(28)

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} B_k(n)\, a^k q^n$$

largest part, part of a consecutive group

Case 1

$$= 1 + \sum_{N=1}^{\infty} a q^N \left(q^{N-1} + a q^{2(N-1)} + a q^{3(N-1)} + \cdots\right)$$
$$\cdot\left(q^{N-2} + a q^{2(N-2)} + \cdots\right)$$
$$\vdots$$
$$\cdot\left(q + a q^2 + a q^3 + \cdots\right)$$

Case 2

$$+ \sum_{N=1}^{\infty} \left(a q^{2N} + a q^{3N} + \cdots\right) \cdot \prod_{j=1}^{N-1}\left(q^j + a q^{2j} + a q^{3j} + \cdots\right)$$

$$= 1 + \sum_{N=1}^{\infty} a q^N \prod_{j=1}^{N-1} q^j\left(1 + \frac{a q^j}{1-q^j}\right)$$
$$+ \sum_{N=1}^{\infty} \frac{a q^{2N}}{1-q^N} \prod_{j=1}^{N-1} q^j \frac{(1-(1-a)q^j)}{(1-q^j)}$$

$$= 1 + \sum_{N=1}^{\infty} a\left(\frac{(1-q^N)q^N + q^{2N}}{(1-q^N)}\right) q^{N(N-1)/2} \frac{((1-a)q)_{N-1}}{\prod_{j=1}^{N-1}(1-q^j)}$$

$$= 1 + \sum_{N=1}^{\infty} \frac{(1-(1-a))((1-a)q)_{N-1}\, q^{N(N+1)/2}}{(q)_N}$$

$$= 1 + \sum_{N=1}^{\infty} \frac{((1-a)q)_N}{(q)_N} q^{N(N+1)/2}$$

$$= ((1-a)q; q^2)_\infty (-q)_\infty \quad \text{(by Lebesgue Identity)}$$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A_k(n)\, a^k q^n.$$ Hence $A_k(n) = B_k(n)$ for all n. ∎