Chapter 3 Ramanujan's Partition Congruences

(See Ch 10 of Text).

In 1919, Ramanujan stated and proved the following congruences:

$$p(5n+4) \equiv 0 \pmod{5}$$

$$p(7n+5) \equiv 0 \pmod{7}$$

and $$p(11n+6) \equiv 0 \pmod{11},$$

for all $n \geq 0$.

He also conjectured that if $\delta = 5^a$, 7^b or 11^c and $24\lambda \equiv 1 \pmod{\delta}$. Then

$$p(\delta n + \lambda) \equiv 0 \pmod{\delta}.$$

Not quite correct. Correction made by Chowla (1934)

$$p(7^b n + \lambda_b) \equiv 0 \pmod{7^{[(b+2)/2]}}$$

where $24\lambda_b \equiv 1 \pmod{7^b}$.

Ramanujan's conjecture was proved by $\boxed{\text{Watson (1938)}}$ for $\delta = 5^a$, 7^b, and for $\delta = 11^c$ by $\boxed{\text{Atkin}}$ (1967).

~~Atkin and O'Brien (196?) found~~

$\cancel{p(11^3 \cdot 13n +}$

$\boxed{\text{Atkin (1969)}}$ found

$$p(59^4 \cdot 13n + 111247) \equiv 0 \pmod{13},$$

$$p(23^3 \cdot 17n + 2623) \equiv 0 \pmod{17}.$$

$\boxed{\text{Kolberg (1959)}}$ proved

$p(n) \equiv 0 \pmod{2}$ for infinitely many n

& $p(n) \equiv 1 \pmod{2}$ for infinitely many n