$$(1-az)F(zq) = (1-az)\sum_{n=0}^{\infty} A_n (qz)^n$$

$$= \sum_{n=0}^{\infty} q^n A_n z^n - a\sum_{n=0}^{\infty} q^n z^{n+1} A_n$$

$$= A_0 + \sum_{n=1}^{\infty} q^n A_n z^n - a\sum_{n=1}^{\infty} q^{n-1} z^n A_{n-1}$$

$$= A_0 + \sum_{n=1}^{\infty} (q^n A_n - a q^{n-1} A_{n-1}) z^n.$$

Hence, for $n \geq 1$,

$$A_n - A_{n-1} = q^n A_n - a q^{n-1} A_{n-1}$$

$$(1-q^n) A_n = (1-aq^{n-1}) A_{n-1}$$

$$A_n = \frac{(1-aq^{n-1})}{(1-q^n)} A_{n-1}$$

$$= \frac{(1-aq^{n-1})}{(1-q^n)} \frac{(1-aq^{n-2})}{(1-q^{n-1})} \frac{(1-aq^{n-3})}{(1-q^{n-2})} \cdots \frac{(1-a)}{(1-q)} A_0$$

$$F(0) = \frac{(0)_\infty}{(0)_\infty} = \frac{1}{1} = A_0.$$

Hence $A_n = \dfrac{(a;q)_n}{(q)_n} = \dfrac{(a)_n}{(q)_n}$, for $n \geq 1$, &

$$F(z) = \frac{(az)}{(z)_\infty} = \sum_{n=0} a_n z^n = \sum_{n=0} \frac{(a)_n}{(q)_n} z^n. \quad \square$$

<u>Why q-binomial Theorem?</u>

Let $a = q^\alpha$ where $\alpha \in \mathbb{Z}$, $\alpha > 0$.

$$\frac{(a)_n}{(q)_n} = \frac{(1-q^\alpha)(1-q^{\alpha+1})\cdots(1-q^{\alpha+n-1})}{(1-q)(1-q^2)\cdots(1-q^n)}$$