(33)

Let V = set of vector partitions $= \{(\pi_1, \pi_2, \pi_3) : \pi_1$ is a partition into distinct parts & π_2, π_3 are partitions$\}$.

<u>Defn</u> Let $N_V(m,n)$ = # of vector partitions of n with crank m counted according to the weight w, ie

$$N_V(m,n) = \sum_{\substack{\pi \in V \\ |\pi| = n \\ \text{crank}(\pi) = m}} W(\pi)$$

[[], [], [1, 1]]	1	-2
[[], [], [2]]	1	-1
[[], [1], [1]]	1	0
[[], [1, 1], []]	1	2
[[], [2], []]	1	1
[[1], [], [1]]	-1	-1
[[1], [1], []]	-1	1
[[2], [], []]	-1	0

There are eight vector partitions of $n=2$.

$N_V(2,2) = 1$

$N_V(1,2) = 1 - 1 = 0$

$N_V(0,2) = 1 - 1 = 0$

$N_V(-1,2) = 1 - 1 = 0$

$N_V(-2,2) = 1$

<u>Theorem</u> Let $|q|<1$ & $|q| < |z| < \frac{1}{|q|}$.

$$\sum_{n=0}^{\infty} \sum_{m} N_V(m,n)\, z^m q^n = \frac{(q)_\infty}{(zq)_\infty (z^{-1}q)_\infty} = \prod_{n=1}^{\infty} \frac{(1-q^n)}{(1-zq^n)(1-z^{-1}q^n)}$$