Lemma: Let $A=\sum_{n=0}^{\infty} a_n q^n$, $B=\sum_{n=0}^{\infty} b_n q^n$,

$C=\sum_{n=0}^{\infty} c_n q^n \in \mathbb{Z}[[q]]$, $D \in \sum_{n=0}^{\infty} d_n q^n \in$

Then

(1) if $A \equiv B \pmod{m}$ and $C \equiv D \pmod{m}$

Then

(i) $A + C \equiv B + D \pmod{m}$

(ii) $AC \equiv BD \pmod{m}$

(2) If $a_0 = 1$ then $\frac{1}{A} \in \mathbb{Z}[[q]]$.

(3) If $A \equiv B \pmod{m}$ & $c_0 = 1$ then

$$\frac{A}{C} \equiv \frac{B}{C} \pmod{m}.$$

(4) If $a_0 = b_0 = 1$ and $A \equiv B \pmod{m}$ Then $\frac{1}{A} \equiv \frac{1}{B} \pmod{m}$

~~Ramanujan's Congruence $p(5n+4) \equiv 0 \pmod{5}$~~

Lemma Let p be prime.

Then

(1) $(1-q)^p \equiv (1-q^p) \pmod{p}$.

(2) $(q)_\infty^{\,p} \equiv (q^p; q^p)_\infty \pmod{p}$.

Proof:

(1) By Binomial Thm,

$$(1-q)^p = \sum_{j=0}^{p} \binom{p}{j} 1^j (-q)^{p-j}$$

Let $1 \le j \le p-1$, then $\binom{p}{j} \in \mathbb{Z}$ and