

This gives rise to a partition
of $N - \frac{k(k+1)}{2}$. Hence

(25)

$$\text{Coeff of } z^k q^N = \text{Coeff of } z^k q^N \text{ in } \frac{z^k q^{k(k+1)/2}}{\prod_{n=1}^{\infty} (1 - q^n)}$$

Similarly result also holds for $k < 0$, & we obtain (**). \square

A generalization of Euler's Theorem that $p(O, n) = p(D, n)$

Theorem (Sylvester)

Let $k \geq 1$.

Let $A_k(n)$ = the number of partitions of n into odd parts (repetition allowed) into exactly k different parts.

Let $B_k(n)$ = the number of partitions of n into exactly k noncontiguous sequences of one or more consecutive integers.

Then $A_k(n) = B_k(n)$ for all n .

Example $n=14, k=3$.

Partitions of $n=14$ into 3 odd parts (repetition allowed) with 3 different parts:

$$\begin{aligned} &9 + 3 + 1 + 1 \\ &7 + 5 + 1 + 1 \\ &7 + 3 + 3 + 1 \\ &7 + 3 + 1 + 1 + 1 + 1 \\ &5 + 5 + 3 + 1 \\ &5 + 3 + 3 + 1 + 1 + 1 \\ &5 + 3 + 1 + 1 + 1 + 1 + 1 \end{aligned}$$

$$\text{So } A_3(14) = 7$$