Dyson (1944) conjectured that exist some statistic he called the "crank" which would be something like the rank and explain Ramanujans congruence $p(11n+6) \equiv 0 \pmod{11}$.

The Andrews-Garvan Crank

For a partition λ let $\ell(\lambda)$ = the largest part, $\omega(\lambda)$ = # of ones in λ and $\mu(\lambda)$ = # of parts of $\lambda > \omega(\lambda)$.

Then

$$\operatorname{crank}(\lambda) = \begin{cases} \ell(\lambda) & \text{if } \omega(\lambda)=0 \\ \mu(\lambda)-\omega(\lambda) & \text{if } \omega(\lambda)>0 \end{cases}$$

Let

$M(m,n)$ = # of partitions of n with crank m

and

$M(k,t,n)$ = # of partitions of n with crank $\equiv k \bmod t$.

Theorem (Andrews – G.)

(1) $M(-m,n) = M(m,n)$ for $n \geq 2$.

(2) $M(k,5,5n+4) = \dfrac{p(5n+4)}{5}$, $0 \leq k \leq 4$;

(3) $M(k,7,7n+5) = \dfrac{p(7n+5)}{7}$, $0 \leq k \leq 6$;

(4) $M(k,11,11n+6) = \dfrac{p(11n+6)}{11}$, $0 \leq k \leq 10$.