

$$= \frac{1}{\left(\frac{q^{-L_n}}{b q^{N-n}}\right)_m (-x q^{N-n})^n q^{(2)}} = \frac{1}{\left(\frac{q^{-L_n}}{b q^{N-n}}\right)_m (-x q^{N-n})^n q^{(2)}} \quad (13)$$

Therefore,

$$\begin{aligned} & \frac{(q)_N}{(q)_{N-n}} \cdot \frac{\left(\frac{c}{ab}\right)_{N-n} \left(\frac{c}{ab}\right)^n}{\left(\frac{c}{ab}\right)_N} \\ &= \frac{(q^{-N})_m (-q)^n \left(\frac{c}{ab}\right)^n}{\left(q^{1-N} abc^{-1}\right)_m \left(-\frac{c}{ab}\right)^n} \\ &= \frac{(q^{-N})_n q^n}{\left(q^{1-N} abc^{-1}\right)_m} \end{aligned}$$

Hence

$$\sum_{n=0}^N \frac{(a)_n (b)_n (q^{-N})_n q^n}{(c)_n (abc^{-1} q^{1-N})_n (q)_n} = \frac{(c/a)_N (c/b)_N}{(c)_N (c/ab)_N}$$

(Jackson's  $q$ -analogue of Feulschutz's Thm.)

Now let

$$a = q^{-M+n}, \quad b = q^{m+n+1}, \quad c = q^{m+1}$$

$$\sum_{r=0}^N \frac{\left(q^{-M+n}\right)_r \left(q^{m+n+1}\right)_r q^r \left(q^{-N}\right)_r}{\left(q^{m+1}\right)_r \left(q^{-N-M+n+1}\right)_r (q)_r} = \frac{\left(q^{M+1}\right)_N \left(q^{-n}\right)_N}{\left(q^{m+1}\right)_N \left(q^{M-n}\right)_N}$$

We use

$$\left(q^{-N}\right)_r = \frac{(q)_N (-1)^r q^{r(r-1)/2 - rN}}{(q)_{N-r}}$$