[\frac{1}{131}, 0\right], \left[\frac{1}{137}, 0\right], \left[\frac{1}{89}, 0\right], \left[\frac{1}{97}, 0\right], \left[\frac{1}{101}, 0\right], \left[\frac{1}{103}, 0\right], \left[\frac{1}{107}, 0\right], \left[\frac{1}{109}, 0\right], \left[\frac{1}{113}, 0\right],$
$\left[\frac{1}{73}, 0\right], \left[\frac{1}{79}, 0\right], \left[\frac{1}{83}, 0\right], \left[\frac{1}{67}, 0\right], \left[\frac{1}{71}, 0\right], \left[\frac{149}{420}, 1\right], \left[\frac{151}{420}, 1\right]\right]$

"TOTAL ORD = ", 0

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"POWER of q CORRECT"
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"All n are divisors of ", 420

"val0=", 0

"which is even."

"valinf=", 2

"which is even."

"It IS a modfunc on Gamma1(", 420, ")"

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"TERM ", 2, "of ", 4, " ****************************************************************"
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$$\text{"XX=", } -2\, q^{5}\, \mathrm{JAC}(1, 420, \infty)\, \mathrm{JAC}(5, 420, \infty)\, \mathrm{JAC}(7, 420, \infty)^{2}\, \mathrm{JAC}(11, 420, \infty)\, \mathrm{JAC}(13, 420, \infty)\, \mathrm{JAC}(17, 420, \infty)\, \mathrm{JAC}(19, 420, \infty)\, \mathrm{JAC}(23, 420, \infty)\, \mathrm{JAC}(25, 420, \infty)\, \mathrm{JAC}(29, 420, \infty)\, \mathrm{JAC}(31, 420, \infty)\, \mathrm{JAC}(35, 420, \infty)^{2}\, \mathrm{JAC}(37, 420, \infty)\, \mathrm{JAC}(41, 420, \infty)\, \mathrm{JAC}(43, 420, \infty)\, \mathrm{JAC}(47, 420, \infty)\, \mathrm{JAC}(49, 420, \infty)^{2}\, \mathrm{JAC}(53, 420, \infty)\, \mathrm{JAC}(55, 420, \infty)\, \mathrm{JAC}(59, 420, \infty)\, \mathrm{JAC}(61, 420, \infty)\, \mathrm{JAC}(65, 420, \infty)\, \mathrm{JAC}(67, 420, \infty)\, \mathrm{JAC}(71, 420, \infty)\, \mathrm{JAC}(73, 420, \infty)\, \mathrm{JAC}(77, 420, \infty)^{2}\, \mathrm{JAC}(79, 420, \infty)\, \mathrm{JAC}(83, 420, \infty)\, \mathrm{JAC}(85, 420, \infty)\, \mathrm{JAC}(89, 420, \infty)\, \mathrm{JAC}(91, 420, \infty)^{2}\, \mathrm{JAC}(95, 420, \infty)\, \mathrm{JAC}(97, 420, \infty)\, \mathrm{JAC}(101, 420, \infty)\, \mathrm{JAC}(103, 420, \infty)\, \mathrm{JAC}(107, 420, \infty)\, \mathrm{JAC}(109, 420, \infty)\, \mathrm{JAC}(113, 420, \infty)\, \mathrm{JAC}(115, 420, \infty)\, \mathrm{JAC}(119, 420, \infty)^{2}\, \mathrm{JAC}(121, 420, \infty)\, \mathrm{JAC}(125, 420, \infty)\, \mathrm{JAC}(127, 420, \infty)\, \mathrm{JAC}(131, 420, \infty)\, \mathrm{JAC}(133, 420, \infty)^{2}\, \mathrm{JAC}(137, 420, \infty)\, \mathrm{JAC}(139, 420, \infty)\, \mathrm{JAC}(143, 420, \infty)\, \mathrm{JAC}(145, 420, \infty)\, \mathrm{JAC}(149, 420, \infty)\, \mathrm{JAC}(151, 420, \infty)\, \mathrm{JAC}(155, 420, \infty)\, \mathrm{JAC}(157, 420, \infty)\, \mathrm{JAC}(161, 420, \infty)^{2}\, \mathrm{JAC}(163, 420, \infty)\, \mathrm{JAC}(167, 420, \infty)\, \mathrm{JAC}(169, 420, \infty)\, \mathrm{JAC}(173, 420, \infty)\, \mathrm{JAC}(175, 420, \infty)^{2}\, \mathrm{JAC}(179, 420, \infty)\, \mathrm{JAC}(181, 420, \infty)\, \mathrm{JAC}(185, 420, \infty)\, \mathrm{JAC}(187, 420, \infty)\, \mathrm{JAC}(191, 420, \infty)\, \mathrm{JAC}(193, 420, \infty)\, \mathrm{JAC}(197, 420, \infty)\, \mathrm{JAC}(199, 420, \infty)\, \mathrm{JAC}(203, 420, \infty)^{2}\, \mathrm{JAC}(205, 420, \infty)\, \mathrm{JAC}(209, 420, \infty) \Big/ \big(\mathrm{JAC}(2, 420, \infty)\, \mathrm{JAC}(6, 420, \infty)\, \mathrm{JAC}(10, 420, \infty)\, \mathrm{JAC}(12, 420, \infty)\, \mathrm{JAC}(14, 420, \infty)^{2}\, \mathrm{JAC}(18, 420, \infty)^{2}\, \mathrm{JAC}(22, 420, \infty)\, \mathrm{JAC}(26, 420, \infty)\, \mathrm{JAC}(28, 420, \infty)$$

$\mathrm{JAC}(30, 420, \infty)\,\mathrm{JAC}(34, 420, \infty)\,\mathrm{JAC}(38, 420, \infty)\,\mathrm{JAC}(42, 420, \infty)^4\,\mathrm{JAC}(46, 420, \infty)$
$\mathrm{JAC}(48, 420, \infty)\,\mathrm{JAC}(50, 420, \infty)\,\mathrm{JAC}(54, 420, \infty)\,\mathrm{JAC}(58, 420, \infty)\,\mathrm{JAC}(62, 420, \infty)$
$\mathrm{JAC}(66, 420, \infty)\,\mathrm{JAC}(70, 420, \infty)^2\,\mathrm{JAC}(72, 420, \infty)\,\mathrm{JAC}(74, 420, \infty)\,\mathrm{JAC}(78, 420, \infty)^2$
$\mathrm{JAC}(82, 420, \infty)\,\mathrm{JAC}(86, 420, \infty)\,\mathrm{JAC}(90, 420, \infty)\,\mathrm{JAC}(94, 420, \infty)\,\mathrm{JAC}(98, 420, \infty)^3$
$\mathrm{JAC}(102, 420, \infty)^2\,\mathrm{JAC}(106, 420, \infty)\,\mathrm{JAC}(108, 420, \infty)\,\mathrm{JAC}(110, 420, \infty)\,\mathrm{JAC}(112, 420, \infty)$
$\mathrm{JAC}(114, 420, \infty)\,\mathrm{JAC}(118, 420, \infty)\,\mathrm{JAC}(122, 420, \infty)\,\mathrm{JAC}(126, 420, \infty)^2\,\mathrm{JAC}(130, 420, \infty)$
$\mathrm{JAC}(132, 420, \infty)\,\mathrm{JAC}(134, 420, \infty)\,\mathrm{JAC}(138, 420, \infty)^2\,\mathrm{JAC}(142, 420, \infty)\,\mathrm{JAC}(146, 420, \infty)$
$\mathrm{JAC}(150, 420, \infty)\,\mathrm{JAC}(154, 420, \infty)^2\,\mathrm{JAC}(158, 420, \infty)\,\mathrm{JAC}(162, 420, \infty)^2\,\mathrm{JAC}(166, 420, \infty)$
$\mathrm{JAC}(168, 420, \infty)^2\,\mathrm{JAC}(170, 420, \infty)\,\mathrm{JAC}(174, 420, \infty)\,\mathrm{JAC}(178, 420, \infty)\,\mathrm{JAC}(182, 420, \infty)^3$
$\mathrm{JAC}(186, 420, \infty)\,\mathrm{JAC}(190, 420, \infty)\,\mathrm{JAC}(192, 420, \infty)\,\mathrm{JAC}(194, 420, \infty)\,\mathrm{JAC}(198, 420, \infty)^2$
$\mathrm{JAC}(202, 420, \infty)\,\mathrm{JAC}(206, 420, \infty)\,\mathrm{JAC}(210, 420, \infty))$

"Cusp ORDS: "

$$\left[[oo, 5], \left[\frac{1}{147}, -10\right], \left[\frac{19}{203}, -2\right], \left[\frac{59}{168}, 5\right], \left[\frac{17}{203}, -2\right], \left[\frac{32}{105}, 2\right], \left[\frac{67}{189}, -10\right], \left[\frac{13}{203}, -2\right], \left[\frac{13}{49}, -2\right],\right.$$
$$\left[\frac{52}{105}, 2\right], \left[\frac{11}{49}, -2\right], \left[\frac{13}{147}, -10\right], \left[\frac{46}{105}, -2\right], \left[\frac{37}{90}, 0\right], \left[\frac{44}{105}, -2\right], \left[\frac{23}{203}, -2\right], \left[\frac{11}{147}, -10\right], \left[\frac{38}{105}, 2\right],$$
$$\left[\frac{34}{105}, -2\right], \left[\frac{74}{105}, -2\right], \left[\frac{13}{14}, -24\right], \left[\frac{11}{14}, -24\right], \left[\frac{47}{147}, -10\right], \left[\frac{41}{147}, -10\right], \left[\frac{68}{105}, 2\right], \left[\frac{83}{168}, 5\right], \left[\frac{37}{147}, -10\right],$$
$$\left[\frac{23}{49}, -2\right], \left[\frac{31}{147}, -10\right], \left[\frac{64}{105}, -2\right], \left[\frac{19}{49}, -2\right], \left[\frac{29}{147}, -10\right], \left[\frac{61}{105}, -2\right], \left[\frac{23}{147}, -10\right], \left[\frac{62}{105}, 2\right], \left[\frac{67}{168}, 5\right],$$
$$\left[\frac{17}{147}, -10\right], \left[\frac{58}{105}, 2\right], \left[\frac{17}{49}, -2\right], \left[\frac{61}{168}, 5\right], \left[\frac{17}{60}, 1\right], \left[\frac{92}{105}, 2\right], \left[\frac{17}{154}, -24\right], \left[\frac{13}{154}, -24\right], \left[\frac{11}{60}, 9\right],$$
$$\left[\frac{88}{105}, 2\right], \left[\frac{123}{154}, -24\right], \left[\frac{1}{150}, 16\right], \left[\frac{86}{105}, -2\right], \left[\frac{169}{420}, 5\right], \left[\frac{1}{154}, -24\right], \left[\frac{149}{168}, 5\right], \left[\frac{9}{14}, -24\right], \left[\frac{82}{105}, 2\right],$$
$$\left[\frac{139}{168}, 5\right], \left[\frac{43}{130}, 4\right], \left[\frac{5}{14}, -24\right], \left[\frac{76}{105}, -2\right], \left[\frac{109}{168}, 5\right], \left[\frac{3}{14}, -24\right], \left[\frac{17}{133}, -2\right], \left[\frac{67}{147}, -10\right], \left[\frac{89}{168}, 5\right],$$
$$\left[\frac{5}{28}, 3\right], \left[\frac{41}{60}, 9\right], \left[\frac{25}{28}, 3\right], \left[\frac{197}{420}, 1\right], \left[\frac{15}{28}, 3\right], \left[\frac{9}{28}, 3\right], \left[\frac{3}{28}, 3\right], \left[\frac{37}{60}, 1\right], \left[\frac{17}{182}, -24\right], \left[\frac{23}{28}, 3\right],$$
$$\left[\frac{193}{420}, 1\right], \left[\frac{69}{182}, -24\right], \left[\frac{17}{28}, 3\right], \left[\frac{19}{28}, 3\right], \left[\frac{13}{28}, 3\right], \left[\frac{67}{105}, 2\right], \left[\frac{31}{60}, 9\right], \left[\frac{191}{420}, 5\right], \left[\frac{187}{420}, 1\right], \left[\frac{11}{28}, 3\right],$$
$$\left[\frac{181}{420}, 5\right], \left[\frac{173}{420}, 1\right], \left[\frac{179}{420}, 5\right], \left[\frac{104}{105}, -2\right], \left[\frac{167}{420}, 1\right], \left[\frac{1}{182}, -24\right], \left[\frac{23}{60}, 1\right], \left[\frac{94}{105}, -2\right], \left[\frac{23}{154}, -24\right],$$
$$\left[\frac{19}{154}, -24\right], \left[\frac{17}{180}, 1\right], \left[\frac{29}{60}, 9\right], \left[\frac{83}{112}, 3\right], \left[\frac{19}{98}, -24\right], \left[\frac{11}{150}, 16\right], \left[\frac{13}{180}, 1\right], \left[\frac{61}{112}, 3\right], \left[\frac{53}{112}, 3\right],$$

$$\left[\frac{11}{210}, 8\right], \left[\frac{43}{112}, 3\right], \left[\frac{11}{180}, 9\right], \left[\frac{31}{112}, 3\right], \left[\frac{37}{112}, 3\right], \left[\frac{17}{98}, -24\right], \left[\frac{1}{180}, 9\right], \left[\frac{1}{210}, 8\right], \left[\frac{13}{98}, -24\right],$$

$$\left[\frac{11}{182}, -24\right], \left[\frac{17}{112}, 3\right], \left[\frac{19}{112}, 3\right], \left[\frac{23}{112}, 3\right], \left[\frac{11}{98}, -24\right], \left[\frac{49}{60}, 9\right], \left[\frac{13}{112}, 3\right], \left[\frac{11}{112}, 3\right], \left[\frac{1}{112}, 3\right],$$

$$\left[\frac{27}{56}, 3\right], \left[\frac{5}{56}, 3\right], \left[\frac{59}{60}, 9\right], \left[\frac{53}{56}, 3\right], \left[\frac{40}{133}, -2\right], \left[\frac{1}{98}, -24\right], \left[\frac{37}{56}, 3\right], \left[\frac{43}{56}, 3\right], \left[\frac{53}{60}, 1\right], \left[\frac{23}{56}, 3\right],$$

$$\left[\frac{31}{56}, 3\right], \left[\frac{19}{56}, 3\right], \left[\frac{17}{56}, 3\right], \left[\frac{13}{56}, 3\right], \left[\frac{47}{60}, 1\right], \left[\frac{23}{182}, -24\right], \left[\frac{11}{56}, 3\right], \left[\frac{19}{182}, -24\right], \left[\frac{43}{60}, 1\right], \left[\frac{199}{420}, 5\right],$$

$$\left[\frac{209}{420}, 5\right], \left[\frac{27}{28}, 3\right], \left[\frac{17}{30}, 0\right], \left[\frac{59}{180}, 9\right], \left[\frac{11}{30}, 16\right], \left[\frac{43}{210}, 0\right], \left[\frac{17}{35}, 8\right], \left[\frac{53}{180}, 1\right], \left[\frac{41}{210}, 8\right], \left[\frac{47}{180}, 1\right],$$

$$\left[\frac{13}{35}, 8\right], \left[\frac{37}{210}, 0\right], \left[\frac{43}{180}, 1\right], \left[\frac{83}{196}, 3\right], \left[\frac{11}{35}, 0\right], \left[\frac{31}{210}, 8\right], \left[\frac{61}{196}, 3\right], \left[\frac{41}{180}, 9\right], \left[\frac{37}{196}, 3\right], \left[\frac{43}{196}, 3\right],$$

$$\left[\frac{53}{196}, 3\right], \left[\frac{37}{180}, 1\right], \left[\frac{23}{196}, 3\right], \left[\frac{31}{196}, 3\right], \left[\frac{29}{210}, 8\right], \left[\frac{31}{180}, 9\right], \left[\frac{23}{210}, 0\right], \left[\frac{29}{180}, 9\right], \left[\frac{17}{196}, 3\right],$$

$$\left[\frac{19}{196}, 3\right], \left[\frac{19}{210}, 8\right], \left[\frac{23}{180}, 1\right], \left[\frac{13}{196}, 3\right], \left[\frac{11}{196}, 3\right], \left[\frac{17}{210}, 0\right], \left[\frac{19}{180}, 9\right], \left[\frac{13}{210}, 0\right], \left[\frac{23}{98}, -24\right],$$

$$\left[\frac{103}{210}, 0\right], \left[\frac{53}{120}, 1\right], \left[\frac{47}{120}, 1\right], \left[\frac{2}{15}, 6\right], \left[\frac{8}{35}, 8\right], \left[\frac{101}{210}, 8\right], \left[\frac{43}{120}, 1\right], \left[\frac{97}{210}, 0\right], \left[\frac{1}{130}, -4\right], \left[\frac{11}{15}, -2\right],$$

$$\left[\frac{41}{120}, 9\right], \left[\frac{89}{210}, 8\right], \left[\frac{37}{120}, 1\right], \left[\frac{2}{35}, 8\right], \left[\frac{31}{120}, 9\right], \left[\frac{6}{35}, 0\right], \left[\frac{53}{105}, 2\right], \left[\frac{83}{210}, 0\right], \left[\frac{29}{120}, 9\right], \left[\frac{43}{105}, 2\right],$$

$$\left[\frac{47}{105}, 2\right], \left[\frac{41}{105}, -2\right], \left[\frac{79}{210}, 8\right], \left[\frac{19}{110}, -4\right], \left[\frac{37}{105}, 2\right], \left[\frac{73}{210}, 0\right], \left[\frac{23}{120}, 1\right], \left[\frac{13}{90}, 0\right], \left[\frac{19}{105}, -2\right],$$

$$\left[\frac{23}{105}, 2\right], \left[\frac{29}{105}, -2\right], \left[\frac{71}{210}, 8\right], \left[\frac{17}{105}, 2\right], \left[\frac{13}{105}, 2\right], \left[\frac{19}{120}, 9\right], \left[\frac{17}{110}, 4\right], \left[\frac{17}{120}, 1\right], \left[\frac{67}{210}, 0\right], \left[\frac{31}{35}, 0\right],$$

$$\left[\frac{11}{105}, -2\right], \left[\frac{29}{90}, 16\right], \left[\frac{61}{210}, 8\right], \left[\frac{1}{105}, -2\right], \left[\frac{23}{90}, 0\right], \left[\frac{13}{120}, 1\right], \left[\frac{59}{210}, 8\right], \left[\frac{19}{90}, 16\right], \left[\frac{11}{120}, 9\right], \left[\frac{17}{90}, 0\right],$$

$$\left[\frac{13}{110}, 4\right], \left[\frac{1}{120}, 9\right], \left[\frac{29}{35}, 0\right], \left[\frac{53}{210}, 0\right], \left[\frac{23}{35}, 8\right], \left[\frac{1}{110}, -4\right], \left[\frac{109}{180}, 9\right], \left[\frac{1}{90}, 16\right], \left[\frac{71}{105}, -2\right], \left[\frac{11}{90}, 16\right],$$

$$\left[\frac{47}{210}, 0\right], \left[\frac{19}{35}, 0\right], \left[\frac{67}{180}, 1\right], \left[\frac{29}{30}, 16\right], \left[\frac{23}{30}, 0\right], \left[\frac{19}{165}, -2\right], \left[\frac{17}{165}, 6\right], \left[\frac{13}{165}, 6\right], \left[\frac{1}{47}, 0\right], \left[\frac{1}{49}, -2\right],$$

$$\left[\frac{1}{51}, -12\right], \left[\frac{1}{53}, 0\right], \left[\frac{1}{57}, -12\right], \left[\frac{1}{59}, 0\right], \left[\frac{1}{170}, -4\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right], \left[\frac{1}{43}, 0\right], \left[\frac{26}{35}, 0\right], \left[\frac{101}{165}, -2\right],$$

$$\left[\frac{13}{150}, 0\right], \left[\frac{1}{165}, -2\right], \left[\frac{109}{120}, 9\right], \left[\frac{1}{27}, -12\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, -12\right], \left[\frac{24}{35}, 0\right], \left[\frac{67}{120}, 1\right], \left[\frac{18}{35}, 8\right],$$

$$\left[\frac{14}{15}, -2\right], \left[\frac{59}{120}, 9\right], \left[\frac{1}{7}, -2\right], \left[\frac{1}{9}, -12\right], \left[\frac{1}{11}, 0\right], \left[\frac{8}{15}, 6\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, -10\right], \left[\frac{1}{23}, 0\right],$$

$$\left[\frac{12}{35}, 8\right], \left[\frac{73}{105}, 2\right], [0, 0], \left[\frac{1}{3}, -12\right], \left[\frac{4}{35}, 0\right], \left[\frac{43}{195}, 6\right], \left[\frac{23}{133}, -2\right], \left[\frac{33}{35}, 8\right], \left[\frac{11}{195}, -2\right], \left[\frac{1}{195}, -2\right],$$

$$\left[\frac{19}{170}, -4\right], \left[\frac{37}{165}, 6\right], \left[\frac{27}{35}, 8\right], \left[\frac{29}{165}, -2\right], \left[\frac{97}{170}, 4\right], \left[\frac{23}{165}, 6\right], \left[\frac{13}{170}, 4\right], \left[\frac{1}{61}, 0\right], \left[\frac{1}{63}, -10\right], \left[\frac{13}{45}, 6\right],$$

$$\left[\frac{1}{58}, -20\right], \left[\frac{1}{62}, -20\right], \left[\frac{1}{4}, 5\right], \left[\frac{59}{190}, -4\right], \left[\frac{1}{8}, 5\right], \left[\frac{1}{54}, -4\right], \left[\frac{1}{42}, 0\right], \left[\frac{22}{35}, 8\right], \left[\frac{11}{45}, -2\right], \left[\frac{17}{190}, 4\right],$$

$$\left[\frac{1}{46}, -20\right], \left[\frac{1}{38}, -20\right], \left[\frac{1}{34}, -20\right], \left[\frac{37}{195}, 6\right], \left[\frac{1}{6}, -4\right], \left[\frac{1}{14}, -24\right], \left[\frac{1}{18}, -4\right], \left[\frac{1}{22}, -20\right], \left[\frac{16}{35}, 0\right],$$

$$\left[\frac{29}{195}, -2\right], \left[\frac{79}{105}, -2\right], \left[\frac{13}{190}, 4\right], \left[\frac{23}{195}, 6\right], \left[\frac{19}{195}, -2\right], \left[\frac{1}{2}, -20\right], \left[\frac{1}{190}, -4\right], \left[\frac{17}{195}, 6\right], \left[\frac{13}{135}, 6\right],$$

$$\left[\frac{1}{48}, 3\right], \left[\frac{1}{56}, 3\right], \left[\frac{1}{64}, 5\right], \left[\frac{11}{135}, -2\right], \left[\frac{1}{44}, 5\right], \left[\frac{1}{28}, 3\right], \left[\frac{1}{32}, 5\right], \left[\frac{1}{36}, 3\right], \left[\frac{13}{175}, 8\right], \left[\frac{17}{50}, 4\right],$$

$$\left[\frac{1}{135}, -2\right], \left[\frac{37}{45}, 6\right], \left[\frac{1}{24}, 3\right], \left[\frac{29}{45}, -2\right], \left[\frac{13}{50}, 4\right], \left[\frac{11}{175}, 0\right], \left[\frac{17}{150}, 0\right], \left[\frac{23}{45}, 6\right], \left[\frac{1}{12}, 3\right], \left[\frac{1}{16}, 5\right],$$

$$\left[\frac{1}{175}, 0\right], \left[\frac{19}{45}, -2\right], \left[\frac{34}{35}, 0\right], \left[\frac{17}{45}, 6\right], \left[\frac{23}{175}, 8\right], \left[\frac{23}{135}, 6\right], \left[\frac{7}{15}, 6\right], \left[\frac{4}{15}, -2\right], \left[\frac{1}{35}, 0\right], \left[\frac{3}{35}, 8\right],$$

$$\left[\frac{19}{135}, -2\right], \left[\frac{19}{175}, 0\right], \left[\frac{83}{105}, 2\right], \left[\frac{1}{15}, -2\right], \left[\frac{13}{15}, 6\right], \left[\frac{19}{50}, -4\right], \left[\frac{1}{5}, 4\right], \left[\frac{3}{5}, 0\right], \left[\frac{2}{5}, 0\right], \left[\frac{4}{5}, 4\right], \left[\frac{17}{175}, 8\right],$$

$$\left[\frac{19}{100}, 1\right], \left[\frac{8}{105}, 2\right], \left[\frac{41}{420}, 5\right], \left[\frac{11}{144}, 3\right], \left[\frac{3}{140}, 1\right], \left[\frac{11}{126}, 0\right], \left[\frac{17}{100}, 5\right], \left[\frac{121}{161}, -2\right], \left[\frac{1}{144}, 3\right],$$

$$\left[\frac{1}{126}, 0\right], \left[\frac{4}{105}, -2\right], \left[\frac{139}{140}, 9\right], \left[\frac{41}{175}, 0\right], \left[\frac{13}{75}, 6\right], \left[\frac{11}{75}, -2\right], \left[\frac{1}{55}, 4\right], \left[\frac{1}{25}, 4\right], \left[\frac{37}{175}, 8\right], \left[\frac{17}{135}, 6\right],$$

$$\left[\frac{37}{189}, -10\right], \left[\frac{41}{189}, -10\right], \left[\frac{31}{175}, 0\right], \left[\frac{32}{35}, 8\right], \left[\frac{9}{35}, 0\right], \left[\frac{1}{45}, -2\right], \left[\frac{37}{135}, 6\right], \left[\frac{29}{175}, 0\right], \left[\frac{29}{135}, -2\right],$$

$$\left[\frac{17}{75}, 6\right], \left[\frac{59}{175}, 0\right], \left[\frac{1}{30}, 16\right], \left[\frac{13}{30}, 0\right], \left[\frac{7}{30}, 0\right], \left[\frac{19}{30}, 16\right], \left[\frac{1}{40}, 1\right], \left[\frac{23}{75}, 6\right], \left[\frac{53}{175}, 8\right], \left[\frac{1}{10}, -4\right],$$

$$\left[\frac{3}{10}, 4\right], \left[\frac{7}{10}, 4\right], \left[\frac{9}{10}, -4\right], \left[\frac{47}{175}, 8\right], \left[\frac{19}{75}, -2\right], \left[\frac{1}{75}, -2\right], \left[\frac{43}{175}, 8\right], \left[\frac{73}{175}, 8\right], \left[\frac{67}{175}, 8\right], \left[\frac{3}{20}, 5\right],$$

$$\left[\frac{7}{20}, 5\right], \left[\frac{9}{20}, 1\right], \left[\frac{1}{20}, 1\right], \left[\frac{37}{75}, 6\right], \left[\frac{7}{60}, 1\right], \left[\frac{19}{60}, 9\right], \left[\frac{1}{50}, -4\right], \left[\frac{7}{40}, 5\right], \left[\frac{1}{60}, 9\right], \left[\frac{13}{60}, 1\right], \left[\frac{1}{161}, -2\right],$$

$$\left[\frac{61}{175}, 0\right], \left[\frac{29}{75}, -2\right], \left[\frac{109}{175}, 0\right], \left[\frac{19}{70}, 12\right], \left[\frac{17}{70}, -4\right], \left[\frac{1}{13}, 0\right], \left[\frac{103}{175}, 8\right], \left[\frac{13}{70}, -4\right], \left[\frac{11}{70}, 12\right], \left[\frac{97}{175}, 8\right],$$

$$\left[\frac{19}{150}, 16\right], \left[\frac{1}{70}, 12\right], \left[\frac{79}{175}, 0\right], \left[\frac{41}{70}, 12\right], \left[\frac{1}{140}, 9\right], \left[\frac{37}{70}, -4\right], \left[\frac{139}{175}, 0\right], \left[\frac{31}{70}, 12\right], \left[\frac{127}{175}, 8\right],$$

$$\left[\frac{29}{70}, 12\right], \left[\frac{121}{175}, 0\right], \left[\frac{23}{70}, -4\right], \left[\frac{89}{105}, -2\right], \left[\frac{53}{70}, -4\right], \left[\frac{97}{105}, 2\right], \left[\frac{47}{70}, -4\right], \left[\frac{13}{140}, 1\right], \left[\frac{23}{150}, 0\right],$$

$$\left[\frac{11}{161}, -2\right], \left[\frac{43}{70}, -4\right], \left[\frac{11}{140}, 9\right], \left[\frac{1}{39}, -12\right], \left[\frac{11}{39}, -12\right], \left[\frac{1}{52}, 5\right], \left[\frac{11}{20}, 1\right], \left[\frac{3}{70}, -4\right], \left[\frac{1}{26}, -20\right], \left[\frac{67}{70}, -4\right],$$

$$\left[\frac{19}{140}, 9\right], \left[\frac{5}{12}, 3\right], \left[\frac{11}{12}, 3\right], \left[\frac{61}{70}, 12\right], \left[\frac{17}{140}, 1\right], \left[\frac{59}{70}, 12\right], \left[\frac{31}{140}, 9\right], \left[\frac{29}{140}, 9\right], \left[\frac{1}{132}, 3\right], \left[\frac{39}{70}, 12\right],$$

$$\left[\frac{33}{70}, -4\right], \left[\frac{7}{12}, 3\right], \left[\frac{19}{20}, 1\right], \left[\frac{27}{70}, -4\right], \left[\frac{23}{140}, 1\right], \left[\frac{17}{20}, 5\right], \left[\frac{9}{70}, 12\right], \left[\frac{11}{52}, 5\right], \left[\frac{13}{20}, 5\right], \left[\frac{1}{156}, 3\right],$$

$$\left[\frac{101}{105}, -2\right], \left[\frac{41}{140}, 9\right], \left[\frac{11}{40}, 1\right], \left[\frac{69}{70}, 12\right], \left[\frac{19}{132}, 3\right], \left[\frac{37}{140}, 1\right], \left[\frac{57}{70}, -4\right], \left[\frac{13}{161}, -2\right], \left[\frac{17}{65}, 0\right], \left[\frac{19}{65}, 4\right],$$

$$\left[\frac{29}{150}, 16\right], \left[\frac{51}{70}, 12\right], \left[\frac{1}{65}, 4\right], \left[\frac{17}{132}, 3\right], \left[\frac{35}{132}, 3\right], \left[\frac{31}{189}, -10\right], \left[\frac{19}{40}, 1\right], \left[\frac{11}{156}, 3\right], \left[\frac{17}{40}, 5\right], \left[\frac{13}{40}, 5\right],$$

$$\left[\frac{43}{140}, 1\right], \left[\frac{19}{130}, -4\right], \left[\frac{19}{156}, 3\right], \left[\frac{29}{40}, 1\right], \left[\frac{17}{130}, 4\right], \left[\frac{17}{156}, 3\right], \left[\frac{53}{140}, 1\right], \left[\frac{23}{40}, 5\right], \left[\frac{47}{140}, 1\right], \left[\frac{67}{140}, 1\right],$$

$$\left[\frac{103}{105}, 2\right], \left[\frac{11}{204}, 3\right], \left[\frac{37}{150}, 0\right], \left[\frac{61}{140}, 9\right], \left[\frac{1}{160}, 1\right], \left[\frac{1}{204}, 3\right], \left[\frac{23}{189}, -10\right], \left[\frac{29}{189}, -10\right], \left[\frac{59}{140}, 9\right],$$

$$\left[\frac{73}{140}, 1\right], \left[\frac{29}{204}, 3\right], \left[\frac{17}{160}, 5\right], \left[\frac{71}{140}, 9\right], \left[\frac{17}{161}, -2\right], \left[\frac{13}{160}, 5\right], \left[\frac{11}{160}, 1\right], \left[\frac{29}{160}, 1\right], \left[\frac{83}{140}, 1\right],$$

$$\left[\frac{23}{160}, 5\right], \left[\frac{79}{140}, 9\right], \left[\frac{19}{204}, 3\right], \left[\frac{19}{160}, 1\right], \left[\frac{1}{200}, 1\right], \left[\frac{89}{140}, 9\right], \left[\frac{11}{42}, 0\right], \left[\frac{47}{160}, 5\right], \left[\frac{101}{140}, 9\right], \left[\frac{17}{420}, 1\right],$$

$$\left[\frac{19}{200}, 1\right], \left[\frac{13}{189}, -10\right], \left[\frac{17}{189}, -10\right], \left[\frac{19}{189}, -10\right], \left[\frac{17}{200}, 5\right], \left[\frac{17}{42}, 0\right], \left[\frac{13}{420}, 1\right], \left[\frac{13}{200}, 5\right], \left[\frac{17}{24}, 3\right],$$

$$\left[\frac{97}{140}, 1\right], \left[\frac{11}{24}, 3\right], \left[\frac{11}{200}, 1\right], \left[\frac{13}{42}, 0\right], \left[\frac{11}{420}, 5\right], \left[\frac{11}{72}, 3\right], \left[\frac{107}{140}, 1\right], \left[\frac{23}{42}, 0\right], \left[\frac{23}{200}, 5\right], \left[\frac{1}{72}, 3\right],$$

$$\left[\frac{103}{140}, 1\right], \left[\frac{19}{42}, 0\right], \left[\frac{19}{24}, 3\right], \left[\frac{29}{63}, -10\right], \left[\frac{31}{63}, -10\right], \left[\frac{37}{63}, -10\right], \left[\frac{41}{63}, -10\right], \left[\frac{29}{200}, 1\right], \left[\frac{29}{42}, 0\right], \left[\frac{109}{140}, 9\right],$$

$$\left[\frac{19}{72}, 3\right], \left[\frac{121}{140}, 9\right], \left[\frac{37}{42}, 0\right], \left[\frac{1}{100}, 1\right], \left[\frac{19}{63}, -10\right], \left[\frac{23}{63}, -10\right], \left[\frac{29}{420}, 5\right], \left[\frac{47}{200}, 5\right], \left[\frac{31}{42}, 0\right], \left[\frac{113}{140}, 1\right],$$

$$\left[\frac{23}{420}, 1\right], \left[\frac{17}{72}, 3\right], \left[\frac{13}{63}, -10\right], \left[\frac{31}{420}, 5\right], \left[\frac{127}{140}, 1\right], \left[\frac{1}{96}, 3\right], \left[\frac{47}{168}, 5\right], \left[\frac{53}{168}, 5\right], \left[\frac{17}{63}, -10\right],$$

$$\left[\frac{11}{63}, -10\right], \left[\frac{19}{420}, 5\right], \left[\frac{11}{100}, 1\right], \left[\frac{43}{168}, 5\right], \left[\frac{2}{105}, 2\right], \left[\frac{41}{42}, 0\right], \left[\frac{19}{96}, 3\right], \left[\frac{137}{140}, 1\right], \left[\frac{25}{42}, 0\right], \left[\frac{17}{96}, 3\right],$$

$$\left[\frac{5}{42}, 0\right], \left[\frac{37}{420}, 1\right], \left[\frac{19}{161}, -2\right], \left[\frac{131}{140}, 9\right], \left[\frac{11}{96}, 3\right], \left[\frac{1}{203}, -2\right], \left[\frac{17}{126}, 0\right], \left[\frac{23}{100}, 5\right], \left[\frac{17}{144}, 3\right], \left[\frac{13}{126}, 0\right],$$

$$\left[\frac{43}{420}, 1\right], \left[\frac{9}{140}, 9\right], \left[\frac{53}{420}, 1\right], \left[\frac{13}{100}, 5\right], \left[\frac{23}{126}, 0\right], \left[\frac{33}{140}, 1\right], \left[\frac{5}{21}, -10\right], \left[\frac{4}{21}, -10\right], \left[\frac{16}{105}, -2\right],$$

$$\left[\frac{47}{100}, 5\right], \left[\frac{29}{100}, 1\right], \left[\frac{47}{420}, 1\right], \left[\frac{19}{126}, 0\right], \left[\frac{27}{140}, 1\right], \left[\frac{19}{144}, 3\right], \left[\frac{13}{80}, 5\right], \left[\frac{31}{126}, 0\right], \left[\frac{17}{36}, 3\right], \left[\frac{51}{140}, 9\right],$$

$\left[\frac{29}{126}, 0\right], \left[\frac{11}{80}, 1\right], \left[\frac{11}{36}, 3\right], \left[\frac{39}{140}, 9\right], \left[\frac{1}{80}, 1\right], \left[\frac{67}{420}, 1\right], \left[\frac{19}{36}, 3\right], \left[\frac{57}{140}, 1\right], \left[\frac{19}{80}, 1\right], \left[\frac{37}{126}, 0\right],$
$\left[\frac{17}{80}, 5\right], \left[\frac{61}{420}, 5\right], \left[\frac{20}{21}, -10\right], \left[\frac{59}{420}, 5\right], \left[\frac{47}{80}, 5\right], \left[\frac{69}{140}, 9\right], \left[\frac{29}{80}, 1\right], \left[\frac{71}{420}, 5\right], \left[\frac{1}{108}, 3\right], \left[\frac{41}{126}, 0\right],$
$\left[\frac{16}{21}, -10\right], \left[\frac{23}{80}, 5\right], \left[\frac{8}{21}, -10\right], \left[\frac{41}{168}, 5\right], \left[\frac{10}{21}, -10\right], \left[\frac{73}{420}, 1\right], \left[\frac{31}{168}, 5\right], \left[\frac{37}{168}, 5\right], \left[\frac{79}{420}, 5\right],$
$\left[\frac{19}{21}, -10\right], \left[\frac{2}{21}, -10\right], \left[\frac{11}{108}, 3\right], \left[\frac{11}{203}, -2\right], \left[\frac{13}{21}, -10\right], \left[\frac{17}{21}, -10\right], \left[\frac{47}{126}, 0\right], \left[\frac{81}{140}, 9\right], \left[\frac{29}{168}, 5\right],$
$\left[\frac{83}{420}, 1\right], \left[\frac{11}{21}, -10\right], \left[\frac{11}{84}, 5\right], \left[\frac{97}{420}, 1\right], \left[\frac{1}{84}, 5\right], \left[\frac{17}{108}, 3\right], \left[\frac{23}{168}, 5\right], \left[\frac{89}{420}, 5\right], \left[\frac{19}{147}, -10\right], \left[\frac{19}{84}, 5\right],$
$\left[\frac{19}{108}, 3\right], \left[\frac{103}{420}, 1\right], \left[\frac{47}{189}, -10\right], \left[\frac{87}{140}, 1\right], \left[\frac{17}{84}, 5\right], \left[\frac{17}{168}, 5\right], \left[\frac{19}{168}, 5\right], \left[\frac{1}{189}, -10\right], \left[\frac{11}{189}, -10\right],$
$\left[\frac{22}{105}, 2\right], \left[\frac{101}{420}, 5\right], \left[\frac{47}{63}, -10\right], \left[\frac{4}{63}, -10\right], \left[\frac{67}{126}, 0\right], \left[\frac{29}{84}, 5\right], \left[\frac{109}{420}, 5\right], \left[\frac{1}{196}, 3\right], \left[\frac{13}{84}, 5\right], \left[\frac{25}{84}, 5\right],$
$\left[\frac{55}{84}, 5\right], \left[\frac{65}{84}, 5\right], \left[\frac{1}{168}, 5\right], \left[\frac{11}{168}, 5\right], \left[\frac{13}{168}, 5\right], \left[\frac{23}{84}, 5\right], \left[\frac{93}{140}, 1\right], \left[\frac{107}{420}, 1\right], \left[\frac{79}{84}, 5\right], \left[\frac{83}{84}, 5\right],$
$\left[\frac{5}{84}, 5\right], \left[\frac{31}{84}, 5\right], \left[\frac{157}{420}, 1\right], \left[\frac{163}{420}, 1\right], \left[\frac{113}{420}, 1\right], \left[\frac{99}{140}, 9\right], \left[\frac{59}{105}, -2\right], \left[\frac{31}{105}, -2\right], \left[\frac{73}{168}, 5\right], \left[\frac{79}{168}, 5\right],$
$\left[\frac{71}{168}, 5\right], \left[\frac{137}{420}, 1\right], \left[\frac{53}{84}, 5\right], \left[\frac{59}{84}, 5\right], \left[\frac{1}{192}, 3\right], \left[\frac{129}{140}, 9\right], \left[\frac{61}{84}, 5\right], \left[\frac{11}{192}, 3\right], \left[\frac{139}{420}, 5\right], \left[\frac{17}{192}, 3\right],$
$\left[\frac{73}{84}, 5\right], \left[\frac{26}{105}, -2\right], \left[\frac{71}{84}, 5\right], \left[\frac{143}{420}, 1\right], \left[\frac{19}{192}, 3\right], \left[\frac{67}{84}, 5\right], \left[\frac{111}{140}, 9\right], \left[\frac{11}{48}, 3\right], \left[\frac{41}{84}, 5\right], \left[\frac{121}{420}, 5\right],$
$\left[\frac{37}{84}, 5\right], \left[\frac{17}{48}, 3\right], \left[\frac{117}{140}, 1\right], \left[\frac{43}{84}, 5\right], \left[\frac{127}{420}, 1\right], \left[\frac{47}{84}, 5\right], \left[\frac{131}{420}, 5\right], \left[\frac{13}{133}, -2\right], \left[\frac{23}{77}, -2\right], \left[\frac{1}{91}, -2\right],$
$\left[\frac{11}{91}, -2\right], \left[\frac{69}{91}, -2\right], \left[\frac{17}{91}, -2\right], \left[\frac{19}{91}, -2\right], \left[\frac{23}{91}, -2\right], \left[\frac{46}{77}, -2\right], \left[\frac{13}{77}, -2\right], \left[\frac{17}{77}, -2\right], \left[\frac{19}{77}, -2\right], \left[\frac{1}{119}, -2\right],$
$\left[\frac{11}{119}, -2\right], \left[\frac{13}{119}, -2\right], \left[\frac{80}{119}, -2\right], \left[\frac{19}{119}, -2\right], \left[\frac{23}{119}, -2\right], \left[\frac{1}{133}, -2\right], \left[\frac{11}{133}, -2\right], \left[\frac{19}{48}, 3\right], \left[\frac{4}{7}, -2\right],$
$\left[\frac{6}{7}, -2\right], \left[\frac{3}{7}, -2\right], \left[\frac{5}{7}, -2\right], \left[\frac{2}{7}, -2\right], \left[\frac{1}{77}, -2\right], \left[\frac{123}{140}, 1\right], \left[\frac{35}{198}, -4\right], \left[\frac{1}{198}, -4\right], \left[\frac{11}{102}, -4\right], \left[\frac{1}{114}, -4\right],$
$\left[\frac{11}{114}, -4\right], \left[\frac{1}{138}, -4\right], \left[\frac{11}{138}, -4\right], \left[\frac{1}{174}, -4\right], \left[\frac{11}{174}, -4\right], \left[\frac{1}{186}, -4\right], \left[\frac{11}{186}, -4\right], \left[\frac{11}{18}, -4\right], \left[\frac{11}{54}, -4\right],$
$\left[\frac{1}{162}, -4\right], \left[\frac{11}{162}, -4\right], \left[\frac{5}{6}, -4\right], \left[\frac{1}{66}, -4\right], \left[\frac{35}{66}, -4\right], \left[\frac{1}{78}, -4\right], \left[\frac{11}{78}, -4\right], \left[\frac{1}{102}, -4\right], \left[\frac{1}{125}, 4\right], \left[\frac{13}{125}, 0\right],$
$\left[\frac{17}{125}, 0\right], \left[\frac{19}{125}, 4\right], \left[\frac{13}{205}, 0\right], \left[\frac{17}{205}, 0\right], \left[\frac{19}{205}, 4\right], \left[\frac{13}{25}, 0\right], \left[\frac{17}{25}, 0\right], \left[\frac{19}{25}, 4\right], \left[\frac{17}{155}, 0\right], \left[\frac{19}{155}, 4\right],$

$$\left[\frac{1}{185}, 4\right], \left[\frac{13}{185}, 0\right], \left[\frac{17}{185}, 0\right], \left[\frac{19}{185}, 4\right], \left[\frac{1}{205}, 4\right], \left[\frac{1}{85}, 4\right], \left[\frac{13}{85}, 0\right], \left[\frac{12}{85}, 0\right], \left[\frac{19}{85}, 4\right], \left[\frac{1}{95}, 4\right],$$

$$\left[\frac{13}{95}, 0\right], \left[\frac{17}{95}, 0\right], \left[\frac{59}{95}, 4\right], \left[\frac{43}{65}, 0\right], \left[\frac{13}{55}, 0\right], \left[\frac{17}{55}, 0\right], \left[\frac{19}{55}, 4\right], \left[\frac{11}{184}, 5\right], \left[\frac{1}{208}, 5\right], \left[\frac{11}{208}, 5\right],$$

$$\left[\frac{1}{176}, 5\right], \left[\frac{79}{176}, 5\right], \left[\frac{1}{184}, 5\right], \left[\frac{11}{104}, 5\right], \left[\frac{1}{128}, 5\right], \left[\frac{11}{128}, 5\right], \left[\frac{1}{136}, 5\right], \left[\frac{11}{136}, 5\right], \left[\frac{1}{152}, 5\right],$$

$$\left[\frac{11}{152}, 5\right], \left[\frac{1}{115}, 4\right], \left[\frac{13}{115}, 0\right], \left[\frac{17}{115}, 0\right], \left[\frac{19}{115}, 4\right], \left[\frac{1}{145}, 4\right], \left[\frac{13}{145}, 0\right], \left[\frac{17}{145}, 0\right], \left[\frac{19}{145}, 4\right],$$

$$\left[\frac{1}{155}, 4\right], \left[\frac{13}{155}, 0\right], \left[\frac{11}{164}, 5\right], \left[\frac{1}{172}, 5\right], \left[\frac{1}{116}, 5\right], \left[\frac{11}{116}, 5\right], \left[\frac{1}{124}, 5\right], \left[\frac{11}{124}, 5\right], \left[\frac{1}{148}, 5\right],$$

$$\left[\frac{11}{148}, 5\right], \left[\frac{1}{164}, 5\right], \left[\frac{35}{44}, 5\right], \left[\frac{1}{68}, 5\right], \left[\frac{11}{68}, 5\right], \left[\frac{1}{76}, 5\right], \left[\frac{11}{76}, 5\right], \left[\frac{1}{92}, 5\right], \left[\frac{11}{92}, 5\right], \left[\frac{3}{4}, 5\right],$$

$$\left[\frac{11}{207}, -12\right], \left[\frac{1}{117}, -12\right], \left[\frac{11}{117}, -12\right], \left[\frac{1}{153}, -12\right], \left[\frac{11}{153}, -12\right], \left[\frac{1}{171}, -12\right], \left[\frac{11}{171}, -12\right], \left[\frac{1}{207}, -12\right],$$

$$\left[\frac{1}{99}, -12\right], \left[\frac{35}{99}, -12\right], \left[\frac{11}{201}, -12\right], \left[\frac{2}{9}, -12\right], \left[\frac{11}{27}, -12\right], \left[\frac{1}{81}, -12\right], \left[\frac{11}{81}, -12\right], \left[\frac{11}{123}, -12\right],$$

$$\left[\frac{1}{129}, -12\right], \left[\frac{11}{129}, -12\right], \left[\frac{1}{141}, -12\right], \left[\frac{11}{141}, -12\right], \left[\frac{1}{87}, -12\right], \left[\frac{11}{87}, -12\right], \left[\frac{1}{93}, -12\right], \left[\frac{11}{93}, -12\right],$$

$$\left[\frac{1}{111}, -12\right], \left[\frac{11}{111}, -12\right], \left[\frac{1}{123}, -12\right], \left[\frac{11}{51}, -12\right], \left[\frac{11}{57}, -12\right], \left[\frac{1}{69}, -12\right], \left[\frac{11}{69}, -12\right], \left[\frac{1}{159}, -12\right],$$

$$\left[\frac{11}{159}, -12\right], \left[\frac{1}{177}, -12\right], \left[\frac{11}{177}, -12\right], \left[\frac{1}{183}, -12\right], \left[\frac{11}{183}, -12\right], \left[\frac{1}{201}, -12\right], \left[\frac{11}{172}, 5\right], \left[\frac{1}{188}, 5\right],$$

$$\left[\frac{11}{188}, 5\right], \left[\frac{3}{8}, 5\right], \left[\frac{11}{16}, 5\right], \left[\frac{11}{32}, 5\right], \left[\frac{11}{64}, 5\right], \left[\frac{1}{88}, 5\right], \left[\frac{79}{88}, 5\right], \left[\frac{1}{104}, 5\right], \left[\frac{2}{3}, -12\right], \left[\frac{2}{33}, -12\right],$$

$$\left[\frac{1}{202}, -20\right], \left[\frac{1}{206}, -20\right], \left[\frac{1}{122}, -20\right], \left[\frac{1}{134}, -20\right], \left[\frac{1}{142}, -20\right], \left[\frac{1}{146}, -20\right], \left[\frac{1}{158}, -20\right], \left[\frac{1}{166}, -20\right],$$

$$\left[\frac{1}{178}, -20\right], \left[\frac{1}{194}, -20\right], \left[\frac{1}{74}, -20\right], \left[\frac{1}{82}, -20\right], \left[\frac{1}{86}, -20\right], \left[\frac{1}{94}, -20\right], \left[\frac{1}{106}, -20\right], \left[\frac{1}{118}, -20\right],$$

$$\left[\frac{1}{181}, 0\right], \left[\frac{1}{187}, 0\right], \left[\frac{1}{191}, 0\right], \left[\frac{1}{193}, 0\right], \left[\frac{1}{197}, 0\right], \left[\frac{1}{199}, 0\right], \left[\frac{1}{209}, 0\right], \left[\frac{1}{169}, 0\right], \left[\frac{1}{173}, 0\right],$$

$$\left[\frac{1}{179}, 0\right], \left[\frac{1}{139}, 0\right], \left[\frac{1}{143}, 0\right], \left[\frac{1}{149}, 0\right], \left[\frac{1}{151}, 0\right], \left[\frac{1}{157}, 0\right], \left[\frac{1}{163}, 0\right], \left[\frac{1}{167}, 0\right], \left[\frac{1}{121}, 0\right],$$

$$\left[\frac{1}{127}, 0\right], \left[\frac{1}{131}, 0\right], \left[\frac{1}{137}, 0\right], \left[\frac{1}{89}, 0\right], \left[\frac{1}{97}, 0\right], \left[\frac{1}{101}, 0\right], \left[\frac{1}{103}, 0\right], \left[\frac{1}{107}, 0\right], \left[\frac{1}{109}, 0\right], \left[\frac{1}{113}, 0\right],$$

$$\left[\frac{1}{73}, 0\right], \left[\frac{1}{79}, 0\right], \left[\frac{1}{83}, 0\right], \left[\frac{1}{67}, 0\right], \left[\frac{1}{71}, 0\right], \left[\frac{149}{420}, 5\right], \left[\frac{151}{420}, 5\right]]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 420

"val0=", 0

"which is even."

"valinf=", 10

"which is even."

"It IS a modfunc on Gamma1(", 420, ")"

```
"TERM ", 3, "of ", 4, " *****************************************************************"
```

"XX=", 1

```
"TERM ", 4, "of ", 4, " *****************************************************************"
```

"XX=", $-\mathrm{JAC}(1,84,\infty)^2\,\mathrm{JAC}(5,84,\infty)^2\,\mathrm{JAC}(7,84,\infty)^4\,\mathrm{JAC}(11,84,\infty)^2\,\mathrm{JAC}(13,84,\infty)^2\,\mathrm{JAC}(17,84,\infty)^2\,\mathrm{JAC}(19,84,\infty)^2\,\mathrm{JAC}(23,84,\infty)^2\,\mathrm{JAC}(25,84,\infty)^2\,\mathrm{JAC}(29,84,\infty)^2\,\mathrm{JAC}(31,84,\infty)^2\,\mathrm{JAC}(35,84,\infty)^4\,\mathrm{JAC}(37,84,\infty)^2\,\mathrm{JAC}(41,84,\infty)^2 \Big/ (\mathrm{JAC}(0,84,\infty)^{24}\,\mathrm{JAC}(2,84,\infty)\,\mathrm{JAC}(10,84,\infty)\,\mathrm{JAC}(14,84,\infty)^2\,\mathrm{JAC}(22,84,\infty)\,\mathrm{JAC}(26,84,\infty)\,\mathrm{JAC}(34,84,\infty)\,\mathrm{JAC}(38,84,\infty))$

"Cusp ORDS: "

$\left[[oo,0],\left[\frac{1}{147},-10\right],\left[\frac{19}{203},10\right],\left[\frac{59}{168},0\right],\left[\frac{17}{203},10\right],\left[\frac{32}{105},-2\right],\left[\frac{67}{189},-10\right],\left[\frac{13}{203},10\right],\left[\frac{13}{49},10\right],\right.$
$\left[\frac{52}{105},-2\right],\left[\frac{11}{49},10\right],\left[\frac{13}{147},-10\right],\left[\frac{46}{105},-2\right],\left[\frac{37}{90},4\right],\left[\frac{44}{105},-2\right],\left[\frac{23}{203},10\right],\left[\frac{11}{147},-10\right],\left[\frac{38}{105},-2\right],$
$\left[\frac{34}{105},-2\right],\left[\frac{74}{105},-2\right],\left[\frac{13}{14},-20\right],\left[\frac{11}{14},-20\right],\left[\frac{47}{147},-10\right],\left[\frac{41}{147},-10\right],\left[\frac{68}{105},-2\right],\left[\frac{83}{168},0\right],$
$\left[\frac{37}{147},-10\right],\left[\frac{23}{49},10\right],\left[\frac{31}{147},-10\right],\left[\frac{64}{105},-2\right],\left[\frac{19}{49},10\right],\left[\frac{29}{147},-10\right],\left[\frac{61}{105},-2\right],\left[\frac{23}{147},-10\right],$
$\left[\frac{62}{105},-2\right],\left[\frac{67}{168},0\right],\left[\frac{17}{147},-10\right],\left[\frac{58}{105},-2\right],\left[\frac{17}{49},10\right],\left[\frac{61}{168},0\right],\left[\frac{17}{60},0\right],\left[\frac{92}{105},-2\right],\left[\frac{17}{154},-20\right],$
$\left[\frac{13}{154},-20\right],\left[\frac{11}{60},0\right],\left[\frac{88}{105},-2\right],\left[\frac{123}{154},-20\right],\left[\frac{1}{150},4\right],\left[\frac{86}{105},-2\right],\left[\frac{169}{420},0\right],\left[\frac{1}{154},-20\right],\left[\frac{149}{168},0\right],$
$\left[\frac{9}{14},-20\right],\left[\frac{82}{105},-2\right],\left[\frac{139}{168},0\right],\left[\frac{43}{130},-4\right],\left[\frac{5}{14},-20\right],\left[\frac{76}{105},-2\right],\left[\frac{109}{168},0\right],\left[\frac{3}{14},-20\right],\left[\frac{17}{133},10\right],$
$\left[\frac{67}{147},-10\right],\left[\frac{89}{168},0\right],\left[\frac{5}{28},0\right],\left[\frac{41}{60},0\right],\left[\frac{25}{28},0\right],\left[\frac{197}{420},0\right],\left[\frac{15}{28},0\right],\left[\frac{9}{28},0\right],\left[\frac{3}{28},0\right],\left[\frac{37}{60},0\right],$
$\left[\frac{17}{182},-20\right],\left[\frac{23}{28},0\right],\left[\frac{193}{420},0\right],\left[\frac{69}{182},-20\right],\left[\frac{17}{28},0\right],\left[\frac{19}{28},0\right],\left[\frac{13}{28},0\right],\left[\frac{67}{105},-2\right],\left[\frac{31}{60},0\right],\left[\frac{191}{420},0\right],$

$$\left[\frac{187}{420}, 0\right], \left[\frac{11}{28}, 0\right], \left[\frac{181}{420}, 0\right], \left[\frac{173}{420}, 0\right], \left[\frac{179}{420}, 0\right], \left[\frac{104}{105}, -2\right], \left[\frac{167}{420}, 0\right], \left[\frac{1}{182}, -20\right], \left[\frac{23}{60}, 0\right],$$

$$\left[\frac{94}{105}, -2\right], \left[\frac{23}{154}, -20\right], \left[\frac{19}{154}, -20\right], \left[\frac{17}{180}, 0\right], \left[\frac{29}{60}, 0\right], \left[\frac{83}{112}, 0\right], \left[\frac{19}{98}, -20\right], \left[\frac{11}{150}, 4\right], \left[\frac{13}{180}, 0\right],$$

$$\left[\frac{61}{112}, 0\right], \left[\frac{53}{112}, 0\right], \left[\frac{11}{210}, 4\right], \left[\frac{43}{112}, 0\right], \left[\frac{11}{180}, 0\right], \left[\frac{31}{112}, 0\right], \left[\frac{37}{112}, 0\right], \left[\frac{17}{98}, -20\right], \left[\frac{1}{180}, 0\right],$$

$$\left[\frac{1}{210}, 4\right], \left[\frac{13}{98}, -20\right], \left[\frac{11}{182}, -20\right], \left[\frac{17}{112}, 0\right], \left[\frac{19}{112}, 0\right], \left[\frac{23}{112}, 0\right], \left[\frac{11}{98}, -20\right], \left[\frac{49}{60}, 0\right], \left[\frac{13}{112}, 0\right],$$

$$\left[\frac{11}{112}, 0\right], \left[\frac{1}{112}, 0\right], \left[\frac{27}{56}, 0\right], \left[\frac{5}{56}, 0\right], \left[\frac{59}{60}, 0\right], \left[\frac{53}{56}, 0\right], \left[\frac{40}{133}, 10\right], \left[\frac{1}{98}, -20\right], \left[\frac{37}{56}, 0\right], \left[\frac{43}{56}, 0\right],$$

$$\left[\frac{53}{60}, 0\right], \left[\frac{23}{56}, 0\right], \left[\frac{31}{56}, 0\right], \left[\frac{19}{56}, 0\right], \left[\frac{17}{56}, 0\right], \left[\frac{13}{56}, 0\right], \left[\frac{47}{60}, 0\right], \left[\frac{23}{182}, -20\right], \left[\frac{11}{56}, 0\right], \left[\frac{19}{182}, -20\right],$$

$$\left[\frac{43}{60}, 0\right], \left[\frac{199}{420}, 0\right], \left[\frac{209}{420}, 0\right], \left[\frac{27}{28}, 0\right], \left[\frac{17}{30}, 4\right], \left[\frac{59}{180}, 0\right], \left[\frac{11}{30}, 4\right], \left[\frac{43}{210}, 4\right], \left[\frac{17}{35}, 2\right], \left[\frac{53}{180}, 0\right],$$

$$\left[\frac{41}{210}, 4\right], \left[\frac{47}{180}, 0\right], \left[\frac{13}{35}, 2\right], \left[\frac{37}{210}, 4\right], \left[\frac{43}{180}, 0\right], \left[\frac{83}{196}, 0\right], \left[\frac{11}{35}, 2\right], \left[\frac{31}{210}, 4\right], \left[\frac{61}{196}, 0\right], \left[\frac{41}{180}, 0\right],$$

$$\left[\frac{37}{196}, 0\right], \left[\frac{43}{196}, 0\right], \left[\frac{53}{196}, 0\right], \left[\frac{37}{180}, 0\right], \left[\frac{23}{196}, 0\right], \left[\frac{31}{196}, 0\right], \left[\frac{29}{210}, 4\right], \left[\frac{31}{180}, 0\right], \left[\frac{23}{210}, 4\right],$$

$$\left[\frac{29}{180}, 0\right], \left[\frac{17}{196}, 0\right], \left[\frac{19}{196}, 0\right], \left[\frac{19}{210}, 4\right], \left[\frac{23}{180}, 0\right], \left[\frac{13}{196}, 0\right], \left[\frac{11}{196}, 0\right], \left[\frac{17}{210}, 4\right], \left[\frac{19}{180}, 0\right],$$

$$\left[\frac{13}{210}, 4\right], \left[\frac{23}{98}, -20\right], \left[\frac{103}{210}, 4\right], \left[\frac{53}{120}, 0\right], \left[\frac{47}{120}, 0\right], \left[\frac{2}{15}, -2\right], \left[\frac{8}{35}, 2\right], \left[\frac{101}{210}, 4\right], \left[\frac{43}{120}, 0\right], \left[\frac{97}{210}, 4\right],$$

$$\left[\frac{1}{130}, -4\right], \left[\frac{11}{15}, -2\right], \left[\frac{41}{120}, 0\right], \left[\frac{89}{210}, 4\right], \left[\frac{37}{120}, 0\right], \left[\frac{2}{35}, 2\right], \left[\frac{31}{120}, 0\right], \left[\frac{6}{35}, 2\right], \left[\frac{53}{105}, -2\right], \left[\frac{83}{210}, 4\right],$$

$$\left[\frac{29}{120}, 0\right], \left[\frac{43}{105}, -2\right], \left[\frac{47}{105}, -2\right], \left[\frac{41}{105}, -2\right], \left[\frac{79}{210}, 4\right], \left[\frac{19}{110}, -4\right], \left[\frac{37}{105}, -2\right], \left[\frac{73}{210}, 4\right], \left[\frac{23}{120}, 0\right],$$

$$\left[\frac{13}{90}, 4\right], \left[\frac{19}{105}, -2\right], \left[\frac{23}{105}, -2\right], \left[\frac{29}{105}, -2\right], \left[\frac{71}{210}, 4\right], \left[\frac{17}{105}, -2\right], \left[\frac{13}{105}, -2\right], \left[\frac{19}{120}, 0\right], \left[\frac{17}{110}, -4\right],$$

$$\left[\frac{17}{120}, 0\right], \left[\frac{67}{210}, 4\right], \left[\frac{31}{35}, 2\right], \left[\frac{11}{105}, -2\right], \left[\frac{29}{90}, 4\right], \left[\frac{61}{210}, 4\right], \left[\frac{1}{105}, -2\right], \left[\frac{23}{90}, 4\right], \left[\frac{13}{120}, 0\right], \left[\frac{59}{210}, 4\right],$$

$$\left[\frac{19}{90}, 4\right], \left[\frac{11}{120}, 0\right], \left[\frac{17}{90}, 4\right], \left[\frac{13}{110}, -4\right], \left[\frac{1}{120}, 0\right], \left[\frac{29}{35}, 2\right], \left[\frac{53}{210}, 4\right], \left[\frac{23}{35}, 2\right], \left[\frac{1}{110}, -4\right], \left[\frac{109}{180}, 0\right],$$

$$\left[\frac{1}{90}, 4\right], \left[\frac{71}{105}, -2\right], \left[\frac{11}{90}, 4\right], \left[\frac{47}{210}, 4\right], \left[\frac{19}{35}, 2\right], \left[\frac{67}{180}, 0\right], \left[\frac{29}{30}, 4\right], \left[\frac{23}{30}, 4\right], \left[\frac{19}{165}, -2\right], \left[\frac{17}{165}, -2\right],$$

$$\left[\frac{13}{165}, -2\right], \left[\frac{1}{47}, 10\right], \left[\frac{1}{49}, 10\right], \left[\frac{1}{51}, -10\right], \left[\frac{1}{53}, 10\right], \left[\frac{1}{57}, -10\right], \left[\frac{1}{59}, 10\right], \left[\frac{1}{170}, -4\right], \left[\frac{1}{37}, 10\right],$$

$$\left[\frac{1}{41}, 10\right], \left[\frac{1}{43}, 10\right], \left[\frac{26}{35}, 2\right], \left[\frac{101}{165}, -2\right], \left[\frac{13}{150}, 4\right], \left[\frac{1}{165}, -2\right], \left[\frac{109}{120}, 0\right], \left[\frac{1}{27}, -10\right], \left[\frac{1}{29}, 10\right],$$

$$\left[\frac{1}{31}, 10\right], \left[\frac{1}{33}, -10\right], \left[\frac{24}{35}, 2\right], \left[\frac{67}{120}, 0\right], \left[\frac{18}{35}, 2\right], \left[\frac{14}{15}, -2\right], \left[\frac{59}{120}, 0\right], \left[\frac{1}{7}, 10\right], \left[\frac{1}{9}, -10\right], \left[\frac{1}{11}, 10\right],$$

$$\left[\frac{8}{15}, -2\right], \left[\frac{1}{17}, 10\right], \left[\frac{1}{19}, 10\right], \left[\frac{1}{21}, -10\right], \left[\frac{1}{23}, 10\right], \left[\frac{12}{35}, 2\right], \left[\frac{73}{105}, -2\right], [0, 10], \left[\frac{1}{3}, -10\right], \left[\frac{4}{35}, 2\right],$$

$$\left[\frac{43}{195}, -2\right], \left[\frac{23}{133}, 10\right], \left[\frac{33}{35}, 2\right], \left[\frac{11}{195}, -2\right], \left[\frac{1}{195}, -2\right], \left[\frac{19}{170}, -4\right], \left[\frac{37}{165}, -2\right], \left[\frac{27}{35}, 2\right], \left[\frac{29}{165}, -2\right],$$

$$\left[\frac{97}{170}, -4\right], \left[\frac{23}{165}, -2\right], \left[\frac{13}{170}, -4\right], \left[\frac{1}{61}, 10\right], \left[\frac{1}{63}, -10\right], \left[\frac{13}{45}, -2\right], \left[\frac{1}{58}, -20\right], \left[\frac{1}{62}, -20\right], \left[\frac{1}{4}, 0\right],$$

$$\left[\frac{59}{190}, -4\right], \left[\frac{1}{8}, 0\right], \left[\frac{1}{54}, 20\right], \left[\frac{1}{42}, 20\right], \left[\frac{22}{35}, 2\right], \left[\frac{11}{45}, -2\right], \left[\frac{17}{190}, -4\right], \left[\frac{1}{46}, -20\right], \left[\frac{1}{38}, -20\right],$$

$$\left[\frac{1}{34}, -20\right], \left[\frac{37}{195}, -2\right], \left[\frac{1}{6}, 20\right], \left[\frac{1}{14}, -20\right], \left[\frac{1}{18}, 20\right], \left[\frac{1}{22}, -20\right], \left[\frac{16}{35}, 2\right], \left[\frac{29}{195}, -2\right], \left[\frac{79}{105}, -2\right],$$

$$\left[\frac{13}{190}, -4\right], \left[\frac{23}{195}, -2\right], \left[\frac{19}{195}, -2\right], \left[\frac{1}{2}, -20\right], \left[\frac{1}{190}, -4\right], \left[\frac{17}{195}, -2\right], \left[\frac{13}{135}, -2\right], \left[\frac{1}{48}, 0\right], \left[\frac{1}{56}, 0\right],$$

$$\left[\frac{1}{64}, 0\right], \left[\frac{11}{135}, -2\right], \left[\frac{1}{44}, 0\right], \left[\frac{1}{28}, 0\right], \left[\frac{1}{32}, 0\right], \left[\frac{1}{36}, 0\right], \left[\frac{13}{175}, 2\right], \left[\frac{17}{50}, -4\right], \left[\frac{1}{135}, -2\right], \left[\frac{37}{45}, -2\right],$$

$$\left[\frac{1}{24}, 0\right], \left[\frac{29}{45}, -2\right], \left[\frac{13}{50}, -4\right], \left[\frac{11}{175}, 2\right], \left[\frac{17}{150}, 4\right], \left[\frac{23}{45}, -2\right], \left[\frac{1}{12}, 0\right], \left[\frac{1}{16}, 0\right], \left[\frac{1}{175}, 2\right], \left[\frac{19}{45}, -2\right],$$

$$\left[\frac{34}{35}, 2\right], \left[\frac{17}{45}, -2\right], \left[\frac{23}{175}, 2\right], \left[\frac{23}{135}, -2\right], \left[\frac{7}{15}, -2\right], \left[\frac{4}{15}, -2\right], \left[\frac{1}{35}, 2\right], \left[\frac{3}{35}, 2\right], \left[\frac{19}{135}, -2\right], \left[\frac{19}{175}, 2\right],$$

$$\left[\frac{83}{105}, -2\right], \left[\frac{1}{15}, -2\right], \left[\frac{13}{15}, -2\right], \left[\frac{19}{50}, -4\right], \left[\frac{1}{5}, 2\right], \left[\frac{3}{5}, 2\right], \left[\frac{2}{5}, 2\right], \left[\frac{4}{5}, 2\right], \left[\frac{17}{175}, 2\right], \left[\frac{19}{100}, 0\right],$$

$$\left[\frac{8}{105}, -2\right], \left[\frac{41}{420}, 0\right], \left[\frac{11}{144}, 0\right], \left[\frac{3}{140}, 0\right], \left[\frac{11}{126}, 20\right], \left[\frac{17}{100}, 0\right], \left[\frac{121}{161}, 10\right], \left[\frac{1}{144}, 0\right], \left[\frac{1}{126}, 20\right],$$

$$\left[\frac{4}{105}, -2\right], \left[\frac{139}{140}, 0\right], \left[\frac{41}{175}, 2\right], \left[\frac{13}{75}, -2\right], \left[\frac{11}{75}, -2\right], \left[\frac{1}{55}, 2\right], \left[\frac{1}{25}, 2\right], \left[\frac{37}{175}, 2\right], \left[\frac{17}{135}, -2\right],$$

$$\left[\frac{37}{189}, -10\right], \left[\frac{41}{189}, -10\right], \left[\frac{31}{175}, 2\right], \left[\frac{32}{35}, 2\right], \left[\frac{9}{35}, 2\right], \left[\frac{1}{45}, -2\right], \left[\frac{37}{135}, -2\right], \left[\frac{29}{175}, 2\right], \left[\frac{29}{135}, -2\right],$$

$$\left[\frac{17}{75}, -2\right], \left[\frac{59}{175}, 2\right], \left[\frac{1}{30}, 4\right], \left[\frac{13}{30}, 4\right], \left[\frac{7}{30}, 4\right], \left[\frac{19}{30}, 4\right], \left[\frac{1}{40}, 0\right], \left[\frac{23}{75}, -2\right], \left[\frac{53}{175}, 2\right], \left[\frac{1}{10}, -4\right],$$

$$\left[\frac{3}{10}, -4\right], \left[\frac{7}{10}, -4\right], \left[\frac{9}{10}, -4\right], \left[\frac{47}{175}, 2\right], \left[\frac{19}{75}, -2\right], \left[\frac{1}{75}, -2\right], \left[\frac{43}{175}, 2\right], \left[\frac{73}{175}, 2\right], \left[\frac{67}{175}, 2\right], \left[\frac{3}{20}, 0\right],$$

$$\left[\frac{7}{20}, 0\right], \left[\frac{9}{20}, 0\right], \left[\frac{1}{20}, 0\right], \left[\frac{37}{75}, -2\right], \left[\frac{7}{60}, 0\right], \left[\frac{19}{60}, 0\right], \left[\frac{1}{50}, -4\right], \left[\frac{7}{40}, 0\right], \left[\frac{1}{60}, 0\right], \left[\frac{13}{60}, 0\right],$$

$$\left[\frac{1}{161}, 10\right], \left[\frac{61}{175}, 2\right], \left[\frac{29}{75}, -2\right], \left[\frac{109}{175}, 2\right], \left[\frac{19}{70}, -4\right], \left[\frac{17}{70}, -4\right], \left[\frac{1}{13}, 10\right], \left[\frac{103}{175}, 2\right], \left[\frac{13}{70}, -4\right], \left[\frac{11}{70}, -4\right],$$

$$\left[\frac{97}{175}, 2\right], \left[\frac{19}{150}, 4\right], \left[\frac{1}{70}, -4\right], \left[\frac{79}{175}, 2\right], \left[\frac{41}{70}, -4\right], \left[\frac{1}{140}, 0\right], \left[\frac{37}{70}, -4\right], \left[\frac{139}{175}, 2\right], \left[\frac{31}{70}, -4\right], \left[\frac{127}{175}, 2\right],$$

$$\left[\frac{29}{70}, -4\right], \left[\frac{121}{175}, 2\right], \left[\frac{23}{70}, -4\right], \left[\frac{89}{105}, -2\right], \left[\frac{53}{70}, -4\right], \left[\frac{97}{105}, -2\right], \left[\frac{47}{70}, -4\right], \left[\frac{13}{140}, 0\right], \left[\frac{23}{150}, 4\right],$$

$$\left[\frac{11}{161}, 10\right], \left[\frac{43}{70}, -4\right], \left[\frac{11}{140}, 0\right], \left[\frac{1}{39}, -10\right], \left[\frac{11}{39}, -10\right], \left[\frac{1}{52}, 0\right], \left[\frac{11}{20}, 0\right], \left[\frac{3}{70}, -4\right], \left[\frac{1}{26}, -20\right],$$

$$\left[\frac{67}{70}, -4\right], \left[\frac{19}{140}, 0\right], \left[\frac{5}{12}, 0\right], \left[\frac{11}{12}, 0\right], \left[\frac{61}{70}, -4\right], \left[\frac{17}{140}, 0\right], \left[\frac{59}{70}, -4\right], \left[\frac{31}{140}, 0\right], \left[\frac{29}{140}, 0\right], \left[\frac{1}{132}, 0\right],$$

$$\left[\frac{39}{70}, -4\right], \left[\frac{33}{70}, -4\right], \left[\frac{7}{12}, 0\right], \left[\frac{19}{20}, 0\right], \left[\frac{27}{70}, -4\right], \left[\frac{23}{140}, 0\right], \left[\frac{17}{20}, 0\right], \left[\frac{9}{70}, -4\right], \left[\frac{11}{52}, 0\right], \left[\frac{13}{20}, 0\right],$$

$$\left[\frac{1}{156}, 0\right], \left[\frac{101}{105}, -2\right], \left[\frac{41}{140}, 0\right], \left[\frac{11}{40}, 0\right], \left[\frac{69}{70}, -4\right], \left[\frac{19}{132}, 0\right], \left[\frac{37}{140}, 0\right], \left[\frac{57}{70}, -4\right], \left[\frac{13}{161}, 10\right], \left[\frac{17}{65}, 2\right],$$

$$\left[\frac{19}{65}, 2\right], \left[\frac{29}{150}, 4\right], \left[\frac{51}{70}, -4\right], \left[\frac{1}{65}, 2\right], \left[\frac{17}{132}, 0\right], \left[\frac{35}{132}, 0\right], \left[\frac{31}{189}, -10\right], \left[\frac{19}{40}, 0\right], \left[\frac{11}{156}, 0\right], \left[\frac{17}{40}, 0\right],$$

$$\left[\frac{13}{40}, 0\right], \left[\frac{43}{140}, 0\right], \left[\frac{19}{130}, -4\right], \left[\frac{19}{156}, 0\right], \left[\frac{29}{40}, 0\right], \left[\frac{17}{130}, -4\right], \left[\frac{17}{156}, 0\right], \left[\frac{53}{140}, 0\right], \left[\frac{23}{40}, 0\right], \left[\frac{47}{140}, 0\right],$$

$$\left[\frac{67}{140}, 0\right], \left[\frac{103}{105}, -2\right], \left[\frac{11}{204}, 0\right], \left[\frac{37}{150}, 4\right], \left[\frac{61}{140}, 0\right], \left[\frac{1}{160}, 0\right], \left[\frac{1}{204}, 0\right], \left[\frac{23}{189}, -10\right], \left[\frac{29}{189}, -10\right],$$

$$\left[\frac{59}{140}, 0\right], \left[\frac{73}{140}, 0\right], \left[\frac{29}{204}, 0\right], \left[\frac{17}{160}, 0\right], \left[\frac{71}{140}, 0\right], \left[\frac{17}{161}, 10\right], \left[\frac{13}{160}, 0\right], \left[\frac{11}{160}, 0\right], \left[\frac{29}{160}, 0\right],$$

$$\left[\frac{83}{140}, 0\right], \left[\frac{23}{160}, 0\right], \left[\frac{79}{140}, 0\right], \left[\frac{19}{204}, 0\right], \left[\frac{19}{160}, 0\right], \left[\frac{1}{200}, 0\right], \left[\frac{89}{140}, 0\right], \left[\frac{11}{42}, 20\right], \left[\frac{47}{160}, 0\right],$$

$$\left[\frac{101}{140}, 0\right], \left[\frac{17}{420}, 0\right], \left[\frac{19}{200}, 0\right], \left[\frac{13}{189}, -10\right], \left[\frac{17}{189}, -10\right], \left[\frac{19}{189}, -10\right], \left[\frac{17}{200}, 0\right], \left[\frac{17}{42}, 20\right], \left[\frac{13}{420}, 0\right],$$

$$\left[\frac{13}{200}, 0\right], \left[\frac{17}{24}, 0\right], \left[\frac{97}{140}, 0\right], \left[\frac{11}{24}, 0\right], \left[\frac{11}{200}, 0\right], \left[\frac{13}{42}, 20\right], \left[\frac{11}{420}, 0\right], \left[\frac{11}{72}, 0\right], \left[\frac{107}{140}, 0\right], \left[\frac{23}{42}, 20\right],$$

$$\left[\frac{23}{200}, 0\right], \left[\frac{1}{72}, 0\right], \left[\frac{103}{140}, 0\right], \left[\frac{19}{42}, 20\right], \left[\frac{19}{24}, 0\right], \left[\frac{29}{63}, -10\right], \left[\frac{31}{63}, -10\right], \left[\frac{37}{63}, -10\right], \left[\frac{41}{63}, -10\right],$$

$$\left[\frac{29}{200}, 0\right], \left[\frac{29}{42}, 20\right], \left[\frac{109}{140}, 0\right], \left[\frac{19}{72}, 0\right], \left[\frac{121}{140}, 0\right], \left[\frac{37}{42}, 20\right], \left[\frac{1}{100}, 0\right], \left[\frac{19}{63}, -10\right], \left[\frac{23}{63}, -10\right],$$

$$\left[\frac{29}{420}, 0\right], \left[\frac{47}{200}, 0\right], \left[\frac{31}{42}, 20\right], \left[\frac{113}{140}, 0\right], \left[\frac{23}{420}, 0\right], \left[\frac{17}{72}, 0\right], \left[\frac{13}{63}, -10\right], \left[\frac{31}{420}, 0\right], \left[\frac{127}{140}, 0\right], \left[\frac{1}{96}, 0\right],$$

$$\left[\frac{47}{168}, 0\right], \left[\frac{53}{168}, 0\right], \left[\frac{17}{63}, -10\right], \left[\frac{11}{63}, -10\right], \left[\frac{19}{420}, 0\right], \left[\frac{11}{100}, 0\right], \left[\frac{43}{168}, 0\right], \left[\frac{2}{105}, -2\right], \left[\frac{41}{42}, 20\right],$$

$$\left[\frac{19}{96}, 0\right], \left[\frac{137}{140}, 0\right], \left[\frac{25}{42}, 20\right], \left[\frac{17}{96}, 0\right], \left[\frac{5}{42}, 20\right], \left[\frac{37}{420}, 0\right], \left[\frac{19}{161}, 10\right], \left[\frac{131}{140}, 0\right], \left[\frac{11}{96}, 0\right], \left[\frac{1}{203}, 10\right],$$

$$\left[\frac{17}{126}, 20\right], \left[\frac{23}{100}, 0\right], \left[\frac{17}{144}, 0\right], \left[\frac{13}{126}, 20\right], \left[\frac{43}{420}, 0\right], \left[\frac{9}{140}, 0\right], \left[\frac{53}{420}, 0\right], \left[\frac{13}{100}, 0\right], \left[\frac{23}{126}, 20\right],$$

$$\left[\frac{33}{140}, 0\right], \left[\frac{5}{21}, -10\right], \left[\frac{4}{21}, -10\right], \left[\frac{16}{105}, -2\right], \left[\frac{47}{100}, 0\right], \left[\frac{29}{100}, 0\right], \left[\frac{47}{420}, 0\right], \left[\frac{19}{126}, 20\right], \left[\frac{27}{140}, 0\right],$$

$$\left[\frac{19}{144}, 0\right], \left[\frac{13}{80}, 0\right], \left[\frac{31}{126}, 20\right], \left[\frac{17}{36}, 0\right], \left[\frac{51}{140}, 0\right], \left[\frac{29}{126}, 20\right], \left[\frac{11}{80}, 0\right], \left[\frac{11}{36}, 0\right], \left[\frac{39}{140}, 0\right], \left[\frac{1}{80}, 0\right],$$

$$\left[\frac{67}{420}, 0\right], \left[\frac{19}{36}, 0\right], \left[\frac{57}{140}, 0\right], \left[\frac{19}{80}, 0\right], \left[\frac{37}{126}, 20\right], \left[\frac{17}{80}, 0\right], \left[\frac{61}{420}, 0\right], \left[\frac{20}{21}, -10\right], \left[\frac{59}{420}, 0\right], \left[\frac{47}{80}, 0\right],$$

$$\left[\frac{69}{140}, 0\right], \left[\frac{29}{80}, 0\right], \left[\frac{71}{420}, 0\right], \left[\frac{1}{108}, 0\right], \left[\frac{41}{126}, 20\right], \left[\frac{16}{21}, -10\right], \left[\frac{23}{80}, 0\right], \left[\frac{8}{21}, -10\right], \left[\frac{41}{168}, 0\right],$$

$$\left[\frac{10}{21}, -10\right], \left[\frac{73}{420}, 0\right], \left[\frac{31}{168}, 0\right], \left[\frac{37}{168}, 0\right], \left[\frac{79}{420}, 0\right], \left[\frac{19}{21}, -10\right], \left[\frac{2}{21}, -10\right], \left[\frac{11}{108}, 0\right], \left[\frac{11}{203}, 10\right],$$

$$\left[\frac{13}{21}, -10\right], \left[\frac{17}{21}, -10\right], \left[\frac{47}{126}, 20\right], \left[\frac{81}{140}, 0\right], \left[\frac{29}{168}, 0\right], \left[\frac{83}{420}, 0\right], \left[\frac{11}{21}, -10\right], \left[\frac{11}{84}, 0\right], \left[\frac{97}{420}, 0\right],$$

$$\left[\frac{1}{84}, 0\right], \left[\frac{17}{108}, 0\right], \left[\frac{23}{168}, 0\right], \left[\frac{89}{420}, 0\right], \left[\frac{19}{147}, -10\right], \left[\frac{19}{84}, 0\right], \left[\frac{19}{108}, 0\right], \left[\frac{103}{420}, 0\right], \left[\frac{47}{189}, -10\right],$$

$$\left[\frac{87}{140}, 0\right], \left[\frac{17}{84}, 0\right], \left[\frac{17}{168}, 0\right], \left[\frac{19}{168}, 0\right], \left[\frac{1}{189}, -10\right], \left[\frac{11}{189}, -10\right], \left[\frac{22}{105}, -2\right], \left[\frac{101}{420}, 0\right], \left[\frac{47}{63}, -10\right],$$

$$\left[\frac{4}{63}, -10\right], \left[\frac{67}{126}, 20\right], \left[\frac{29}{84}, 0\right], \left[\frac{109}{420}, 0\right], \left[\frac{1}{196}, 0\right], \left[\frac{13}{84}, 0\right], \left[\frac{25}{84}, 0\right], \left[\frac{55}{84}, 0\right], \left[\frac{65}{84}, 0\right], \left[\frac{1}{168}, 0\right],$$

$$\left[\frac{11}{168}, 0\right], \left[\frac{13}{168}, 0\right], \left[\frac{23}{84}, 0\right], \left[\frac{93}{140}, 0\right], \left[\frac{107}{420}, 0\right], \left[\frac{79}{84}, 0\right], \left[\frac{83}{84}, 0\right], \left[\frac{5}{84}, 0\right], \left[\frac{31}{84}, 0\right], \left[\frac{157}{420}, 0\right],$$

$$\left[\frac{163}{420}, 0\right], \left[\frac{113}{420}, 0\right], \left[\frac{99}{140}, 0\right], \left[\frac{59}{105}, -2\right], \left[\frac{31}{105}, -2\right], \left[\frac{73}{168}, 0\right], \left[\frac{79}{168}, 0\right], \left[\frac{71}{168}, 0\right], \left[\frac{137}{420}, 0\right],$$

$$\left[\frac{53}{84}, 0\right], \left[\frac{59}{84}, 0\right], \left[\frac{1}{192}, 0\right], \left[\frac{129}{140}, 0\right], \left[\frac{61}{84}, 0\right], \left[\frac{11}{192}, 0\right], \left[\frac{139}{420}, 0\right], \left[\frac{17}{192}, 0\right], \left[\frac{73}{84}, 0\right], \left[\frac{26}{105}, -2\right],$$

$$\left[\frac{71}{84}, 0\right], \left[\frac{143}{420}, 0\right], \left[\frac{19}{192}, 0\right], \left[\frac{67}{84}, 0\right], \left[\frac{111}{140}, 0\right], \left[\frac{11}{48}, 0\right], \left[\frac{41}{84}, 0\right], \left[\frac{121}{420}, 0\right], \left[\frac{37}{84}, 0\right], \left[\frac{17}{48}, 0\right],$$

$$\left[\frac{117}{140}, 0\right], \left[\frac{43}{84}, 0\right], \left[\frac{127}{420}, 0\right], \left[\frac{47}{84}, 0\right], \left[\frac{131}{420}, 0\right], \left[\frac{13}{133}, 10\right], \left[\frac{23}{77}, 10\right], \left[\frac{1}{91}, 10\right], \left[\frac{11}{91}, 10\right], \left[\frac{69}{91}, 10\right],$$

$$\left[\frac{17}{91}, 10\right], \left[\frac{19}{91}, 10\right], \left[\frac{23}{91}, 10\right], \left[\frac{46}{77}, 10\right], \left[\frac{13}{77}, 10\right], \left[\frac{17}{77}, 10\right], \left[\frac{19}{77}, 10\right], \left[\frac{1}{119}, 10\right], \left[\frac{11}{119}, 10\right],$$

$$\left[\frac{13}{119}, 10\right], \left[\frac{80}{119}, 10\right], \left[\frac{19}{119}, 10\right], \left[\frac{23}{119}, 10\right], \left[\frac{1}{133}, 10\right], \left[\frac{11}{133}, 10\right], \left[\frac{19}{48}, 0\right], \left[\frac{4}{7}, 10\right], \left[\frac{6}{7}, 10\right],$$

$$\left[\frac{3}{7}, 10\right], \left[\frac{5}{7}, 10\right], \left[\frac{2}{7}, 10\right], \left[\frac{1}{77}, 10\right], \left[\frac{123}{140}, 0\right], \left[\frac{35}{198}, 20\right], \left[\frac{1}{198}, 20\right], \left[\frac{11}{102}, 20\right], \left[\frac{1}{114}, 20\right],$$

$$\left[\frac{11}{114}, 20\right], \left[\frac{1}{138}, 20\right], \left[\frac{11}{138}, 20\right], \left[\frac{1}{174}, 20\right], \left[\frac{11}{174}, 20\right], \left[\frac{1}{186}, 20\right], \left[\frac{11}{186}, 20\right], \left[\frac{11}{18}, 20\right], \left[\frac{11}{54}, 20\right],$$

$$\left[\frac{1}{162}, 20\right], \left[\frac{11}{162}, 20\right], \left[\frac{5}{6}, 20\right], \left[\frac{1}{66}, 20\right], \left[\frac{35}{66}, 20\right], \left[\frac{1}{78}, 20\right], \left[\frac{11}{78}, 20\right], \left[\frac{1}{102}, 20\right], \left[\frac{1}{125}, 2\right],$$

$$\left[\frac{13}{125}, 2\right], \left[\frac{17}{125}, 2\right], \left[\frac{19}{125}, 2\right], \left[\frac{13}{205}, 2\right], \left[\frac{17}{205}, 2\right], \left[\frac{19}{205}, 2\right], \left[\frac{13}{25}, 2\right], \left[\frac{17}{25}, 2\right], \left[\frac{19}{25}, 2\right], \left[\frac{17}{155}, 2\right],$$

$$\left[\frac{19}{155}, 2\right], \left[\frac{1}{185}, 2\right], \left[\frac{13}{185}, 2\right], \left[\frac{17}{185}, 2\right], \left[\frac{19}{185}, 2\right], \left[\frac{1}{205}, 2\right], \left[\frac{1}{85}, 2\right], \left[\frac{13}{85}, 2\right], \left[\frac{12}{85}, 2\right], \left[\frac{19}{85}, 2\right],$$

$$\left[\frac{1}{95}, 2\right], \left[\frac{13}{95}, 2\right], \left[\frac{17}{95}, 2\right], \left[\frac{59}{95}, 2\right], \left[\frac{43}{65}, 2\right], \left[\frac{13}{55}, 2\right], \left[\frac{17}{55}, 2\right], \left[\frac{19}{55}, 2\right], \left[\frac{11}{184}, 0\right], \left[\frac{1}{208}, 0\right],$$

$$\left[\frac{11}{208}, 0\right], \left[\frac{1}{176}, 0\right], \left[\frac{79}{176}, 0\right], \left[\frac{1}{184}, 0\right], \left[\frac{11}{104}, 0\right], \left[\frac{1}{128}, 0\right], \left[\frac{11}{128}, 0\right], \left[\frac{1}{136}, 0\right], \left[\frac{11}{136}, 0\right],$$

$$\left[\frac{1}{152}, 0\right], \left[\frac{11}{152}, 0\right], \left[\frac{1}{115}, 2\right], \left[\frac{13}{115}, 2\right], \left[\frac{17}{115}, 2\right], \left[\frac{19}{115}, 2\right], \left[\frac{1}{145}, 2\right], \left[\frac{13}{145}, 2\right], \left[\frac{17}{145}, 2\right],$$

$$\left[\frac{19}{145}, 2\right], \left[\frac{1}{155}, 2\right], \left[\frac{13}{155}, 2\right], \left[\frac{11}{164}, 0\right], \left[\frac{1}{172}, 0\right], \left[\frac{1}{116}, 0\right], \left[\frac{11}{116}, 0\right], \left[\frac{1}{124}, 0\right], \left[\frac{11}{124}, 0\right],$$

$$\left[\frac{1}{148}, 0\right], \left[\frac{11}{148}, 0\right], \left[\frac{1}{164}, 0\right], \left[\frac{35}{44}, 0\right], \left[\frac{1}{68}, 0\right], \left[\frac{11}{68}, 0\right], \left[\frac{1}{76}, 0\right], \left[\frac{11}{76}, 0\right], \left[\frac{1}{92}, 0\right], \left[\frac{11}{92}, 0\right], \left[\frac{3}{4}, 0\right],$$

$$\left[\frac{11}{207}, -10\right], \left[\frac{1}{117}, -10\right], \left[\frac{11}{117}, -10\right], \left[\frac{1}{153}, -10\right], \left[\frac{11}{153}, -10\right], \left[\frac{1}{171}, -10\right], \left[\frac{11}{171}, -10\right], \left[\frac{1}{207}, -10\right],$$

$$\left[\frac{1}{99}, -10\right], \left[\frac{35}{99}, -10\right], \left[\frac{11}{201}, -10\right], \left[\frac{2}{9}, -10\right], \left[\frac{11}{27}, -10\right], \left[\frac{1}{81}, -10\right], \left[\frac{11}{81}, -10\right], \left[\frac{11}{123}, -10\right],$$

$$\left[\frac{1}{129}, -10\right], \left[\frac{11}{129}, -10\right], \left[\frac{1}{141}, -10\right], \left[\frac{11}{141}, -10\right], \left[\frac{1}{87}, -10\right], \left[\frac{11}{87}, -10\right], \left[\frac{1}{93}, -10\right], \left[\frac{11}{93}, -10\right],$$

$$\left[\frac{1}{111}, -10\right], \left[\frac{11}{111}, -10\right], \left[\frac{1}{123}, -10\right], \left[\frac{11}{51}, -10\right], \left[\frac{11}{57}, -10\right], \left[\frac{1}{69}, -10\right], \left[\frac{11}{69}, -10\right], \left[\frac{1}{159}, -10\right],$$

$$\left[\frac{11}{159}, -10\right], \left[\frac{1}{177}, -10\right], \left[\frac{11}{177}, -10\right], \left[\frac{1}{183}, -10\right], \left[\frac{11}{183}, -10\right], \left[\frac{1}{201}, -10\right], \left[\frac{11}{172}, 0\right], \left[\frac{1}{188}, 0\right],$$

$$\left[\frac{11}{188}, 0\right], \left[\frac{3}{8}, 0\right], \left[\frac{11}{16}, 0\right], \left[\frac{11}{32}, 0\right], \left[\frac{11}{64}, 0\right], \left[\frac{1}{88}, 0\right], \left[\frac{79}{88}, 0\right], \left[\frac{1}{104}, 0\right], \left[\frac{2}{3}, -10\right], \left[\frac{2}{33}, -10\right],$$

$$\left[\frac{1}{202}, -20\right], \left[\frac{1}{206}, -20\right], \left[\frac{1}{122}, -20\right], \left[\frac{1}{134}, -20\right], \left[\frac{1}{142}, -20\right], \left[\frac{1}{146}, -20\right], \left[\frac{1}{158}, -20\right], \left[\frac{1}{166}, -20\right],$$

$$\left[\frac{1}{178}, -20\right], \left[\frac{1}{194}, -20\right], \left[\frac{1}{74}, -20\right], \left[\frac{1}{82}, -20\right], \left[\frac{1}{86}, -20\right], \left[\frac{1}{94}, -20\right], \left[\frac{1}{106}, -20\right], \left[\frac{1}{118}, -20\right],$$

$$\left[\frac{1}{181}, 10\right], \left[\frac{1}{187}, 10\right], \left[\frac{1}{191}, 10\right], \left[\frac{1}{193}, 10\right], \left[\frac{1}{197}, 10\right], \left[\frac{1}{199}, 10\right], \left[\frac{1}{209}, 10\right], \left[\frac{1}{169}, 10\right],$$
$$\left[\frac{1}{173}, 10\right], \left[\frac{1}{179}, 10\right], \left[\frac{1}{139}, 10\right], \left[\frac{1}{143}, 10\right], \left[\frac{1}{149}, 10\right], \left[\frac{1}{151}, 10\right], \left[\frac{1}{157}, 10\right], \left[\frac{1}{163}, 10\right],$$
$$\left[\frac{1}{167}, 10\right], \left[\frac{1}{121}, 10\right], \left[\frac{1}{127}, 10\right], \left[\frac{1}{131}, 10\right], \left[\frac{1}{137}, 10\right], \left[\frac{1}{89}, 10\right], \left[\frac{1}{97}, 10\right], \left[\frac{1}{101}, 10\right], \left[\frac{1}{103}, 10\right],$$
$$\left[\frac{1}{107}, 10\right], \left[\frac{1}{109}, 10\right], \left[\frac{1}{113}, 10\right], \left[\frac{1}{73}, 10\right], \left[\frac{1}{79}, 10\right], \left[\frac{1}{83}, 10\right], \left[\frac{1}{67}, 10\right], \left[\frac{1}{71}, 10\right], \left[\frac{149}{420}, 0\right],$$
$$\left[\frac{151}{420}, 0\right]\Bigg]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 420

"val0=", 20

"which is even."

"valinf=", 0

"which is even."

"It IS a modfunc on Gamma1(", 420, ")"

"min inf ord=", 0

```
"mintotord = ", -2688
"TO PROVE the identity we need to show that v[oo](ID) > ", 2688
"*** There were NO errors. ***"
"*** WARNING: some terms were constants. ***"
"See array CONTERMS."
To prove the identity we will need to verify if up to
q^(2688).
Do you want to prove the identity? (yes/no)
```

> no

```
You did not enter yes.
```

> ramid3:=expand(ramid3);

$$ramid3 := \frac{\mathrm{JAC}(0, 10, \infty)^2\,\mathrm{JAC}(0, 65, \infty)^2}{\mathrm{JAC}(2, 10, \infty)^2\,\mathrm{JAC}(13, 65, \infty)^2} + \frac{2\,\mathrm{JAC}(0, 10, \infty)^2\,\mathrm{JAC}(0, 65, \infty)^2\,q^3}{\mathrm{JAC}(2, 10, \infty)\,\mathrm{JAC}(13, 65, \infty)\,\mathrm{JAC}(4, 10, \infty)\,\mathrm{JAC}(26, 65, \infty)} + \frac{q^6\,\mathrm{JAC}(0, 10, \infty)^2\,\mathrm{JAC}(0, 65, \infty)^2}{\mathrm{JAC}(4, 10, \infty)^2\,\mathrm{JAC}(26, 65, \infty)^2} - \frac{\mathrm{JAC}(0, 13, \infty)\,\mathrm{JAC}(0, 2, \infty)}{\mathrm{JAC}(0, 26, \infty)\,\mathrm{JAC}(0, 1, \infty)} + \frac{q\,\mathrm{JAC}(0, 1, \infty)\,\mathrm{JAC}(0, 26, \infty)}{\mathrm{JAC}(0, 2, \infty)\,\mathrm{JAC}(0, 13, \infty)}$$

> ramid3a:=expand(ramid3/op(1,ramid3));

$$ramid3a := 1+\frac{2\,\mathrm{JAC}(2,10,\infty)\,\mathrm{JAC}(13,65,\infty)\,q^3}{\mathrm{JAC}(4,10,\infty)\,\mathrm{JAC}(26,65,\infty)}+\frac{\mathrm{JAC}(2,10,\infty)^2\,\mathrm{JAC}(13,65,\infty)^2\,q^6}{\mathrm{JAC}(4,10,\infty)^2\,\mathrm{JAC}(26,65,\infty)^2}$$
$$-\frac{\mathrm{JAC}(2,10,\infty)^2\,\mathrm{JAC}(13,65,\infty)^2\,\mathrm{JAC}(0,13,\infty)\,\mathrm{JAC}(0,2,\infty)}{\mathrm{JAC}(0,10,\infty)^2\,\mathrm{JAC}(0,65,\infty)^2\,\mathrm{JAC}(0,26,\infty)\,\mathrm{JAC}(0,1,\infty)}$$
$$+\frac{\mathrm{JAC}(2,10,\infty)^2\,\mathrm{JAC}(13,65,\infty)^2\,q\,\mathrm{JAC}(0,1,\infty)\,\mathrm{JAC}(0,26,\infty)}{\mathrm{JAC}(0,10,\infty)^2\,\mathrm{JAC}(0,65,\infty)^2\,\mathrm{JAC}(0,2,\infty)\,\mathrm{JAC}(0,13,\infty)}$$

```
> series(jac2series(ramid3a,300),q,300);
```

$$O(q^{300})$$

```
> ramid3b:=mixedjac2jac(ramid3a,300);
"term ", 1, "of ", 5
"term ", 2, "of ", 5
"term ", 3, "of ", 5
"term ", 4, "of ", 5
"term ", 5, "of ", 5
```

$$ramid3b := 1+2\,q^3\,\mathrm{JAC}(2,130,\infty)\,\mathrm{JAC}(8,130,\infty)\,\mathrm{JAC}(12,130,\infty)\,\mathrm{JAC}(13,130,\infty)$$
$$\mathrm{JAC}(18,130,\infty)\,\mathrm{JAC}(22,130,\infty)\,\mathrm{JAC}(28,130,\infty)\,\mathrm{JAC}(32,130,\infty)\,\mathrm{JAC}(38,130,\infty)$$
$$\mathrm{JAC}(42,130,\infty)\,\mathrm{JAC}(48,130,\infty)\,\mathrm{JAC}(52,130,\infty)^2\,\mathrm{JAC}(58,130,\infty)\,\mathrm{JAC}(62,130,\infty)\Big/\Big($$
$$\mathrm{JAC}(4,130,\infty)\,\mathrm{JAC}(6,130,\infty)\,\mathrm{JAC}(14,130,\infty)\,\mathrm{JAC}(16,130,\infty)\,\mathrm{JAC}(24,130,\infty)$$
$$\mathrm{JAC}(26,130,\infty)^2\,\mathrm{JAC}(34,130,\infty)\,\mathrm{JAC}(36,130,\infty)\,\mathrm{JAC}(39,130,\infty)\,\mathrm{JAC}(44,130,\infty)$$
$$\mathrm{JAC}(46,130,\infty)\,\mathrm{JAC}(54,130,\infty)\,\mathrm{JAC}(56,130,\infty)\,\mathrm{JAC}(64,130,\infty)\Big)+q^6\,\mathrm{JAC}(2,130,\infty)^2$$
$$\mathrm{JAC}(8,130,\infty)^2\,\mathrm{JAC}(12,130,\infty)^2\,\mathrm{JAC}(13,130,\infty)^2\,\mathrm{JAC}(18,130,\infty)^2\,\mathrm{JAC}(22,130,\infty)^2$$
$$\mathrm{JAC}(28,130,\infty)^2\,\mathrm{JAC}(32,130,\infty)^2\,\mathrm{JAC}(38,130,\infty)^2\,\mathrm{JAC}(42,130,\infty)^2\,\mathrm{JAC}(48,130,\infty)^2$$
$$\mathrm{JAC}(52,130,\infty)^4\,\mathrm{JAC}(58,130,\infty)^2\,\mathrm{JAC}(62,130,\infty)^2\Big/\Big(\mathrm{JAC}(4,130,\infty)^2\,\mathrm{JAC}(6,130,\infty)^2$$
$$\mathrm{JAC}(14,130,\infty)^2\,\mathrm{JAC}(16,130,\infty)^2\,\mathrm{JAC}(24,130,\infty)^2\,\mathrm{JAC}(26,130,\infty)^4\,\mathrm{JAC}(34,130,\infty)^2$$
$$\mathrm{JAC}(36,130,\infty)^2\,\mathrm{JAC}(39,130,\infty)^2\,\mathrm{JAC}(44,130,\infty)^2\,\mathrm{JAC}(46,130,\infty)^2\,\mathrm{JAC}(54,130,\infty)^2$$
$$\mathrm{JAC}(56,130,\infty)^2\,\mathrm{JAC}(64,130,\infty)^2\Big)-\mathrm{JAC}(2,130,\infty)^2\,\mathrm{JAC}(8,130,\infty)^2\,\mathrm{JAC}(12,130,\infty)^2$$
$$\mathrm{JAC}(13,130,\infty)^2\,\mathrm{JAC}(18,130,\infty)^2\,\mathrm{JAC}(22,130,\infty)^2\,\mathrm{JAC}(28,130,\infty)^2\,\mathrm{JAC}(32,130,\infty)^2$$
$$\mathrm{JAC}(38,130,\infty)^2\,\mathrm{JAC}(42,130,\infty)^2\,\mathrm{JAC}(48,130,\infty)^2\,\mathrm{JAC}(52,130,\infty)^4\,\mathrm{JAC}(58,130,\infty)^2$$
$$\mathrm{JAC}(62,130,\infty)^2/\big(\mathrm{JAC}(1,130,\infty)\,\mathrm{JAC}(3,130,\infty)\,\mathrm{JAC}(5,130,\infty)\,\mathrm{JAC}(7,130,\infty)$$
$$\mathrm{JAC}(9,130,\infty)\,\mathrm{JAC}(11,130,\infty)\,\mathrm{JAC}(15,130,\infty)\,\mathrm{JAC}(17,130,\infty)\,\mathrm{JAC}(19,130,\infty)$$
$$\mathrm{JAC}(21,130,\infty)\,\mathrm{JAC}(23,130,\infty)\,\mathrm{JAC}(25,130,\infty)\,\mathrm{JAC}(27,130,\infty)\,\mathrm{JAC}(29,130,\infty)$$
$$\mathrm{JAC}(31,130,\infty)\,\mathrm{JAC}(33,130,\infty)\,\mathrm{JAC}(35,130,\infty)\,\mathrm{JAC}(37,130,\infty)\,\mathrm{JAC}(41,130,\infty)$$
$$\mathrm{JAC}(43,130,\infty)\,\mathrm{JAC}(45,130,\infty)\,\mathrm{JAC}(47,130,\infty)\,\mathrm{JAC}(49,130,\infty)\,\mathrm{JAC}(51,130,\infty)$$
$$\mathrm{JAC}(53,130,\infty)\,\mathrm{JAC}(55,130,\infty)\,\mathrm{JAC}(57,130,\infty)\,\mathrm{JAC}(59,130,\infty)\,\mathrm{JAC}(61,130,\infty)$$
$$\mathrm{JAC}(63,130,\infty)\big)+q\,\mathrm{JAC}(1,130,\infty)\,\mathrm{JAC}(2,130,\infty)^2\,\mathrm{JAC}(3,130,\infty)\,\mathrm{JAC}(5,130,\infty)$$
$$\mathrm{JAC}(7,130,\infty)\,\mathrm{JAC}(8,130,\infty)^2\,\mathrm{JAC}(9,130,\infty)\,\mathrm{JAC}(11,130,\infty)\,\mathrm{JAC}(12,130,\infty)^2$$

$$\mathrm{JAC}(13, 130, \infty)^2\, \mathrm{JAC}(15, 130, \infty)\, \mathrm{JAC}(17, 130, \infty)\, \mathrm{JAC}(18, 130, \infty)^2\, \mathrm{JAC}(19, 130, \infty)$$
$$\mathrm{JAC}(21, 130, \infty)\, \mathrm{JAC}(22, 130, \infty)^2\, \mathrm{JAC}(23, 130, \infty)\, \mathrm{JAC}(25, 130, \infty)\, \mathrm{JAC}(27, 130, \infty)$$
$$\mathrm{JAC}(28, 130, \infty)^2\, \mathrm{JAC}(29, 130, \infty)\, \mathrm{JAC}(31, 130, \infty)\, \mathrm{JAC}(32, 130, \infty)^2\, \mathrm{JAC}(33, 130, \infty)$$
$$\mathrm{JAC}(35, 130, \infty)\, \mathrm{JAC}(37, 130, \infty)\, \mathrm{JAC}(38, 130, \infty)^2\, \mathrm{JAC}(41, 130, \infty)\, \mathrm{JAC}(42, 130, \infty)^2$$
$$\mathrm{JAC}(43, 130, \infty)\, \mathrm{JAC}(45, 130, \infty)\, \mathrm{JAC}(47, 130, \infty)\, \mathrm{JAC}(48, 130, \infty)^2\, \mathrm{JAC}(49, 130, \infty)$$
$$\mathrm{JAC}(51, 130, \infty)\, \mathrm{JAC}(52, 130, \infty)^4\, \mathrm{JAC}(53, 130, \infty)\, \mathrm{JAC}(55, 130, \infty)\, \mathrm{JAC}(57, 130, \infty)$$
$$\mathrm{JAC}(58, 130, \infty)^2\, \mathrm{JAC}(59, 130, \infty)\, \mathrm{JAC}(61, 130, \infty)\, \mathrm{JAC}(62, 130, \infty)^2\, \mathrm{JAC}(63, 130, \infty) \Big/$$
$$\mathrm{JAC}(0, 130, \infty)^{60}$$

We calculate a set of inequivalent cusps for $\Gamma_1(130)$
and the width of each cusp. Note: oo is the first cusp in the list.

```
> cusps130:=cuspmake1(130):
> cusp130:=cusps130 minus {[1,0]}:
> cusps130:=convert(cusp130,list):
> wids130:=map(x->cuspwid1(x[1],x[2],130),cusps130):
> wids130:=[1,op(wids130)]:
> CUSPS130:=map(x->x[1]/x[2],cusps130):
> CUSPS130:=[oo,op(CUSPS130)];
```

$$CUSPS130 := \left[oo, \frac{57}{130}, \frac{59}{130}, \frac{63}{130}, \frac{61}{130}, \frac{9}{52}, \frac{43}{130}, \frac{1}{47}, \frac{1}{49}, \frac{1}{51}, \frac{1}{53}, \frac{1}{57}, \frac{1}{59}, \frac{1}{37}, \frac{1}{41}, \frac{1}{43}, \frac{1}{27}, \frac{1}{29}, \frac{1}{31}, \frac{1}{33}, \frac{1}{7},\right.$$
$$\frac{1}{9}, \frac{1}{11}, \frac{1}{17}, \frac{1}{19}, \frac{1}{21}, \frac{1}{23}, 0, \frac{1}{3}, \frac{1}{61}, \frac{1}{63}, \frac{1}{58}, \frac{1}{62}, \frac{1}{4}, \frac{1}{8}, \frac{1}{54}, \frac{1}{42}, \frac{1}{46}, \frac{1}{38}, \frac{1}{34}, \frac{1}{6}, \frac{1}{14}, \frac{1}{18}, \frac{1}{22}, \frac{7}{52}, \frac{1}{2}, \frac{1}{48}, \frac{1}{56}, \frac{1}{64},$$
$$\frac{1}{44}, \frac{1}{28}, \frac{1}{32}, \frac{1}{36}, \frac{1}{24}, \frac{1}{12}, \frac{1}{16}, \frac{9}{130}, \frac{7}{15}, \frac{4}{15}, \frac{1}{35}, \frac{3}{35}, \frac{1}{15}, \frac{13}{15}, \frac{1}{5}, \frac{3}{5}, \frac{2}{5}, \frac{4}{5}, \frac{51}{130}, \frac{3}{25}, \frac{7}{25}, \frac{9}{25}, \frac{7}{45}, \frac{4}{45}, \frac{1}{55}, \frac{3}{55}, \frac{9}{55},$$
$$\frac{1}{25}, \frac{32}{35}, \frac{9}{35}, \frac{1}{45}, \frac{43}{45}, \frac{1}{30}, \frac{13}{30}, \frac{7}{30}, \frac{19}{30}, \frac{1}{40}, \frac{3}{40}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \frac{47}{130}, \frac{3}{20}, \frac{7}{20}, \frac{9}{20}, \frac{9}{50}, \frac{1}{20}, \frac{7}{60}, \frac{19}{60}, \frac{1}{50}, \frac{3}{50}, \frac{7}{50},$$
$$\frac{7}{40}, \frac{9}{40}, \frac{1}{60}, \frac{13}{60}, \frac{9}{13}, \frac{11}{13}, \frac{4}{13}, \frac{1}{13}, \frac{3}{13}, \frac{7}{13}, \frac{2}{13}, \frac{8}{13}, \frac{10}{13}, \frac{5}{13}, \frac{9}{26}, \frac{6}{13}, \frac{25}{39}, \frac{19}{39}, \frac{34}{39}, \frac{23}{39}, \frac{31}{39}, \frac{2}{39}, \frac{12}{13}, \frac{1}{39}, \frac{16}{39}, \frac{7}{39},$$
$$\frac{22}{39}, \frac{11}{39}, \frac{17}{39}, \frac{7}{26}, \frac{15}{26}, \frac{25}{26}, \frac{1}{52}, \frac{17}{26}, \frac{19}{26}, \frac{21}{26}, \frac{23}{26}, \frac{1}{26}, \frac{3}{26}, \frac{23}{52}, \frac{31}{52}, \frac{41}{52}, \frac{51}{52}, \frac{11}{52}, \frac{17}{52}, \frac{19}{52}, \frac{21}{52}, \frac{8}{65}, \frac{12}{65}, \frac{6}{65}, \frac{5}{26}, \frac{4}{65},$$
$$\frac{14}{65}, \frac{16}{65}, \frac{18}{65}, \frac{22}{65}, \frac{24}{65}, \frac{29}{65}, \frac{31}{65}, \frac{2}{65}, \frac{11}{65}, \frac{17}{65}, \frac{19}{65}, \frac{21}{65}, \frac{23}{65}, \frac{27}{65}, \frac{1}{65}, \frac{3}{65}, \frac{7}{65}, \frac{9}{65}, \frac{28}{65}, \frac{32}{65}, \frac{27}{130}, \frac{29}{130}, \frac{31}{130}, \frac{33}{130},$$
$$\left.\frac{37}{130}, \frac{41}{130}, \frac{19}{130}, \frac{21}{130}, \frac{23}{130}, \frac{7}{55}, \frac{11}{130}, \frac{17}{130}, \frac{3}{130}, \frac{7}{130}, \frac{3}{52}, \frac{11}{26}, \frac{49}{130}, \frac{53}{130}\right]$$

```
> nops(CUSPS130);
```

192

```
> provemodfuncid(ramid3b,CUSPS130,wids130,130);
"TERM ", 1, "of ", 5, " ***********************************************************
```

```
********"
```

"XX=", 1

```
"TERM ", 2, "of ", 5, " ************************************************************
********"
```

"XX=", $2\, q^3\, \mathrm{JAC}(2, 130, \infty)\, \mathrm{JAC}(8, 130, \infty)\, \mathrm{JAC}(12, 130, \infty)\, \mathrm{JAC}(13, 130, \infty)\, \mathrm{JAC}(18, 130, \infty)\, \mathrm{JAC}(22, 130, \infty)\, \mathrm{JAC}(28, 130, \infty)\, \mathrm{JAC}(32, 130, \infty)\, \mathrm{JAC}(38, 130, \infty)\, \mathrm{JAC}(42, 130, \infty)\, \mathrm{JAC}(48, 130, \infty)\, \mathrm{JAC}(52, 130, \infty)^2\, \mathrm{JAC}(58, 130, \infty)\, \mathrm{JAC}(62, 130, \infty) \Big/ (\mathrm{JAC}(4, 130, \infty)\, \mathrm{JAC}(6, 130, \infty)\, \mathrm{JAC}(14, 130, \infty)\, \mathrm{JAC}(16, 130, \infty)\, \mathrm{JAC}(24, 130, \infty)\, \mathrm{JAC}(26, 130, \infty)^2\, \mathrm{JAC}(34, 130, \infty)\, \mathrm{JAC}(36, 130, \infty)\, \mathrm{JAC}(39, 130, \infty)\, \mathrm{JAC}(44, 130, \infty)\, \mathrm{JAC}(46, 130, \infty)\, \mathrm{JAC}(54, 130, \infty)\, \mathrm{JAC}(56, 130, \infty)\, \mathrm{JAC}(64, 130, \infty))$

"Cusp ORDS: "

$$\left[[oo, 3], \left[\frac{57}{130}, -3\right], \left[\frac{59}{130}, 3\right], \left[\frac{63}{130}, -3\right], \left[\frac{61}{130}, 3\right], \left[\frac{9}{52}, 0\right], \left[\frac{43}{130}, -3\right], \left[\frac{1}{47}, 0\right], \left[\frac{1}{49}, 0\right], \left[\frac{1}{51}, 0\right],\right.$$

$$\left[\frac{1}{53}, 0\right], \left[\frac{1}{57}, 0\right], \left[\frac{1}{59}, 0\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right], \left[\frac{1}{43}, 0\right], \left[\frac{1}{27}, 0\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, 0\right], \left[\frac{1}{7}, 0\right],$$

$$\left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, 0\right], \left[\frac{1}{23}, 0\right], [0, 0], \left[\frac{1}{3}, 0\right], \left[\frac{1}{61}, 0\right], \left[\frac{1}{63}, 0\right], \left[\frac{1}{58}, 0\right],$$

$$\left[\frac{1}{62}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{8}, 0\right], \left[\frac{1}{54}, 0\right], \left[\frac{1}{42}, 0\right], \left[\frac{1}{46}, 0\right], \left[\frac{1}{38}, 0\right], \left[\frac{1}{34}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{14}, 0\right], \left[\frac{1}{18}, 0\right],$$

$$\left[\frac{1}{22}, 0\right], \left[\frac{7}{52}, 0\right], \left[\frac{1}{2}, 0\right], \left[\frac{1}{48}, 0\right], \left[\frac{1}{56}, 0\right], \left[\frac{1}{64}, 0\right], \left[\frac{1}{44}, 0\right], \left[\frac{1}{28}, 0\right], \left[\frac{1}{32}, 0\right], \left[\frac{1}{36}, 0\right], \left[\frac{1}{24}, 0\right],$$

$$\left[\frac{1}{12}, 0\right], \left[\frac{1}{16}, 0\right], \left[\frac{9}{130}, 3\right], \left[\frac{7}{15}, 3\right], \left[\frac{4}{15}, -3\right], \left[\frac{1}{35}, -3\right], \left[\frac{3}{35}, 3\right], \left[\frac{1}{15}, -3\right], \left[\frac{13}{15}, 3\right], \left[\frac{1}{5}, -3\right], \left[\frac{3}{5}, 3\right],$$

$$\left[\frac{2}{5}, 3\right], \left[\frac{4}{5}, -3\right], \left[\frac{51}{130}, 3\right], \left[\frac{3}{25}, 3\right], \left[\frac{7}{25}, 3\right], \left[\frac{9}{25}, -3\right], \left[\frac{7}{45}, 3\right], \left[\frac{4}{45}, -3\right], \left[\frac{1}{55}, -3\right], \left[\frac{3}{55}, 3\right], \left[\frac{9}{55}, -3\right],$$

$$\left[\frac{1}{25}, -3\right], \left[\frac{32}{35}, 3\right], \left[\frac{9}{35}, -3\right], \left[\frac{1}{45}, -3\right], \left[\frac{43}{45}, 3\right], \left[\frac{1}{30}, 5\right], \left[\frac{13}{30}, -5\right], \left[\frac{7}{30}, -5\right], \left[\frac{19}{30}, 5\right], \left[\frac{1}{40}, 5\right],$$

$$\left[\frac{3}{40}, -5\right], \left[\frac{1}{10}, 5\right], \left[\frac{3}{10}, -5\right], \left[\frac{7}{10}, -5\right], \left[\frac{9}{10}, 5\right], \left[\frac{47}{130}, -3\right], \left[\frac{3}{20}, -5\right], \left[\frac{7}{20}, -5\right], \left[\frac{9}{20}, 5\right], \left[\frac{9}{50}, 5\right],$$

$$\left[\frac{1}{20}, 5\right], \left[\frac{7}{60}, -5\right], \left[\frac{19}{60}, 5\right], \left[\frac{1}{50}, 5\right], \left[\frac{3}{50}, -5\right], \left[\frac{7}{50}, -5\right], \left[\frac{7}{40}, -5\right], \left[\frac{9}{40}, 5\right], \left[\frac{1}{60}, 5\right], \left[\frac{13}{60}, -5\right],$$

$$\left[\frac{9}{13}, 0\right], \left[\frac{11}{13}, 0\right], \left[\frac{4}{13}, 0\right], \left[\frac{1}{13}, 0\right], \left[\frac{3}{13}, 0\right], \left[\frac{7}{13}, 0\right], \left[\frac{2}{13}, 0\right], \left[\frac{8}{13}, 0\right], \left[\frac{10}{13}, 0\right], \left[\frac{5}{13}, 0\right], \left[\frac{9}{26}, 0\right],$$

$$\left[\frac{6}{13}, 0\right], \left[\frac{25}{39}, 0\right], \left[\frac{19}{39}, 0\right], \left[\frac{34}{39}, 0\right], \left[\frac{23}{39}, 0\right], \left[\frac{31}{39}, 0\right], \left[\frac{2}{39}, 0\right], \left[\frac{12}{13}, 0\right], \left[\frac{1}{39}, 0\right], \left[\frac{16}{39}, 0\right], \left[\frac{7}{39}, 0\right],$$

$$\left[\frac{22}{39}, 0\right], \left[\frac{11}{39}, 0\right], \left[\frac{17}{39}, 0\right], \left[\frac{7}{26}, 0\right], \left[\frac{15}{26}, 0\right], \left[\frac{25}{26}, 0\right], \left[\frac{1}{52}, 0\right], \left[\frac{17}{26}, 0\right], \left[\frac{19}{26}, 0\right], \left[\frac{21}{26}, 0\right], \left[\frac{23}{26}, 0\right],$$

$\left[\frac{1}{26}, 0\right], \left[\frac{3}{26}, 0\right], \left[\frac{23}{52}, 0\right], \left[\frac{31}{52}, 0\right], \left[\frac{41}{52}, 0\right], \left[\frac{51}{52}, 0\right], \left[\frac{11}{52}, 0\right], \left[\frac{17}{52}, 0\right], \left[\frac{19}{52}, 0\right], \left[\frac{21}{52}, 0\right], \left[\frac{8}{65}, -5\right],$
$\left[\frac{12}{65}, -5\right], \left[\frac{6}{65}, 5\right], \left[\frac{5}{26}, 0\right], \left[\frac{4}{65}, 5\right], \left[\frac{14}{65}, 5\right], \left[\frac{16}{65}, 5\right], \left[\frac{18}{65}, -5\right], \left[\frac{22}{65}, -5\right], \left[\frac{24}{65}, 5\right], \left[\frac{29}{65}, 5\right], \left[\frac{31}{65}, 5\right],$
$\left[\frac{2}{65}, -5\right], \left[\frac{11}{65}, 5\right], \left[\frac{17}{65}, -5\right], \left[\frac{19}{65}, 5\right], \left[\frac{21}{65}, 5\right], \left[\frac{23}{65}, -5\right], \left[\frac{27}{65}, -5\right], \left[\frac{1}{65}, 5\right], \left[\frac{3}{65}, -5\right], \left[\frac{7}{65}, -5\right],$
$\left[\frac{9}{65}, 5\right], \left[\frac{28}{65}, -5\right], \left[\frac{32}{65}, -5\right], \left[\frac{27}{130}, -3\right], \left[\frac{29}{130}, 3\right], \left[\frac{31}{130}, 3\right], \left[\frac{33}{130}, -3\right], \left[\frac{37}{130}, -3\right], \left[\frac{41}{130}, 3\right],$
$\left[\frac{19}{130}, 3\right], \left[\frac{21}{130}, 3\right], \left[\frac{23}{130}, -3\right], \left[\frac{7}{55}, 3\right], \left[\frac{11}{130}, 3\right], \left[\frac{17}{130}, -3\right], \left[\frac{3}{130}, -3\right], \left[\frac{7}{130}, -3\right], \left[\frac{3}{52}, 0\right], \left[\frac{11}{26}, 0\right],$
$\left[\frac{49}{130}, 3\right], \left[\frac{53}{130}, -3\right]\Big]$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 130

"val0=", 0

"which is even."

"valinf=", 6

"which is even."

"It IS a modfunc on Gamma1(", 130, ")"

```
"TERM ", 3, "of ", 5, " *************************************************************
********"
```

"XX=", q^6 JAC(2, 130, ∞)^2 JAC(8, 130, ∞)^2 JAC(12, 130, ∞)^2 JAC(13, 130, ∞)^2
JAC(18, 130, ∞)^2 JAC(22, 130, ∞)^2 JAC(28, 130, ∞)^2 JAC(32, 130, ∞)^2 JAC(38, 130, ∞)^2
JAC(42, 130, ∞)^2 JAC(48, 130, ∞)^2 JAC(52, 130, ∞)^4 JAC(58, 130, ∞)^2 JAC(62, 130, ∞)^2 / (
JAC(4, 130, ∞)^2 JAC(6, 130, ∞)^2 JAC(14, 130, ∞)^2 JAC(16, 130, ∞)^2 JAC(24, 130, ∞)^2
JAC(26, 130, ∞)^4 JAC(34, 130, ∞)^2 JAC(36, 130, ∞)^2 JAC(39, 130, ∞)^2 JAC(44, 130, ∞)^2
JAC(46, 130, ∞)^2 JAC(54, 130, ∞)^2 JAC(56, 130, ∞)^2 JAC(64, 130, ∞)^2)

"Cusp ORDS: "

$\Big[[oo, 6], \left[\frac{57}{130}, -6\right], \left[\frac{59}{130}, 6\right], \left[\frac{63}{130}, -6\right], \left[\frac{61}{130}, 6\right], \left[\frac{9}{52}, 0\right], \left[\frac{43}{130}, -6\right], \left[\frac{1}{47}, 0\right], \left[\frac{1}{49}, 0\right], \left[\frac{1}{51}, 0\right],$
$\left[\frac{1}{53}, 0\right], \left[\frac{1}{57}, 0\right], \left[\frac{1}{59}, 0\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right], \left[\frac{1}{43}, 0\right], \left[\frac{1}{27}, 0\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, 0\right], \left[\frac{1}{7}, 0\right],$
$\left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, 0\right], \left[\frac{1}{23}, 0\right], [0, 0], \left[\frac{1}{3}, 0\right], \left[\frac{1}{61}, 0\right], \left[\frac{1}{63}, 0\right], \left[\frac{1}{58}, 0\right],$

$$\left[\frac{1}{62}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{8}, 0\right], \left[\frac{1}{54}, 0\right], \left[\frac{1}{42}, 0\right], \left[\frac{1}{46}, 0\right], \left[\frac{1}{38}, 0\right], \left[\frac{1}{34}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{14}, 0\right], \left[\frac{1}{18}, 0\right],$$

$$\left[\frac{1}{22}, 0\right], \left[\frac{7}{52}, 0\right], \left[\frac{1}{2}, 0\right], \left[\frac{1}{48}, 0\right], \left[\frac{1}{56}, 0\right], \left[\frac{1}{64}, 0\right], \left[\frac{1}{44}, 0\right], \left[\frac{1}{28}, 0\right], \left[\frac{1}{32}, 0\right], \left[\frac{1}{36}, 0\right], \left[\frac{1}{24}, 0\right],$$

$$\left[\frac{1}{12}, 0\right], \left[\frac{1}{16}, 0\right], \left[\frac{9}{130}, 6\right], \left[\frac{7}{15}, 6\right], \left[\frac{4}{15}, -6\right], \left[\frac{1}{35}, -6\right], \left[\frac{3}{35}, 6\right], \left[\frac{1}{15}, -6\right], \left[\frac{13}{15}, 6\right], \left[\frac{1}{5}, -6\right], \left[\frac{3}{5}, 6\right],$$

$$\left[\frac{2}{5}, 6\right], \left[\frac{4}{5}, -6\right], \left[\frac{51}{130}, 6\right], \left[\frac{3}{25}, 6\right], \left[\frac{7}{25}, 6\right], \left[\frac{9}{25}, -6\right], \left[\frac{7}{45}, 6\right], \left[\frac{4}{45}, -6\right], \left[\frac{1}{55}, -6\right], \left[\frac{3}{55}, 6\right], \left[\frac{9}{55}, -6\right],$$

$$\left[\frac{1}{25}, -6\right], \left[\frac{32}{35}, 6\right], \left[\frac{9}{35}, -6\right], \left[\frac{1}{45}, -6\right], \left[\frac{43}{45}, 6\right], \left[\frac{1}{30}, 10\right], \left[\frac{13}{30}, -10\right], \left[\frac{7}{30}, -10\right], \left[\frac{19}{30}, 10\right], \left[\frac{1}{40}, 10\right],$$

$$\left[\frac{3}{40}, -10\right], \left[\frac{1}{10}, 10\right], \left[\frac{3}{10}, -10\right], \left[\frac{7}{10}, -10\right], \left[\frac{9}{10}, 10\right], \left[\frac{47}{130}, -6\right], \left[\frac{3}{20}, -10\right], \left[\frac{7}{20}, -10\right], \left[\frac{9}{20}, 10\right],$$

$$\left[\frac{9}{50}, 10\right], \left[\frac{1}{20}, 10\right], \left[\frac{7}{60}, -10\right], \left[\frac{19}{60}, 10\right], \left[\frac{1}{50}, 10\right], \left[\frac{3}{50}, -10\right], \left[\frac{7}{50}, -10\right], \left[\frac{7}{40}, -10\right], \left[\frac{9}{40}, 10\right],$$

$$\left[\frac{1}{60}, 10\right], \left[\frac{13}{60}, -10\right], \left[\frac{9}{13}, 0\right], \left[\frac{11}{13}, 0\right], \left[\frac{4}{13}, 0\right], \left[\frac{1}{13}, 0\right], \left[\frac{3}{13}, 0\right], \left[\frac{7}{13}, 0\right], \left[\frac{2}{13}, 0\right], \left[\frac{8}{13}, 0\right], \left[\frac{10}{13}, 0\right],$$

$$\left[\frac{5}{13}, 0\right], \left[\frac{9}{26}, 0\right], \left[\frac{6}{13}, 0\right], \left[\frac{25}{39}, 0\right], \left[\frac{19}{39}, 0\right], \left[\frac{34}{39}, 0\right], \left[\frac{23}{39}, 0\right], \left[\frac{31}{39}, 0\right], \left[\frac{2}{39}, 0\right], \left[\frac{12}{13}, 0\right], \left[\frac{1}{39}, 0\right],$$

$$\left[\frac{16}{39}, 0\right], \left[\frac{7}{39}, 0\right], \left[\frac{22}{39}, 0\right], \left[\frac{11}{39}, 0\right], \left[\frac{17}{39}, 0\right], \left[\frac{7}{26}, 0\right], \left[\frac{15}{26}, 0\right], \left[\frac{25}{26}, 0\right], \left[\frac{1}{52}, 0\right], \left[\frac{17}{26}, 0\right], \left[\frac{19}{26}, 0\right],$$

$$\left[\frac{21}{26}, 0\right], \left[\frac{23}{26}, 0\right], \left[\frac{1}{26}, 0\right], \left[\frac{3}{26}, 0\right], \left[\frac{23}{52}, 0\right], \left[\frac{31}{52}, 0\right], \left[\frac{41}{52}, 0\right], \left[\frac{51}{52}, 0\right], \left[\frac{11}{52}, 0\right], \left[\frac{17}{52}, 0\right], \left[\frac{19}{52}, 0\right],$$

$$\left[\frac{21}{52}, 0\right], \left[\frac{8}{65}, -10\right], \left[\frac{12}{65}, -10\right], \left[\frac{6}{65}, 10\right], \left[\frac{5}{26}, 0\right], \left[\frac{4}{65}, 10\right], \left[\frac{14}{65}, 10\right], \left[\frac{16}{65}, 10\right], \left[\frac{18}{65}, -10\right],$$

$$\left[\frac{22}{65}, -10\right], \left[\frac{24}{65}, 10\right], \left[\frac{29}{65}, 10\right], \left[\frac{31}{65}, 10\right], \left[\frac{2}{65}, -10\right], \left[\frac{11}{65}, 10\right], \left[\frac{17}{65}, -10\right], \left[\frac{19}{65}, 10\right], \left[\frac{21}{65}, 10\right],$$

$$\left[\frac{23}{65}, -10\right], \left[\frac{27}{65}, -10\right], \left[\frac{1}{65}, 10\right], \left[\frac{3}{65}, -10\right], \left[\frac{7}{65}, -10\right], \left[\frac{9}{65}, 10\right], \left[\frac{28}{65}, -10\right], \left[\frac{32}{65}, -10\right], \left[\frac{27}{130}, -6\right],$$

$$\left[\frac{29}{130}, 6\right], \left[\frac{31}{130}, 6\right], \left[\frac{33}{130}, -6\right], \left[\frac{37}{130}, -6\right], \left[\frac{41}{130}, 6\right], \left[\frac{19}{130}, 6\right], \left[\frac{21}{130}, 6\right], \left[\frac{23}{130}, -6\right], \left[\frac{7}{55}, 6\right],$$

$$\left[\frac{11}{130}, 6\right], \left[\frac{17}{130}, -6\right], \left[\frac{3}{130}, -6\right], \left[\frac{7}{130}, -6\right], \left[\frac{3}{52}, 0\right], \left[\frac{11}{26}, 0\right], \left[\frac{49}{130}, 6\right], \left[\frac{53}{130}, -6\right]\right]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 130

"val0=", 0

"which is even."

"valinf=", 12

"which is even."

"It IS a modfunc on Gamma1(", 130, ")"

```
"TERM ", 4, "of ", 5, " ****************************************************************
********"
```

$$"XX=", -\mathrm{JAC}(2, 130, \infty)^2\, \mathrm{JAC}(8, 130, \infty)^2\, \mathrm{JAC}(12, 130, \infty)^2\, \mathrm{JAC}(13, 130, \infty)^2\, \mathrm{JAC}(18, 130, \infty)^2\, \mathrm{JAC}(22, 130, \infty)^2\, \mathrm{JAC}(28, 130, \infty)^2\, \mathrm{JAC}(32, 130, \infty)^2\, \mathrm{JAC}(38, 130, \infty)^2\, \mathrm{JAC}(42, 130, \infty)^2\, \mathrm{JAC}(48, 130, \infty)^2\, \mathrm{JAC}(52, 130, \infty)^4\, \mathrm{JAC}(58, 130, \infty)^2\, \mathrm{JAC}(62, 130, \infty)^2 / (\mathrm{JAC}(1, 130, \infty)\, \mathrm{JAC}(3, 130, \infty)\, \mathrm{JAC}(5, 130, \infty)\, \mathrm{JAC}(7, 130, \infty)\, \mathrm{JAC}(9, 130, \infty)\, \mathrm{JAC}(11, 130, \infty)\, \mathrm{JAC}(15, 130, \infty)\, \mathrm{JAC}(17, 130, \infty)\, \mathrm{JAC}(19, 130, \infty)\, \mathrm{JAC}(21, 130, \infty)\, \mathrm{JAC}(23, 130, \infty)\, \mathrm{JAC}(25, 130, \infty)\, \mathrm{JAC}(27, 130, \infty)\, \mathrm{JAC}(29, 130, \infty)\, \mathrm{JAC}(31, 130, \infty)\, \mathrm{JAC}(33, 130, \infty)\, \mathrm{JAC}(35, 130, \infty)\, \mathrm{JAC}(37, 130, \infty)\, \mathrm{JAC}(41, 130, \infty)\, \mathrm{JAC}(43, 130, \infty)\, \mathrm{JAC}(45, 130, \infty)\, \mathrm{JAC}(47, 130, \infty)\, \mathrm{JAC}(49, 130, \infty)\, \mathrm{JAC}(51, 130, \infty)\, \mathrm{JAC}(53, 130, \infty)\, \mathrm{JAC}(55, 130, \infty)\, \mathrm{JAC}(57, 130, \infty)\, \mathrm{JAC}(59, 130, \infty)\, \mathrm{JAC}(61, 130, \infty)\, \mathrm{JAC}(63, 130, \infty))$$

"Cusp ORDS: "

$$\left[[oo, 0], \left[\frac{57}{130}, -6\right], \left[\frac{59}{130}, 0\right], \left[\frac{63}{130}, -6\right], \left[\frac{61}{130}, 0\right], \left[\frac{9}{52}, 0\right], \left[\frac{43}{130}, -6\right], \left[\frac{1}{47}, 0\right], \left[\frac{1}{49}, 0\right], \left[\frac{1}{51}, 0\right],\right.$$

$$\left[\frac{1}{53}, 0\right], \left[\frac{1}{57}, 0\right], \left[\frac{1}{59}, 0\right], \left[\frac{1}{37}, 0\right], \left[\frac{1}{41}, 0\right], \left[\frac{1}{43}, 0\right], \left[\frac{1}{27}, 0\right], \left[\frac{1}{29}, 0\right], \left[\frac{1}{31}, 0\right], \left[\frac{1}{33}, 0\right], \left[\frac{1}{7}, 0\right],$$

$$\left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{17}, 0\right], \left[\frac{1}{19}, 0\right], \left[\frac{1}{21}, 0\right], \left[\frac{1}{23}, 0\right], [0, 0], \left[\frac{1}{3}, 0\right], \left[\frac{1}{61}, 0\right], \left[\frac{1}{63}, 0\right], \left[\frac{1}{58}, 7\right],$$

$$\left[\frac{1}{62}, 7\right], \left[\frac{1}{4}, 7\right], \left[\frac{1}{8}, 7\right], \left[\frac{1}{54}, 7\right], \left[\frac{1}{42}, 7\right], \left[\frac{1}{46}, 7\right], \left[\frac{1}{38}, 7\right], \left[\frac{1}{34}, 7\right], \left[\frac{1}{6}, 7\right], \left[\frac{1}{14}, 7\right], \left[\frac{1}{18}, 7\right],$$

$$\left[\frac{1}{22}, 7\right], \left[\frac{7}{52}, 0\right], \left[\frac{1}{2}, 7\right], \left[\frac{1}{48}, 7\right], \left[\frac{1}{56}, 7\right], \left[\frac{1}{64}, 7\right], \left[\frac{1}{44}, 7\right], \left[\frac{1}{28}, 7\right], \left[\frac{1}{32}, 7\right], \left[\frac{1}{36}, 7\right], \left[\frac{1}{24}, 7\right],$$

$$\left[\frac{1}{12}, 7\right], \left[\frac{1}{16}, 7\right], \left[\frac{9}{130}, 0\right], \left[\frac{7}{15}, 0\right], \left[\frac{4}{15}, -6\right], \left[\frac{1}{35}, -6\right], \left[\frac{3}{35}, 0\right], \left[\frac{1}{15}, -6\right], \left[\frac{13}{15}, 0\right], \left[\frac{1}{5}, -6\right], \left[\frac{3}{5}, 0\right],$$

$$\left[\frac{2}{5}, 0\right], \left[\frac{4}{5}, -6\right], \left[\frac{51}{130}, 0\right], \left[\frac{3}{25}, 0\right], \left[\frac{7}{25}, 0\right], \left[\frac{9}{25}, -6\right], \left[\frac{7}{45}, 0\right], \left[\frac{4}{45}, -6\right], \left[\frac{1}{55}, -6\right], \left[\frac{3}{55}, 0\right], \left[\frac{9}{55}, -6\right],$$

$$\left[\frac{1}{25}, -6\right], \left[\frac{32}{35}, 0\right], \left[\frac{9}{35}, -6\right], \left[\frac{1}{45}, -6\right], \left[\frac{43}{45}, 0\right], \left[\frac{1}{30}, 1\right], \left[\frac{13}{30}, -9\right], \left[\frac{7}{30}, -9\right], \left[\frac{19}{30}, 1\right], \left[\frac{1}{40}, 1\right],$$

$$\left[\frac{3}{40}, -9\right], \left[\frac{1}{10}, 1\right], \left[\frac{3}{10}, -9\right], \left[\frac{7}{10}, -9\right], \left[\frac{9}{10}, 1\right], \left[\frac{47}{130}, -6\right], \left[\frac{3}{20}, -9\right], \left[\frac{7}{20}, -9\right], \left[\frac{9}{20}, 1\right], \left[\frac{9}{50}, 1\right],$$

$$\left[\frac{1}{20}, 1\right], \left[\frac{7}{60}, -9\right], \left[\frac{19}{60}, 1\right], \left[\frac{1}{50}, 1\right], \left[\frac{3}{50}, -9\right], \left[\frac{7}{50}, -9\right], \left[\frac{7}{40}, -9\right], \left[\frac{9}{40}, 1\right], \left[\frac{1}{60}, 1\right], \left[\frac{13}{60}, -9\right],$$

$$\left[\frac{9}{13}, 7\right], \left[\frac{11}{13}, 7\right], \left[\frac{4}{13}, 7\right], \left[\frac{1}{13}, 7\right], \left[\frac{3}{13}, 7\right], \left[\frac{7}{13}, 7\right], \left[\frac{2}{13}, 7\right], \left[\frac{8}{13}, 7\right], \left[\frac{10}{13}, 7\right], \left[\frac{5}{13}, 7\right], \left[\frac{9}{26}, 0\right],$$

$$\left[\frac{6}{13}, 7\right], \left[\frac{25}{39}, 7\right], \left[\frac{19}{39}, 7\right], \left[\frac{34}{39}, 7\right], \left[\frac{23}{39}, 7\right], \left[\frac{31}{39}, 7\right], \left[\frac{2}{39}, 7\right], \left[\frac{12}{13}, 7\right], \left[\frac{1}{39}, 7\right], \left[\frac{16}{39}, 7\right], \left[\frac{7}{39}, 7\right],$$

$$\left[\frac{22}{39}, 7\right], \left[\frac{11}{39}, 7\right], \left[\frac{17}{39}, 7\right], \left[\frac{7}{26}, 0\right], \left[\frac{15}{26}, 0\right], \left[\frac{25}{26}, 0\right], \left[\frac{1}{52}, 0\right], \left[\frac{17}{26}, 0\right], \left[\frac{19}{26}, 0\right], \left[\frac{21}{26}, 0\right], \left[\frac{23}{26}, 0\right],$$

$$\left[\frac{1}{26}, 0\right], \left[\frac{3}{26}, 0\right], \left[\frac{23}{52}, 0\right], \left[\frac{31}{52}, 0\right], \left[\frac{41}{52}, 0\right], \left[\frac{51}{52}, 0\right], \left[\frac{11}{52}, 0\right], \left[\frac{17}{52}, 0\right], \left[\frac{19}{52}, 0\right], \left[\frac{21}{52}, 0\right], \left[\frac{8}{65}, -9\right],$$

$$\left[\frac{12}{65}, -9\right], \left[\frac{6}{65}, 1\right], \left[\frac{5}{26}, 0\right], \left[\frac{4}{65}, 1\right], \left[\frac{14}{65}, 1\right], \left[\frac{16}{65}, 1\right], \left[\frac{18}{65}, -9\right], \left[\frac{22}{65}, -9\right], \left[\frac{24}{65}, 1\right], \left[\frac{29}{65}, 1\right], \left[\frac{31}{65}, 1\right],$$

$$\left[\frac{2}{65}, -9\right], \left[\frac{11}{65}, 1\right], \left[\frac{17}{65}, -9\right], \left[\frac{19}{65}, 1\right], \left[\frac{21}{65}, 1\right], \left[\frac{23}{65}, -9\right], \left[\frac{27}{65}, -9\right], \left[\frac{1}{65}, 1\right], \left[\frac{3}{65}, -9\right], \left[\frac{7}{65}, -9\right],$$

$$\left[\frac{9}{65}, 1\right], \left[\frac{28}{65}, -9\right], \left[\frac{32}{65}, -9\right], \left[\frac{27}{130}, -6\right], \left[\frac{29}{130}, 0\right], \left[\frac{31}{130}, 0\right], \left[\frac{33}{130}, -6\right], \left[\frac{37}{130}, -6\right], \left[\frac{41}{130}, 0\right],$$

$$\left[\frac{19}{130}, 0\right], \left[\frac{21}{130}, 0\right], \left[\frac{23}{130}, -6\right], \left[\frac{7}{55}, 0\right], \left[\frac{11}{130}, 0\right], \left[\frac{17}{130}, -6\right], \left[\frac{3}{130}, -6\right], \left[\frac{7}{130}, -6\right], \left[\frac{3}{52}, 0\right], \left[\frac{11}{26}, 0\right],$$

$$\left[\frac{49}{130}, 0\right], \left[\frac{53}{130}, -6\right]\Big]\Big]$$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 130

"val0=", 0

"which is even."

"valinf=", 0

"which is even."

"It IS a modfunc on Gamma1(", 130, ")"

"TERM ", 5, "of ", 5, " ***"

$$\text{"XX=", } q\, \mathrm{JAC}(1, 130, \infty)\, \mathrm{JAC}(2, 130, \infty)^2\, \mathrm{JAC}(3, 130, \infty)\, \mathrm{JAC}(5, 130, \infty)\, \mathrm{JAC}(7, 130, \infty)$$
$$\mathrm{JAC}(8, 130, \infty)^2\, \mathrm{JAC}(9, 130, \infty)\, \mathrm{JAC}(11, 130, \infty)\, \mathrm{JAC}(12, 130, \infty)^2\, \mathrm{JAC}(13, 130, \infty)^2$$
$$\mathrm{JAC}(15, 130, \infty)\, \mathrm{JAC}(17, 130, \infty)\, \mathrm{JAC}(18, 130, \infty)^2\, \mathrm{JAC}(19, 130, \infty)\, \mathrm{JAC}(21, 130, \infty)$$
$$\mathrm{JAC}(22, 130, \infty)^2\, \mathrm{JAC}(23, 130, \infty)\, \mathrm{JAC}(25, 130, \infty)\, \mathrm{JAC}(27, 130, \infty)\, \mathrm{JAC}(28, 130, \infty)^2$$
$$\mathrm{JAC}(29, 130, \infty)\, \mathrm{JAC}(31, 130, \infty)\, \mathrm{JAC}(32, 130, \infty)^2\, \mathrm{JAC}(33, 130, \infty)\, \mathrm{JAC}(35, 130, \infty)$$
$$\mathrm{JAC}(37, 130, \infty)\, \mathrm{JAC}(38, 130, \infty)^2\, \mathrm{JAC}(41, 130, \infty)\, \mathrm{JAC}(42, 130, \infty)^2\, \mathrm{JAC}(43, 130, \infty)$$
$$\mathrm{JAC}(45, 130, \infty)\, \mathrm{JAC}(47, 130, \infty)\, \mathrm{JAC}(48, 130, \infty)^2\, \mathrm{JAC}(49, 130, \infty)\, \mathrm{JAC}(51, 130, \infty)$$
$$\mathrm{JAC}(52, 130, \infty)^4\, \mathrm{JAC}(53, 130, \infty)\, \mathrm{JAC}(55, 130, \infty)\, \mathrm{JAC}(57, 130, \infty)\, \mathrm{JAC}(58, 130, \infty)^2$$

$$\mathrm{JAC}(59, 130, \infty)\, \mathrm{JAC}(61, 130, \infty)\, \mathrm{JAC}(62, 130, \infty)^2\, \mathrm{JAC}(63, 130, \infty) \Big/ \mathrm{JAC}(0, 130, \infty)^{60}$$

"Cusp ORDS: "

$$\left[[oo, 1], \left[\frac{57}{130}, -5\right], \left[\frac{59}{130}, 1\right], \left[\frac{63}{130}, -5\right], \left[\frac{61}{130}, 1\right], \left[\frac{9}{52}, 5\right], \left[\frac{43}{130}, -5\right], \left[\frac{1}{47}, 5\right], \left[\frac{1}{49}, 5\right], \left[\frac{1}{51}, 5\right], \right.$$

$$\left[\frac{1}{53}, 5\right], \left[\frac{1}{57}, 5\right], \left[\frac{1}{59}, 5\right], \left[\frac{1}{37}, 5\right], \left[\frac{1}{41}, 5\right], \left[\frac{1}{43}, 5\right], \left[\frac{1}{27}, 5\right], \left[\frac{1}{29}, 5\right], \left[\frac{1}{31}, 5\right], \left[\frac{1}{33}, 5\right], \left[\frac{1}{7}, 5\right],$$

$$\left[\frac{1}{9}, 5\right], \left[\frac{1}{11}, 5\right], \left[\frac{1}{17}, 5\right], \left[\frac{1}{19}, 5\right], \left[\frac{1}{21}, 5\right], \left[\frac{1}{23}, 5\right], [0, 5], \left[\frac{1}{3}, 5\right], \left[\frac{1}{61}, 5\right], \left[\frac{1}{63}, 5\right], \left[\frac{1}{58}, 2\right],$$

$$\left[\frac{1}{62}, 2\right], \left[\frac{1}{4}, 2\right], \left[\frac{1}{8}, 2\right], \left[\frac{1}{54}, 2\right], \left[\frac{1}{42}, 2\right], \left[\frac{1}{46}, 2\right], \left[\frac{1}{38}, 2\right], \left[\frac{1}{34}, 2\right], \left[\frac{1}{6}, 2\right], \left[\frac{1}{14}, 2\right], \left[\frac{1}{18}, 2\right],$$

$$\left[\frac{1}{22}, 2\right], \left[\frac{7}{52}, 5\right], \left[\frac{1}{2}, 2\right], \left[\frac{1}{48}, 2\right], \left[\frac{1}{56}, 2\right], \left[\frac{1}{64}, 2\right], \left[\frac{1}{44}, 2\right], \left[\frac{1}{28}, 2\right], \left[\frac{1}{32}, 2\right], \left[\frac{1}{36}, 2\right], \left[\frac{1}{24}, 2\right],$$

$$\left[\frac{1}{12}, 2\right], \left[\frac{1}{16}, 2\right], \left[\frac{9}{130}, 1\right], \left[\frac{7}{15}, 1\right], \left[\frac{4}{15}, -5\right], \left[\frac{1}{35}, -5\right], \left[\frac{3}{35}, 1\right], \left[\frac{1}{15}, -5\right], \left[\frac{13}{15}, 1\right], \left[\frac{1}{5}, -5\right], \left[\frac{3}{5}, 1\right],$$

$$\left[\frac{2}{5}, 1\right], \left[\frac{4}{5}, -5\right], \left[\frac{51}{130}, 1\right], \left[\frac{3}{25}, 1\right], \left[\frac{7}{25}, 1\right], \left[\frac{9}{25}, -5\right], \left[\frac{7}{45}, 1\right], \left[\frac{4}{45}, -5\right], \left[\frac{1}{55}, -5\right], \left[\frac{3}{55}, 1\right], \left[\frac{9}{55}, -5\right],$$

$$\left[\frac{1}{25}, -5\right], \left[\frac{32}{35}, 1\right], \left[\frac{9}{35}, -5\right], \left[\frac{1}{45}, -5\right], \left[\frac{43}{45}, 1\right], \left[\frac{1}{30}, 0\right], \left[\frac{13}{30}, -10\right], \left[\frac{7}{30}, -10\right], \left[\frac{19}{30}, 0\right], \left[\frac{1}{40}, 0\right],$$

$$\left[\frac{3}{40}, -10\right], \left[\frac{1}{10}, 0\right], \left[\frac{3}{10}, -10\right], \left[\frac{7}{10}, -10\right], \left[\frac{9}{10}, 0\right], \left[\frac{47}{130}, -5\right], \left[\frac{3}{20}, -10\right], \left[\frac{7}{20}, -10\right], \left[\frac{9}{20}, 0\right],$$

$$\left[\frac{9}{50}, 0\right], \left[\frac{1}{20}, 0\right], \left[\frac{7}{60}, -10\right], \left[\frac{19}{60}, 0\right], \left[\frac{1}{50}, 0\right], \left[\frac{3}{50}, -10\right], \left[\frac{7}{50}, -10\right], \left[\frac{7}{40}, -10\right], \left[\frac{9}{40}, 0\right], \left[\frac{1}{60}, 0\right],$$

$$\left[\frac{13}{60}, -10\right], \left[\frac{9}{13}, 2\right], \left[\frac{11}{13}, 2\right], \left[\frac{4}{13}, 2\right], \left[\frac{1}{13}, 2\right], \left[\frac{3}{13}, 2\right], \left[\frac{7}{13}, 2\right], \left[\frac{2}{13}, 2\right], \left[\frac{8}{13}, 2\right], \left[\frac{10}{13}, 2\right], \left[\frac{5}{13}, 2\right],$$

$$\left[\frac{9}{26}, 5\right], \left[\frac{6}{13}, 2\right], \left[\frac{25}{39}, 2\right], \left[\frac{19}{39}, 2\right], \left[\frac{34}{39}, 2\right], \left[\frac{23}{39}, 2\right], \left[\frac{31}{39}, 2\right], \left[\frac{2}{39}, 2\right], \left[\frac{12}{13}, 2\right], \left[\frac{1}{39}, 2\right], \left[\frac{16}{39}, 2\right],$$

$$\left[\frac{7}{39}, 2\right], \left[\frac{22}{39}, 2\right], \left[\frac{11}{39}, 2\right], \left[\frac{17}{39}, 2\right], \left[\frac{7}{26}, 5\right], \left[\frac{15}{26}, 5\right], \left[\frac{25}{26}, 5\right], \left[\frac{1}{52}, 5\right], \left[\frac{17}{26}, 5\right], \left[\frac{19}{26}, 5\right], \left[\frac{21}{26}, 5\right],$$

$$\left[\frac{23}{26}, 5\right], \left[\frac{1}{26}, 5\right], \left[\frac{3}{26}, 5\right], \left[\frac{23}{52}, 5\right], \left[\frac{31}{52}, 5\right], \left[\frac{41}{52}, 5\right], \left[\frac{51}{52}, 5\right], \left[\frac{11}{52}, 5\right], \left[\frac{17}{52}, 5\right], \left[\frac{19}{52}, 5\right], \left[\frac{21}{52}, 5\right],$$

$$\left[\frac{8}{65}, -10\right], \left[\frac{12}{65}, -10\right], \left[\frac{6}{65}, 0\right], \left[\frac{5}{26}, 5\right], \left[\frac{4}{65}, 0\right], \left[\frac{14}{65}, 0\right], \left[\frac{16}{65}, 0\right], \left[\frac{18}{65}, -10\right], \left[\frac{22}{65}, -10\right], \left[\frac{24}{65}, 0\right],$$

$$\left[\frac{29}{65}, 0\right], \left[\frac{31}{65}, 0\right], \left[\frac{2}{65}, -10\right], \left[\frac{11}{65}, 0\right], \left[\frac{17}{65}, -10\right], \left[\frac{19}{65}, 0\right], \left[\frac{21}{65}, 0\right], \left[\frac{23}{65}, -10\right], \left[\frac{27}{65}, -10\right], \left[\frac{1}{65}, 0\right],$$

$$\left[\frac{3}{65}, -10\right], \left[\frac{7}{65}, -10\right], \left[\frac{9}{65}, 0\right], \left[\frac{28}{65}, -10\right], \left[\frac{32}{65}, -10\right], \left[\frac{27}{130}, -5\right], \left[\frac{29}{130}, 1\right], \left[\frac{31}{130}, 1\right], \left[\frac{33}{130}, -5\right],$$

$$\left[\frac{37}{130}, -5\right], \left[\frac{41}{130}, 1\right], \left[\frac{19}{130}, 1\right], \left[\frac{21}{130}, 1\right], \left[\frac{23}{130}, -5\right], \left[\frac{7}{55}, 1\right], \left[\frac{11}{130}, 1\right], \left[\frac{17}{130}, -5\right], \left[\frac{3}{130}, -5\right],$$
$$\left[\frac{7}{130}, -5\right], \left[\frac{3}{52}, 5\right], \left[\frac{11}{26}, 5\right], \left[\frac{49}{130}, 1\right], \left[\frac{53}{130}, -5\right]\right]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 130

"val0=", 10

"which is even."

"valinf=", 2

"which is even."

"It IS a modfunc on Gamma1(", 130, ")"

"min inf ord=", 0

```
"mintotord = ", -384
"TO PROVE the identity we need to show that v[oo](ID) > ", 384
"*** There were NO errors. ***"
"*** WARNING: some terms were constants. ***"
"See array CONTERMS."
To prove the identity we will need to verify if up to
q^(384).
Do you want to prove the identity? (yes/no)
```

```
> yes
```

```
You entered yes.
We verify the identity to O(q^(644)).
```

0

```
0 was returned and this proves the identity.
```

==
==========

EXAMPLE 3: The Rogers-Ramanujan Continued Fraction

```
> rr:=1+q^10:
> for j from 9 by -1 to 1 do
>    rr:= 1 + q^j/rr:
> od:
> rr:=1/rr;
```

$$rr := \cfrac{1}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cfrac{q^4}{1+\cfrac{q^5}{1+\cfrac{q^6}{1+\cfrac{q^7}{1+\cfrac{q^8}{1+\cfrac{q^9}{1+q^{10}}}}}}}}}}}$$

```
> prodmake(rr,q,10);
```

$$\frac{(1-q)\,(1-q^4)\,(1-q^6)\,(1-q^9)}{(1-q^2)\,(1-q^3)\,(1-q^7)\,(1-q^8)}$$

```
> R5:=JAC(5,25,infinity)/JAC(10,25,infinity)*q;
```

$$R5 := \frac{\mathrm{JAC}(5, 25, \infty)\, q}{\mathrm{JAC}(10, 25, \infty)}$$

```
> ROGRAMID:=1/R5 - 1 - R5 - JAC(0,1,infinity)/JAC(0,25,infinity)/q;
```

$$ROGRAMID := \frac{\mathrm{JAC}(10, 25, \infty)}{\mathrm{JAC}(5, 25, \infty)\, q} - 1 - \frac{\mathrm{JAC}(5, 25, \infty)\, q}{\mathrm{JAC}(10, 25, \infty)} - \frac{\mathrm{JAC}(0, 1, \infty)}{\mathrm{JAC}(0, 25, \infty)\, q}$$

```
> ROGRAMIDa:=mixedjac2jac(ROGRAMID,100);
"term ", 1, "of ", 4
"term ", 2, "of ", 4
"term ", 3, "of ", 4
"term ", 4, "of ", 4
```

$$ROGRAMIDa := \frac{\mathrm{JAC}(10, 25, \infty)}{\mathrm{JAC}(5, 25, \infty)\, q} - 1 - \frac{\mathrm{JAC}(5, 25, \infty)\, q}{\mathrm{JAC}(10, 25, \infty)} - \mathrm{JAC}(1, 25, \infty)\, \mathrm{JAC}(2, 25, \infty)\, \mathrm{JAC}(3, 25, \infty)\, \mathrm{JAC}(4, 25, \infty)\, \mathrm{JAC}(5, 25, \infty)\, \mathrm{JAC}(6, 25, \infty)\, \mathrm{JAC}(7, 25, \infty)\, \mathrm{JAC}(8, 25, \infty)\, \mathrm{JAC}(9, 25, \infty)\, \mathrm{JAC}(10, 25, \infty)\, \mathrm{JAC}(11, 25, \infty)\, \mathrm{JAC}(12, 25, \infty) \,/\, (q\, \mathrm{JAC}(0, 25, \infty)^{12})$$

We calculate a set of inequivalent cusps for $\Gamma_1(25)$

and the width of each cusp. Note: oo is the first cusp in the list.

```
> cusps25:=cuspmake1(25):
> cusp25:=cusps25 minus {[1,0]}:
> cusps25:=convert(cusp25,list):
> wids25:=map(x->cuspwid1(x[1],x[2],25),cusps25):
> wids25:=[1,op(wids25)]:
> CUSPS25:=map(x->x[1]/x[2],cusps25):
> CUSPS25:=[oo,op(CUSPS25)];
```

$CUSPS25 :=$

$$\left[oo, \frac{12}{25}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{1}{4}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{11}, \frac{1}{12}, 0, \frac{1}{2}, \frac{1}{3}, \frac{7}{25}, \frac{8}{25}, \frac{9}{25}, \frac{11}{25}, \frac{2}{25}, \frac{3}{25}, \frac{4}{25}, \frac{6}{25}, \frac{3}{10}, \frac{9}{10}, \frac{4}{5}, \frac{1}{10}, \frac{7}{10}\right]$$

```
> nops(CUSPS25);
```

28

```
> provemodfuncid(ROGRAMIDa,CUSPS25,wids25,25);
"TERM ", 1, "of ", 4, " *****************************************************************"
```

$$\text{"XX=",}\ \frac{\mathrm{JAC}(10, 25, \infty)}{\mathrm{JAC}(5, 25, \infty)\, q}$$

"Cusp ORDS: "

$$\left[[oo, -1], \left[\frac{12}{25}, 1\right], \left[\frac{1}{5}, 0\right], \left[\frac{2}{5}, 0\right], \left[\frac{3}{5}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{7}, 0\right], \left[\frac{1}{8}, 0\right], \left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{12}, 0\right], [0, 0], \left[\frac{1}{2}, 0\right], \left[\frac{1}{3}, 0\right], \left[\frac{7}{25}, 1\right], \left[\frac{8}{25}, 1\right], \left[\frac{9}{25}, -1\right], \left[\frac{11}{25}, -1\right], \left[\frac{2}{25}, 1\right], \left[\frac{3}{25}, 1\right], \left[\frac{4}{25}, -1\right], \left[\frac{6}{25}, -1\right], \left[\frac{3}{10}, 0\right], \left[\frac{9}{10}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{1}{10}, 0\right], \left[\frac{7}{10}, 0\right]\right]$$

"TOTAL ORD = ", 0

```
"POWER of q CORRECT"
```

"All n are divisors of ", 25

"val0=", 0

"which is even."

"valinf=", -2

"which is even."

"It IS a modfunc on Gamma1(", 25, ")"

```
"TERM ", 2, "of ", 4, " *****************************************************************"
```

"XX=", -1

```
"TERM ", 3, "of ", 4, " *****************************************************************"
```

$$\text{"XX=",}\ -\frac{\mathrm{JAC}(5, 25, \infty)\, q}{\mathrm{JAC}(10, 25, \infty)}$$

"Cusp ORDS: "

$$\left[[oo, 1], \left[\frac{12}{25}, -1\right], \left[\frac{1}{5}, 0\right], \left[\frac{2}{5}, 0\right], \left[\frac{3}{5}, 0\right], \left[\frac{1}{4}, 0\right], \left[\frac{1}{6}, 0\right], \left[\frac{1}{7}, 0\right], \left[\frac{1}{8}, 0\right], \left[\frac{1}{9}, 0\right], \left[\frac{1}{11}, 0\right], \left[\frac{1}{12}, 0\right], [0, 0], \left[\frac{1}{2}, 0\right], \left[\frac{1}{3}, 0\right], \left[\frac{7}{25}, -1\right], \left[\frac{8}{25}, -1\right], \left[\frac{9}{25}, 1\right], \left[\frac{11}{25}, 1\right], \left[\frac{2}{25}, -1\right], \left[\frac{3}{25}, -1\right], \left[\frac{4}{25}, 1\right], \left[\frac{6}{25}, 1\right], \left[\frac{3}{10}, 0\right], \left[\frac{9}{10}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{1}{10}, 0\right], \left[\frac{7}{10}, 0\right]\right]$$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 25

"val0=", 0

"which is even."

"valinf=", 2

"which is even."

"It IS a modfunc on Gamma1(", 25, ")"

"TERM ", 4, "of ", 4, " **"

$$"XX=", -\mathrm{JAC}(1, 25, \infty)\,\mathrm{JAC}(2, 25, \infty)\,\mathrm{JAC}(3, 25, \infty)\,\mathrm{JAC}(4, 25, \infty)\,\mathrm{JAC}(5, 25, \infty)\,\mathrm{JAC}(6, 25, \infty)\,\mathrm{JAC}(7, 25, \infty)\,\mathrm{JAC}(8, 25, \infty)\,\mathrm{JAC}(9, 25, \infty)\,\mathrm{JAC}(10, 25, \infty)\,\mathrm{JAC}(11, 25, \infty)\,\mathrm{JAC}(12, 25, \infty) / (q\,\mathrm{JAC}(0, 25, \infty)^{12})$$

"Cusp ORDS: "

$$\left[[oo, -1], \left[\frac{12}{25}, -1\right], \left[\frac{1}{5}, 0\right], \left[\frac{2}{5}, 0\right], \left[\frac{3}{5}, 0\right], \left[\frac{1}{4}, 1\right], \left[\frac{1}{6}, 1\right], \left[\frac{1}{7}, 1\right], \left[\frac{1}{8}, 1\right], \left[\frac{1}{9}, 1\right], \left[\frac{1}{11}, 1\right], \left[\frac{1}{12}, 1\right], [0, 1], \left[\frac{1}{2}, 1\right], \left[\frac{1}{3}, 1\right], \left[\frac{7}{25}, -1\right], \left[\frac{8}{25}, -1\right], \left[\frac{9}{25}, -1\right], \left[\frac{11}{25}, -1\right], \left[\frac{2}{25}, -1\right], \left[\frac{3}{25}, -1\right], \left[\frac{4}{25}, -1\right], \left[\frac{6}{25}, -1\right], \left[\frac{3}{10}, 0\right], \left[\frac{9}{10}, 0\right], \left[\frac{4}{5}, 0\right], \left[\frac{1}{10}, 0\right], \left[\frac{7}{10}, 0\right]\right]$$

"TOTAL ORD = ", 0

"POWER of q CORRECT"

"All n are divisors of ", 25

"val0=", 2

"which is even."

"valinf=", -2

"which is even."

"It IS a modfunc on Gamma1(", 25, ")"

"min inf ord=", -1

```
"mintotord = ", -9
"TO PROVE the identity we need to show that v[oo](ID) > ", 9
"*** There were NO errors. ***"
"*** WARNING: some terms were constants. ***"
"See array CONTERMS."
To prove the identity we will need to verify if up to
q^(9).
Do you want to prove the identity? (yes/no)
> yes
You entered yes.
We verify the identity to O(q^(59)).
```

0

```
0 was returned and this proves the identity.
```

>