so (19)

$$F(a,b,q) = A_0(b,q)\sum_n (-1)^n q^{3n(n-1)/2} a^{3n}$$

$$+ A_1(b,q)\Big(\sum_n (-1)^n q^{n(3n-1)/2} a^{3n+1}$$

$$+\sum_n (-1)^{n+1} q^{n(3n+1)/2} a^{3n+2}\Big)$$

$$F(q, b, q^3) = (q;q^3)_\infty (q^2;q^3)_\infty \; (b;q^3)_\infty (q^3/b;q^3)_\infty$$
$$(q/b;q^3)_\infty (q^2 b;q^3)_\infty \; (qb;q^3)_\infty \; (q^2/b;q^3)_\infty$$
$$(q^3;q^3)_\infty^2$$

$$= (q)_\infty (b)_\infty (q/b)_\infty \quad (q^3;q^3)_\infty$$

$$= A_0(b,q^3)\sum_n (-1)^n q^{3n(3n-1)/2}$$

$$+ A_1(b,q^3)\Big(q\sum_n (-1)^n q^{3n(3n+1)/2}$$

$$+q^2\sum_n (-1)^{n+1} q^{\frac{9}{2}n(n+1)}\Big)$$

Note By JTP,

$$(z)_\infty (q/z)_\infty (q)_\infty = \sum_n (-1)^n z^n q^{n(n-1)/2}$$

Hence

$$(q)_\infty^2 (1)_\infty = 0 = \sum_n (-1)^n q^{n(n+1)/2}$$

&

$$\sum_n (-1)^{n+1} q^{\frac{9}{2}n(n+1)} = 0.$$