$$\left(\frac{d}{da}\right)^3\left(a^{-3i}-a^{3i+3}\right)\Big]_{a=1} = (-3i)(-3i-1)(-3i-2) - (3i+3)(3i+2)(3i+1)$$

$$= -3(3i+2)(3i+1)(2i+1)$$

$$\left(\frac{d}{da}\right)^3\left(a^{-3j+1}-a^{3j+2}\right)\Big]_{a=1} = (-3j+1)(-3j)(-3j-1)$$

$$-(3j+2)(3j+1)(3j)$$

$$= -3j(3j+1)(6j+1).$$

Hence,

$$(q)_\infty^{10} = \sum_{i=0}^{\infty}\sum_{j=-\infty}^{\infty} \frac{(-1)^{i+j}}{2}\Big((6j+1)(2i+1)(3i+2)(3i+1)$$

$$-(2i+1)(3j)(3j+1)(6j+1)\Big)$$

$$\cdot q^{\frac{3}{2}i(i+1)+j(3j+1)/2}$$

$$= \sum_{i=0}^{\infty}\sum_{j=-\infty}^{\infty} (-1)^{i+j}\Big((2i+1)(6j+1)\Big)$$

$$\left(\frac{(3i+1)(3i+2)}{2} - \frac{(3j)(3j+1)}{2}\right)$$

$$\cdot q^{\frac{3}{2}i(i+1)+j(3j+1)/2}.$$