$(b)_\infty = (1-b)(bq)_\infty$. We divide both sides of Winquist's identity by $1-b$ & let $b \to 1$.

$$\frac{b^{-3j} - b^{3j+1}}{1-b} = b^{-3j}\frac{(1-b^{6j+1})}{1-b} \longrightarrow 6j+1$$

$$\frac{b^{3i+2} - b^{-3i-1}}{1-b} = b^{-3i-1}\frac{(b^{6i+3}-1)}{1-b} \longrightarrow -(6i+3) = -3(2i+1)$$

We obtain

$$(a)_\infty^3 (q/a)_\infty^3 (q)_\infty^4$$

$$\cancel{= \sum_j (-1)^j}$$

$$= \sum_{i=0}^{\infty} \sum_{j=-\infty}^{\infty} (-1)^{i+j} \Big((6j+1)(a^{-3i} - a^{3i+3})$$
$$- 3(2i+1)(a^{-3j+1} - a^{3j+2})\Big)$$
$$\cdot\, q^{\frac{3}{2}i(i+1) + j(3j+1)/2}.$$

Next, $(a)_\infty^3 = (1-a)^3 (aq)_\infty^3$, ~~we~~ ~~divide both sides by ... & let a→1.~~

$LHS = (1-a)^3 \left((q)_\infty^{10} + c_1(a-1) + c_2(a-1)^2 + \cdots \right)$

$= -(q)_\infty^{10} (a-1)^3 + c_1 (a-1)^4 + \cdots$ (since analytic near $a=1$)

~~Rea~~ Let $R(a)$ = RHS as a function of a near $a=1$.

Then $R(1) = R'(1) = R''(1) = 0$.

Coeff of $(a-1)^3 = \dfrac{R'''(a)}{3!}$