$$= \frac{(1-q^{\alpha})(1-q^{\alpha+1})\cdots(1-q^{\alpha+n-1})}{(1-q)(1-q)\cdots(1-q)}$$

$$\cdot \frac{1}{1\cdot(1+q)(1+q+q^2)\cdots(1+q+\cdots q^{n-1})}$$

$$\lim_{q\to 1}\frac{1-q^j}{1-q} = \lim_{q\to 1}\frac{-jq^{j-1}}{-1} \quad (\text{by } L'H)$$

$$= j \qquad (\text{if } j\in\mathbb{Z}).$$

Hence $\lim_{q\to 1^-} \dfrac{(a)_n}{(q)_n} = \dfrac{(\alpha)(\alpha+1)\cdots(\alpha+n-1)}{(1)\ (2)\cdots(n)}$

$$\frac{(az)_\infty}{(z)_\infty} = \frac{(1-q^{\alpha}z)(1-q^{\alpha+1}z)\cdots\cdots}{(1-z)(1-zq)\cdots(1-zq^{\alpha})\cdots}$$

$$= \frac{1}{(1-z)(1-zq)\cdots(1-zq^{\alpha-1})}$$

$$\lim_{q\to 1^-}\frac{(q^{\alpha}z)}{(z)_\infty} = \frac{1}{(1-z)^{\alpha}}.$$

Hence (formally), we have

$$1+\sum_{n=1}^{\infty}\frac{\alpha(\alpha+1)\cdots(\alpha+n-1)}{n!}z^n = (1-z)^{-\alpha} \quad \text{for } |z|<1.$$

(Gen. Binomial Thm.).