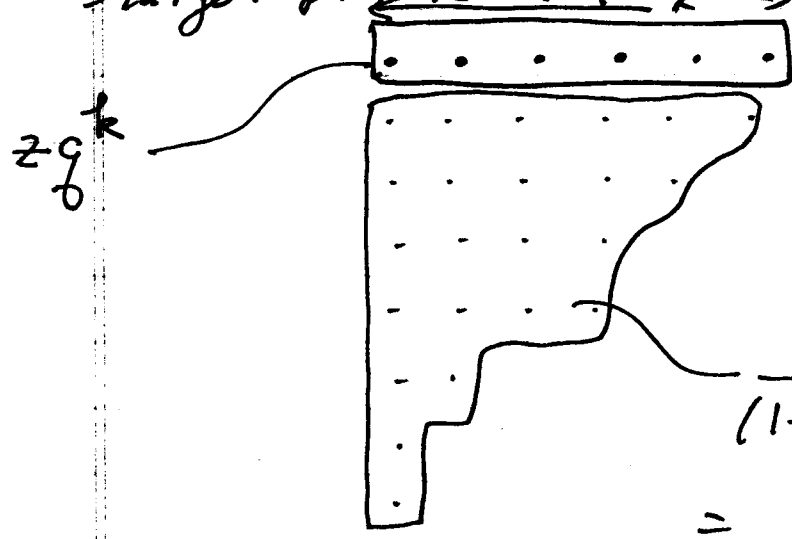


Another identity for $1/(zq)_\infty$

(10A)

Let $k > 1$. Let $\tilde{\mathcal{P}}_k$ be the set of partitions whose largest part is k . $k \rightarrow$



If we remove the largest part, what remains is a partition into parts $\leq k$

$$\frac{1}{(1-zq)} \frac{1}{(1-zq^2)} \cdots \frac{1}{(1-zq^k)} = \frac{1}{(zq)_k}$$

Hence

$$\sum_{\lambda \in \tilde{\mathcal{P}}_k} z^{|\lambda|} q^{|\lambda|} = \frac{zq^k}{(zq)_k}$$

Since

$$\mathcal{P} = \cup \tilde{\mathcal{P}}_k \quad (\text{disjoint})$$

$$\sum_{\lambda \in \mathcal{P}} z^{|\lambda|} q^{|\lambda|} = \sum_{k \geq 0} \sum_{\lambda \in \tilde{\mathcal{P}}_k} z^{|\lambda|} q^{|\lambda|}$$

$$\& \quad 1 + \sum_{k=1}^{\infty} \frac{zq^k}{(zq)_k} = \frac{1}{(zq)_\infty} \quad \text{for } |q| < 1 \text{ (} |zq| < 1 \text{)}$$