Also,

$$(q^3;q^3)_\infty = \sum_n (-1)^n q^{3n(3n-1)/2} = \sum_n (-1)^n q^{3n(3n+1)/2}$$

and

$$(b)_\infty (q/b)_\infty (q)_\infty = A_0(b,q^3) + q A_1(b,q^3)$$

$$\sum_n (-1)^n b^n q^{n(n-1)/2} = \underbrace{A_0(b,q^3) + q A_1(b,q^3)}_{\text{3-dissection (in } q)}$$

n	$\frac{n(n-1)}{2}$ (mod 3)
0	0
1	0
2	1

Hence,

$A_0(b,q^3)$ contains only terms of the form q^{3m} in its q-expan

so

$$A_0(b,q^3) = \sum_{n \equiv 0,1 \pmod 3} (-1)^n b^n q^{n(n-1)/2}$$

$$= \sum_j (-1)^{3j} b^{3j} q^{3j(3j-1)/2}$$

$$+ \sum_j (-1)^{3j+1} b^{3j+1} q^{(3j+1)(3j)/2}$$

$$= \sum_j (-1)^j b^{-3j} q^{3j(j+1)/2} - \sum_j (-1)^j b^{3j+1} q^{3j(3j+1)/2}$$

Hence $A_0(b,q^{\bullet}) = \sum_j (-1)^j (b^{-3j} - b^{3j+1}) q^{j(3j+1)/2}$.