Multinomial coefficient:

Note $\binom{n}{r} = \binom{n}{r, n-r}$

Let $n_1, n_2, \ldots, n_k \geq 0$.

$$\binom{n_1+n_2+\cdots+n_k}{n_1, n_2, \ldots, n_k} := \frac{(n_1+n_2+\cdots+n_k)!}{n_1!\, n_2! \cdots n_k!}$$

Multinomial Thm:

$$(a_1+a_2+\cdots+a_k)^n = \sum_{\substack{n_1, n_2, \ldots, n_k \geq 0 \\ n_1+n_2+\cdots+n_k = n}} \binom{n}{n_1, n_2, \ldots, n_k} a_1^{n_1} a_2^{n_2} \cdots a_k^{n_k}.$$

q-Multinomial Coefficient (or Gaussian multinomial coeff.)

$$\begin{bmatrix} n_1+n_2+\cdots+n_k \\ n_1, n_2, \ldots, n_k \end{bmatrix} := \frac{(q)_{n_1+n_2+\cdots+n_k}}{(q)_{n_1} (q)_{n_2} \cdots (q)_{n_k}}.$$

Definition A multiset is a "set" of possibly repeated elements.

Eg $\{1,1,1,2,2,3,3,3,3\} = \{1^3, 2^2, 3^4\}$.

A permutation of a multiset is a just a rearrangement of the elements. For example

$1\,2\;\;1\,3\;\;3\;\;2\,3\,1\,3$ is a permutation of $1\,1\,1\,2\,2\,3\,3\,3\,3$

Defn Let $m = m_1+m_2+\cdots+m_r$.

Let $inv(m_1, m_2, \ldots, m_r; n)$ = # of permutations $x_1, x_2, \ldots, x_m$ of the multiset $\{1^{m_1}, 2^{m_2}, \ldots, r^{m_r}\}$ in which there are exactly n pairs (x_i, x_j) where $i<j$ but $x_i > x_j$

(such a pair is called an inversion)