COMBINATORIAL PROOF of JTP We write JTP in the equivalent form

$$(-zq;q)_\infty(-z^{-1};q)_\infty = \frac{1}{(q)_\infty}\sum_{n=-\infty}^{\infty} z^n q^{n(n+1)/2}$$

or

$$(**)\quad \prod_{n=1}^{\infty}(1+zq^n)(1+z^{-1}q^{n-1}) = \frac{1}{\prod_{m=1}^{\infty}(1-q^m)}\sum_{n=-\infty}^{\infty} z^n q^{n(n+1)/2}$$

The coeff of $z^k q^N$ on LHS

= # of pairs $(a_1, a_2, \ldots, a_m)$, $(b_1, b_2, \ldots, b_l)$
such that $a_1 > a_2 > \cdots > a_m \geq 1$
$b_1 > b_2 > \cdots > b_l \geq 0$
and $m - l = k$ and
$(a_1 + a_2 + \cdots + a_m) + (b_1 + b_2 + \cdots + b_l) = N$.

Suppose $k \geq 0$. For each such pair we form a diagram