$$\sum_{n=0}^{\infty} p(n) q^n = \frac{1}{E(q)} = \frac{E(\zeta q)\,E(\zeta^2 q)\,E(\zeta^3 q)\,E(\zeta^4 q)}{E(q)\,E(\zeta q)\,E(\zeta^2 q)\,E(\zeta^3 q)\,E(\zeta^4 q)}$$

$$= \left(\frac{E(q^{25})}{E(q^5)^6}\right)\left(E(q^{25})\right)^4$$

$$\Big[(J_0^4 - 3q^5 J_0^{-1}) + q(J_0^3 + 2q^5 J_0^{-2}) + q^2(2J_0^2 - q^5 J_0^{-2}) + q^3(3J_0 + q^5 J_0^{-4}) + 5q^4 \Big]$$

Hence

$$\sum_{n=0}^{\infty} p(5n+4) q^{5n+4} = 5q^4 \frac{E(q^{25})^5}{E(q^5)^6},$$

$$\sum_{n=0}^{\infty} p(5n+4) q^{5n} = 5 \frac{E(q^{25})^5}{E(q^5)^6}$$

$$\sum_{n=0}^{\infty} p(5n+4) q^{n} = \frac{5\,E(q^5)^5}{E(q)^6} = 5\frac{(q^5;q^5)_\infty^5}{(q)_\infty^6}$$

$$= 5 \prod_{n=1}^{\infty} \frac{(1-q^{5n})^5}{(1-q^n)^6}. \qquad \square$$