

Example of (1) ( $n=0$ )

Partition of $k$	Rank	Rank (mod 5)
$k$	$k-1=3$	3
$3+1$	$3-2=1$	1
$2+2$	$2-2=0$	0
$2+1+1$	$2-3=-1$	4
$1+1+1+1$	$1-4=-3$	2

$$N(0, 5, 4) = N(1, 5, 4) = N(2, 5, 4) = N(3, 5, 4) \\ = N(4, 5, 4) = 1$$

$$N(0, 5, 4) + \dots + N(4, 5, 4) = p(4) = 5 N(0, 5, 4)$$

$$\text{So } N(m, 5, 4) = p(4)/5.$$

The Dyson Conjectures were proved by Atkin & Swinnerton-Dyer in 1954. Their proof is analytic and depends on elliptic function and  $q$ -series identities. No combinatorial proof is known.

Note: ~~The analogy of the Dyson conjecture for the analog of (1)-(2)~~ does not hold for partitions of  $11n+6$ .

Example

Partitions of 6	Rank	Rank (mod 11)
6	$6-1=5$	5
$5+1$	$5-2=3$	3
$4+2$	$4-2=2$	2
$4+1+1$	$4-3=1$	1