Now,

$$\sum_{n\geq 0} \sum_{m} N_V(m,n)\zeta^m q^n$$

$$= \sum_{n\geq 0}\left(\sum_{k=0}^{4} \sum_{m\equiv k \pmod 5} N_V(m,n)\zeta^m\right) q^n$$

$$= \sum_{n\geq 0}\left(\sum_{k=0}^{4} \sum_{m\equiv k \pmod 5} N_V(m,n)\zeta^k\right) q^n$$

$$= \sum_{n\geq 0}\left(\sum_{k=0}^{4}\left(\sum_{m\equiv k \pmod 5} N_V(m,n)\right)\zeta^k\right) q^n$$

$$= \sum_{n\geq 0}\left(\sum_{k=0}^{4} N_V(k,5,n)\zeta^k\right) q^n$$

Therefore,

$$(*)\quad \sum_{n\geq 0}\left(\sum_{k=0}^{4} N_V(k,5,n)\zeta^k\right) q^n = \frac{(q)_\infty}{(\zeta q)(\zeta^4 q)_\infty} \quad \text{since } \zeta^5=1 \text{ \& } \zeta^{-1}=\zeta^4$$

$$= \frac{(q)_\infty (q)_\infty (\zeta^2 q)_\infty (\zeta^3 q)_\infty}{(q)_\infty (\zeta q)_\infty (\zeta^2 q)_\infty (\zeta^3 q)_\infty (\zeta^4 q)_\infty}$$

$$= \frac{(q)_\infty (\zeta^2 q)_\infty (\zeta^{-2} q)_\infty (q)_\infty}{(q^5;q^5)_\infty} \quad \text{since } \zeta^5=1 \text{ \& } \zeta^{-2}=\zeta^3$$