$$= 1 + (z-1) q^{1} + \sum_{\substack{\omega(\pi)=0 \\ \pi \in P \\ |\pi| \geq 2}} z^{\text{crank}(\pi)} q^{|\pi|}.$$

Therefore,

$$\sum_{n=0}^{\infty} \sum_{m} N_V(m,n)\, z^m q^n = \frac{(q)_\infty}{(zq)_\infty (z^{-1}q)_\infty}$$

$$= 1 + (-1 + z + z^{-1}) q + \sum_{\substack{\pi \in P \\ |\pi| \geq 2}} z^{\text{crank}(\pi)} q^{|\pi|}$$

$$= 1 + (-1 + z + z^{-1}) q + \sum_{n=2}^{\infty} \sum_{m} M(m,n)\, z^m q^n$$

and

$$N_V(m,n) = M(m,n)$$

for $n \geq 2$. $\Box$

Corollary (Andrews & G.)

(1) $M(-m, n) = M(m,n)$ for $n \geq 2$.

(2) $M(k, 5, 5n+4) = \dfrac{p(5n+4)}{5}$ for $0 \leq k \leq 4$.

(3) $M(k, 7, 7n+5) = \dfrac{p(7n+5)}{7}$ for $0 \leq k \leq 6$.

(4) $M(k, 11, 11n+6) = \dfrac{p(11n+6)}{11}$ for $0 \leq k \leq 11$.