$$\sum_{n=0}^{\infty} \sum_{m} N_V(m,n)\, z^m q^n = \frac{(q)_\infty}{(zq)_\infty (q/z)_\infty}$$

$$= \frac{(1-q)}{(zq)_\infty} \frac{(q^2;q)_\infty}{(q/z)_\infty}$$

$$= \frac{(1-q)}{(zq)_\infty} \sum_{j=0}^{\infty} \frac{(zq)_j (q/z)^j}{(q)_j} \qquad \left(z \to q/z \text{ in } (*) \right) \quad a \to qz$$

(since $|q|<1$ & $|\frac{q}{z}|<1$)

$$= \frac{(1-q)}{(zq)_\infty} \left(1 + \sum_{j=1}^{\infty} \frac{(zq)_j (q/z)^j}{(q)_j} \right)$$

$$= \frac{(1-q)}{(zq)_\infty} \left(1 + \sum_{j=1}^{\infty} \frac{(zq)_j (q/z)^j}{(1-q)(q^2;q)_{j-1}} \right)$$

$$= \frac{(1-q)}{(zq)_\infty} + \sum_{j=1}^{\infty} \frac{q^j z^{-j}}{(q^2;q)_{j-1} (zq^{j+1})_\infty}$$

$$= \frac{(1-q)}{(zq)_\infty} + \underbrace{\sum_{j=1} \frac{z^{-j} q^{\overbrace{1+1+\cdots+1}^{j\text{-times}}}}{(1-q^2)(1-q^3)\cdots(1-q^j)(1-zq^{j+1})(1-zq^{j+2})\cdots}}$$

G.F. for ptns with at least one 1
in which the power of z counts
# of parts larger than the number of ones − # of ones