

(9)

$$n = 5k + r \quad (r=0,1,2,3,4).$$

$$\frac{n(3n-1)}{2} = \frac{75k^2}{2} + 15kr - \frac{5k}{2} + \frac{3r^2}{2} - \frac{1}{2}r$$

$$n = 5k \quad \frac{n(3n-1)}{2} = \frac{75k^2 - 5k}{2} = \frac{5k(15k-1)}{2}$$

$$n = 5k+1 \quad \frac{n(3n-1)}{2} = \frac{75k^2 + 25k + 1}{2} = 25k \frac{(3k+1)}{2} + 1$$

$$n = 5k+2 \quad \frac{n(3n-1)}{2} = \frac{75k^2 + 55k + 5}{2} = \frac{5k(15k+11)}{2} + 5$$

$$n = 5k+3 \quad \frac{n(3n-1)}{2} = \frac{75k^2 + 85k + 12}{2} = \frac{5k(15k+17)}{2} + 12$$

$$n = 5k-1 \quad \frac{n(3n-1)}{2} = \frac{75k^2 - 35k + 2}{2} = \frac{5k(15k-7)}{2} + 2$$

$$\begin{aligned} (q)_\infty &= \sum_k (-1)^{5k} q^{\frac{5k(15k-1)}{2}} + \sum_k (-1)^{5k+2} q^{\frac{5k(15k+11)}{2} + 5} \\ &\quad + \sum_k (-1)^{5k+1} q^{\frac{5k+1}{2} \cdot \frac{25k(3k+1)}{2} + 1} \\ &\quad + \sum_k (-1)^{5k+3} q^{\frac{5k+3}{2} \cdot \frac{5k(15k+17)}{2} + 12} + \sum_k (-1)^{5k-1} q^{\frac{5k-1}{2} \cdot \frac{5k(15k-7)}{2} + 2} \\ &= J_0(q^5) - q \sum_k (-1)^k (q^{25})^{\frac{k(3k+1)}{2}} + q^2 J_2(q^5) \end{aligned}$$

$$(q)_\infty = J_0(q^5) - q (q^{25}; q^{25})_\infty + q^2 J_2(q^5)$$

$$(q)_\infty = J_0(q^5) - q + q^2 J_2(q^5)$$

$$(q^{25}; q^{25})_\infty \text{ where } J_0(q^5) = \frac{j_0(q^5)}{(q^{25}; q^{25})_\infty}, \quad J_2(q^5) = \frac{j_2(q^5)}{(q^{25}; q^{25})_\infty}.$$