$$A_{3n+2} = (-1)^{n+1} A_1 q_b^{3n(n-1)/2+2n}$$

$$= (-1)^{n+1} A_1 q_b^{n(3n+1)/2} \quad \text{for } n \geq 0.$$

Let $n > 0$,

$$A_{-3n+1} = A_{-(3n-1)} = q_b^{3n-1} A_{3n-1}$$

$$= q_b^{3n-1} A_{3(n-1)+2} = q_b^{3n-1} (-1)^n A_1 q_b^{(n-1)(3n-2)/2}$$

$$= (-1)^n A_1 q_b^{n(3n+1)/2}$$

Hence $A_{3n+1} = (-1)^n A_1 q_b^{n(3n+1)/2}$ for all n.

Let $n > 0$,

$$A_{-3n+2} = A_{-(3n-2)} = q_b^{3n-2} A_{3n-2}$$

$$= q_b^{3n-2} A_{3(n-1)+1} = q_b^{3n-2} A_1 q_b^{(n-1)(3n-4)/2} (-1)^{n-1}$$

$$= (-1)^{n-1} A_1 q_b^{n(3n-1)/2}, \quad \&$$

$$A_{3n+2} = (-1)^{n+1} A_1 q_b^{n(3n+1)/2} \quad \text{for all } n.$$

Therefore

$$F(a,b,q) = \sum_n A_{3n} a^{3n} + \sum_n A_{3n+1} a^{3n+1} + \sum_n A_{3n+2} a^{3n+2}$$

$$= \sum_n A_0 (-1)^n q_b^{3n(n-1)/2} a^{3n} + \sum_n (-1)^n A_1 q_b^{n(3n+1)/2} a^{3n+1}$$

$$+ \sum_n (-1)^{n+1} A_1 q_b^{n(3n+1)/2} a^{3n+2}$$