Similarly

$$\lim_{b\to 0} \frac{(az)_\infty}{(bz)_\infty} = \frac{(az)_\infty}{(0)_\infty} = (az)_\infty.$$

Hence

$$\sum_{n=0}^{\infty} \frac{(-az)^n q^{n(n-1)/2}}{(q)_n} = (az)_\infty.$$

Replacing az by z gives the result. $\square$

<u>Combinatorial Proof of (1)</u> Replacing z by zq we write (1) in the equivalent form

$$\sum_{k=0}^{\infty} \frac{q^k z^k}{(q)_k} = \frac{1}{(zq)_\infty} = \prod_{k=1}^{\infty} \frac{1}{1-zq^k},$$

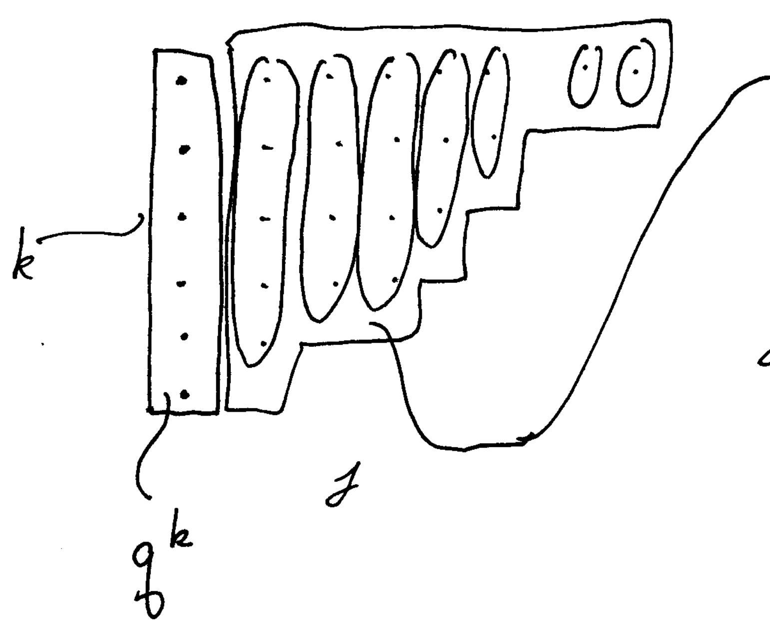
(provided $|q|<1$ & $|zq|<1$).

Recall

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} p(m,n) z^m q^n = \prod_{k=1}^{\infty} \frac{1}{1-zq^k}$$

$$\sum_{\lambda\in\mathcal{P}} z^{\#(\lambda)} q^{|\lambda|}$$

Let $k\geq 1$. Let $\hat{\mathcal{P}}_k$ be the set of partitions into k parts.



reading columns to the right of the first col. we obtain a partition with parts $\leq k$

& $GF = \frac{1}{(1-q)(1-q^2)}\cdots\frac{1}{(1-q^k)}$

$$= \frac{1}{(q)_k}$$