$$= \frac{\left(\frac{c}{a}\right)_\infty \left(\frac{c}{b}\right)_\infty}{(c)_\infty \left(\frac{c}{ab}\right)_\infty}.$$

Result holds for general b ($b \neq 0$) by analytic continuation. $\square$

<u>Corollary</u> (Bailey) If $|q| < \min(1, |b|)$ then

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{\left(\frac{aq}{b}\right)_n (q)_n} \left(\frac{-q}{b}\right)^n = \frac{(aq;q^2)_\infty (-q;q)_\infty \left(\frac{aq^2}{b^2};q^2\right)_\infty}{\left(\frac{aq}{b};q\right)_\infty \left(-\frac{q}{b};q\right)_\infty}$$

Proof:

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n \left(\frac{-q}{b}\right)^n}{\left(\frac{aq}{b}\right)_n (q)_n} = \sum_{n=0}^{\infty} \frac{(b)_n (a)_n \left(\frac{-q}{b}\right)^n}{\left(\frac{aq}{b}\right)_n (q)_n}$$

$$= \frac{(a)_\infty (-q)_\infty}{\left(\frac{aq}{b}\right)_\infty \left(\frac{-q}{b}\right)_\infty} \sum_{n=0}^{\infty} \frac{\left(\frac{q}{b}\right)_n \left(\frac{-q}{b}\right)_n}{(-q)_n (q)_n} a^n$$

(if $|q| < 1$, $\left|\frac{q}{b}\right| < 1$ & $|a| < 1$)

$$= \frac{(a)_\infty (-q)_\infty}{\left(\frac{aq}{b}\right)_\infty \left(\frac{-q}{b}\right)_\infty} \sum_{n=0}^{\infty} \frac{\left(\frac{q^2}{b^2}; q^2\right)_n}{(q^2;q^2)_n} a^n$$

$$= \frac{(a)_\infty (-q)_\infty}{\left(\frac{aq}{b}\right)_\infty \left(\frac{-q}{b}\right)_\infty} \frac{\left(\frac{aq^2}{b^2}; q^2\right)_\infty}{(a;q^2)_\infty}$$