The <u>minimal polynomial</u> of an algebraic number α is the unique (irreducible) monic polynomial of smallest degree $p(x)$ with rational coefficients such that
$$p(\alpha)=0.$$

<u>Example</u>: The minimal polynomial of $\sqrt{2}$ is x^2-2.

<u>Theorem</u> Let p be prime. The minimal polynomial of $\zeta_p = \exp(2\pi i/p) = \cos\left(\frac{2\pi}{p}\right) + i\sin\left(\frac{2\pi}{p}\right)$ is
$$p(x) = 1+x+x^2+\cdots+x^{p-1}.$$

<u>Definition</u>: A <u>vector partition</u> π is a triple
$$\pi = (\pi_1, \pi_2, \pi_3)$$
where π_1 is a partition into distinct parts, and π_2, π_3 ~~is~~ are partitions ~~into~~ unrestricted).
We say π is a vector partition of n if
$$n = |\pi_1| + |\pi_2| + |\pi_3|.$$

<u>Example</u> $\pi = ((5,3,2), (2,2), (5,1,1))$
is a ~~p~~ vector partition of 21.

<u>Definition</u>: ~~We define~~ For a vector partition $\pi = (\pi_1, \pi_2, \pi_3)$ we define a weight ~~W~~(π) and crank(π) by
$$W(\pi) = (-1)^{\#(\pi_1)}$$
and
$$\mathrm{crank}(\pi) = \#(\pi_2) - \#(\pi_3).$$