Partitions of $n=14$ into exactly k noncontiguous sequences of one or more consecutive integers:

$10+3+1$

$9+4+1$

$8+4+2$

$8+5+1$

$7+5+2$

$7+4+\underline{2+1}$

$6+\underline{4+3}+1$

Suppose $|q|<1$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_k(n)\, a^k q^n$$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} A_k(n)\, a^k\right) q^n$$

$$= \prod_{j=1}^{\infty}\left(1 + aq^{2j-1} + aq^{2(2j-1)} + aq^{3(2j-1)} + \cdots\right)$$

$$= \prod_{j=1}^{\infty}\left(1 + \frac{aq^{2j-1}}{1-q^{2j-1}}\right)$$

$$= \prod_{j=1}^{\infty}\frac{(1-q^{2j-1}+aq^{2j-1})}{(1-q^{2j-1})} = \prod_{j=1}^{\infty}\frac{1-(1-a)q^{2j-1}}{1-q^{2j-1}}$$

$$= \frac{((1-a)q;q^2)_\infty}{(q;q^2)_\infty} = ((1-a)q;q^2)_\infty(-q)_\infty.$$