$$= \frac{(b)_\infty}{(c)_\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(c/b)_m\, b^m}{(q)_m} \frac{(a)_n (q^m t)^n}{(q)_n}$$

$$= \frac{(b)_\infty}{(c)_\infty} \sum_{m=0}^{\infty} \frac{(c/b)_m\, b^m}{(q)_m} \sum_{n=0}^{\infty} \frac{(a)_n (q^m t)^n}{(q)_n}$$

$$= \frac{(b)_\infty}{(c)_\infty} \sum_{m=0}^{\infty} \frac{(c/b)_m\, b^m}{(q)_m} \frac{(aq^m t)_\infty}{(tq^m)_\infty}$$

$$= \frac{(b)_\infty}{(c)_\infty} \sum_{m=0}^{\infty} \frac{(c/b)_m\, b^m}{(q)_m} \frac{(at)_\infty}{(at)_m} \cdot \frac{(t)_m}{(t)_\infty}$$

$$= \frac{(b)_\infty (at)_\infty}{(c)_\infty (t)_\infty} \sum_{m=0}^{\infty} \frac{(c/b)_m (t)_m\, b^m}{(at)_m (q)_m}. \qquad \Box$$

<u>Corollary</u> (Heine) Suppose $|q|<1$ & $|c|<|ab|$. Then

$$1 + \sum_{n=1}^{\infty} \frac{(a)_m (b)_m}{(c)_m (q)_n} \left(\frac{c}{ab}\right)^n = \frac{(c/a)_\infty (c/b)_\infty}{(c)_\infty (c/(ab))_\infty}$$

<u>Proof:</u> By Heine's transformation,

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (q)_n} \left(\frac{c}{ab}\right)^n = \frac{(b)_\infty \left(\frac{c}{b}\right)_\infty}{(c)_\infty \left(\frac{c}{ab}\right)_\infty} \sum_{m=0}^{\infty} \frac{\cancel{\left(\frac{c}{b}\right)_n} \left(\frac{c}{ab}\right)_n b^n}{\cancel{\left(\frac{c}{b}\right)_m} (q)_n}$$

(if $|q|<1$, $|c|<|ab|$ & $|b|<1$)

$$= \frac{(b)_\infty \left(\frac{c}{b}\right)_\infty}{(c)_\infty \left(\frac{c}{ab}\right)_\infty} \frac{\left(\frac{c}{a}\right)_\infty}{(b)_\infty} \qquad \text{(by q-bin. Thm)}$$