since $k + l = h$

$$nl + \frac{k(k-1)}{2} + \frac{l(l-1)}{2} = \frac{k(k-1)}{2} + \frac{(h-k)(h-k-1)}{2} + n(h-k)$$

$$= \frac{k(k-1)}{2} - \frac{h(k+1)}{2} - \frac{kh}{2} + \frac{h^2}{2} + \frac{k(k+1)}{2} + n(h-k)$$

$$= k^2 - hk + \frac{h^2}{2} + nh - nk - h/2$$

$$= \frac{h(h-1)}{2} + k^2 - hk + nh - nk$$

$$= \frac{h(h-1)}{2} + (n-k)(h-k).$$

The result follows by comparing the coeff of z^h on both sides of $(*)$.

(4) By Heine's transformation

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n t^n}{(q)_n (c)_n} = \frac{(b)_\infty (at)_\infty}{(c)_\infty (t)_\infty} \sum_{n=0}^{\infty} \frac{(c/b)_n (t)_n b^n}{(at)_n (q)_n}$$

$$= \frac{(b)_\infty (at)_\infty}{(c)_\infty (t)_\infty} \sum_{n=0}^{\infty} \frac{(t)_n (c/b)_n b^n}{(at)_n (q)_n}$$

$$= \frac{\cancel{(b)_\infty}\, \cancel{(at)_\infty}}{(c)_\infty (t)_\infty} \cdot \frac{(c/b)_\infty (bt)_\infty}{\cancel{(at)_\infty}\, \cancel{(b)_\infty}} \sum_{n=0}^{\infty} \frac{(atb/c)_n (b)_n (c/b)^n}{(bt)_n (q)_n}$$

(by Heine's trf)

$$= \frac{(c/b)_\infty (bt)_\infty}{(c)_\infty (t)_\infty} \sum_{n=0}^{\infty} \frac{(b)_n (atb/c)_n (c/b)^n}{(bt)_n (q)_n}$$

$$= \frac{\cancel{(c/b)_\infty}\,\cancel{(bt)_\infty}}{\cancel{(c)_\infty}(t)_\infty} \frac{(atb/c)_\infty \cancel{(c)_\infty}}{\cancel{(bt)_\infty}\,\cancel{(c/b)_\infty}} \sum_{n=0}^{\infty} \frac{(c/a)_n (c/b)_n (abt/c)^n}{(c)_n (q)_n} \quad \text{(by Heine)}$$