Chapter 2 Infinite Series Generating Functions & Basic Hypergeometric Series

Let $p(m,n)$ = # of partitions of n into m parts.

~~Let $p_k(m,n)$ = # of partitions of n into m parts $\le k$~~

Note If $m>n$ then $p(m,n)=0$.

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} p(m,n)\, z^m q^n = \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n} p(m,n)\, z^m\right) q^n$$

$$= 1 + (z^1 q^1) + (z^1 q^2 + z^2 q^{1+1})$$

$$+ (z q^3 + z^2 q^{2+1} + z^3 q^{1+1+1}) + \cdots$$

$$= \sum_{\lambda\in P} z^{\#(\lambda)} q^{|\lambda|}$$

$$= 1 + (zq) + (z+z^2)q^2 + (z+z^2+z^3)q^3 + \cdots$$

$$\cancel{= \sum z^{a_1+a_2+\cdots+a_k}}$$

$$\cancel{= (1 + zq^1 + z^2 q^{1+1} + z^3 q^{1+1+1} + \cdots)}$$

$$\cancel{(1 + zq^2 + z^2 q^{2+2} + z^3 q^{2+2+2} + \cdots)}$$

$\vdots$

Let $p_k(m,n)$ = # of ptns of n into m parts $\le k$

Each ptn of n into m parts $\le k$ can be be uniquely written ~~as~~

$$\lambda : n = 1a_1 + 2a_2 + \cdots + k a_k \quad \text{where } a_i \ge 0$$

where $a_1 + a_2 + \cdots + a_k = m$

Let P_k = set of ptns into parts $\le k$