$$= \frac{(q;q^2)_\infty \, (\frac{aq}{b};q^2)_\infty \, (-q)_\infty \, (\frac{aq^2}{b^2};q^2)_\infty}{(\frac{aq}{b})_\infty \, (-q/b)_\infty \, (q;q^2)_\infty}$$

$$= \frac{(\frac{aq}{b};q^2)_\infty \, (-q;q)_\infty \, (aq^2/b^2;q^2)_\infty}{(\frac{aq}{b};q)_\infty \, (-\frac{q}{b})_\infty} .$$

Result holds for $|q| < \min(1, |b|)$ & general a (by analytic continuation in a). $\square$

<u>Corollary</u> Suppose $|q|<1$.

(1) $$\sum_{n=0}^{\infty} \frac{z^n q^{n^2-n}}{(q)_n (z)_n} = \frac{1}{(z)_\infty} \qquad \text{(Cauchy)}$$

(2) $$\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q)_n^2} = \frac{1}{(q)_\infty} \qquad \text{(Euler)}$$

<u>Proof</u>: In Heine's Corollary, let $a=\alpha^{-1}$, $b=\beta^{-1}$, $c=z$.

$$\sum_{n=0}^{\infty} \frac{(\frac{1}{\alpha})_n (\frac{1}{\beta})_n \alpha^n \beta^n z^n}{(z)_n \, (q)_n} = \frac{(z\alpha)_\infty \, (z\beta)_\infty}{(z)_\infty \, (z\alpha\beta)_\infty}$$

provided $|q|<1$ & $|\alpha\beta z|<1$.

~~$\lim_{\alpha,\beta\to 0} (\frac{1}{\alpha})_n (\frac{1}{\beta})_n \alpha^n \beta^n = \lim_{\alpha,\beta\to 0} (\alpha-1) \ldots$~~