<u>Corollary</u> If $|q|<1$, then

$$\sum_{n=0}^{\infty} \frac{(a)_n \, q^{n(n+1)/2}}{(q)_n} = (aq;q^2)_\infty (-q;q)_\infty$$

<u>Proof:</u> Suppose $|q|<1$. In Corollary (Bailey), let $b=\beta^{-1}$

$$\sum_{n=0}^{\infty} \frac{(a)_n \left(\frac{1}{\beta}\right)_n (-q\beta)^n}{(q)_n \, (a\beta q)_n} = \frac{(aq;q^2)_\infty (-q)_\infty (a\beta^2 q^2;q^2)_\infty}{(a\beta q)_\infty (-\beta q)_\infty}$$

$$\lim_{\beta\to 0} \left(\tfrac{1}{\beta}\right)_n (-\beta)^n = \lim_{\beta\to\infty} (\beta-1)(\beta-q)\cdots(\beta-q^{n-1})(-1)^n$$

$$= (-1)^n q^{n(n-1)/2}(-1)^n = q^{n(n-1)/2}$$

so

$$\sum_{n=0}^{\infty} \frac{(a)_n \, q^{n(n+1)/2}}{(q)_n} = (aq;q^2)_\infty (-q)_\infty . \quad \square$$

<u>JACOBI'S TRIPLE PRODUCT IDENTITY</u>

If $z\neq 0$, $|q|<1$, then

$$\sum_{n=-\infty}^{\infty} z^n q^{n^2} = (-zq;q^2)_\infty (-z^{-1}q;q^2)_\infty (q^2;q^2)_\infty .$$

<u>Note:</u> $$\sum_{n=-\infty}^{\infty} z^n q^{n^2} = 1 + \sum_{n=1}^{\infty} z^n q^{n^2} + \sum_{n=-1}^{-\infty} z^n q^{n^2}$$

$$= 1 + \sum_{n=1}^{\infty} z^n q^{n^2} + \sum_{n=1}^{\infty} z^{-n} q^{(-n)^2}$$

$$= 1 + \sum_{n=1}^{\infty} (z^n + z^{-n}) q^{n^2}.$$