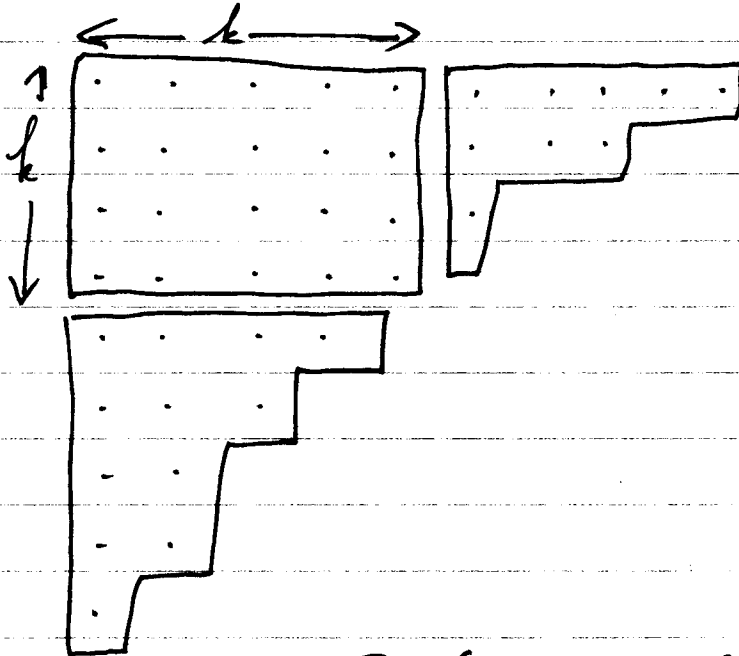


(18)

For a partition λ we let k be the side of the largest square
 Let k be the side of the largest square inside the top
 left part of the Ferrers diagram of λ .



We say λ has a Durfee square of side k .
 Let k be fixed. Let $\mathcal{P}(k)$ be set of partitions
 with Durfee square of side k . The right
 of the Durfee square we have a part with parts $\leq k$
 by reading columns. Below the Durfee square we
 have a partition with part $\leq k$.

$$\sum_{\lambda \in \mathcal{P}(k)} z^{|\lambda|} q^{|\lambda|} = z^k q^{k^2} \times \frac{1}{(q)_k} \times \frac{1}{(zq)_k}$$

Since $\mathcal{P} = \cup \mathcal{P}(k)$ (disjoint).

We have

$$\frac{1}{(zq)_\infty} = \sum_{\lambda \in \mathcal{P}} z^{|\lambda|} q^{|\lambda|} = \sum_{k=0}^{\infty} \sum_{\lambda \in \mathcal{P}(k)} z^{|\lambda|} q^{|\lambda|}$$

$$= \sum_{k=0}^{\infty} \frac{z^k q^{k^2}}{(zq)_k (q)_k} \quad \text{D}$$