$$\frac{(q)_\infty^3}{(q^{2};q^{25})_\infty^3} = \left[J_0 - q + q^2 J_2\right]^3 \tag{10}$$

$$= \left(J_0^3 - 3q^5 J_2^2\right) + \left(-3J_0^2 q + q^6 J_2^3\right)$$

$$+ \left(3q^2 J_0^2 J_2 + 3J_0 q^2\right) + \left(-6q^3 J_0 J_2 - q^3\right)$$

$$+ \left(3J_0 q^4 J_2^2 + 3q^4 J_2\right)$$

$$= \left(J_0^3 - 3q^5 J_2^2\right) + q\left(q^5 J_2^3 - 3J_0^2\right)$$

$$+ 3q^2 (J_0)(J_0 J_2 + 1) - q^3 (1 + 6J_0 J_2)$$

$$+ 3J_2 q^4 (J_0 J_2 + 1).$$

Now $(q)_\infty^3 = \sum_{n \geq 0} (-1)^n (2n+1) q^{n(n+1)/2}$.

n	$n(n+1)/2 \pmod 5$
0	0
1	1
2	3
3	$6 \equiv 1$
4	0

Hence the powers series expansion of $(q)_\infty^3$ does not have terms involving q^{5n+2} & q^{5n+4}.