$$N(k,t,n) = \sum_a{}' N(at+k, n)$$ (b/c sum has

(where we note that this sum has only finitely many nonzero terms.)

$$= \sum_a{}' N(-at-k, n)$$

$$= \sum_b N(-(1-b)t-k, n)$$ (since as b ranges thru $\mathbb{Z}$ so does $a = 1-b$ and vice-versa)

$$= \sum_b N(bt+(t-k), n)$$

$$= N(t-k, t, n). \quad \Box$$

<u>Dyson's Conjectures (1944)</u>

(1) $N(m, 5, 5n+4) = \frac{1}{5}\, p(5n+4), \quad 0 \le m \le 4.$

(2) $N(m, 7, 7n+5) = \frac{1}{7}\, p(7n+5), \quad 0 \le m \le 6$

<u>Note:</u>

(1) means that the residue of the rank mod 5 divides the partitions of 5n+4 into five equal classes.

(2) . . . . mod 7 . . . .

. . . . . 7n+5 . . . seven . . . . .