(**) $g(N,0;q) = \dfrac{(q)_N}{(q)_0 (q)_N} = \dfrac{(q)_N}{(q)_N} = 1$, &

similarly $g(0,M;q) = 1$.

Suppose $N$ & $M \geq 1$.

$$g(N,M;q) - g(N,M-1;q) = \frac{(q)_{M+N}}{(q)_M (q)_N} - \frac{(q)_{M+N-1}}{(q)_{M-1}(q)_N}$$

$$= \frac{(q)_{M+N-1}}{(q)_M (q)_N}\left( (1-q^{M+N}) - (1-q^M) \right)$$

$$= \frac{(q)_{M+N-1}}{(q)_M (q)_N}\left( q^M - q^{M+N} \right) = q^M \frac{(1-q^N)(q)_{M+N-1}}{(q)_M \ (q)_N}$$

$$= q^M \frac{(q)_{M+N-1}}{(q)_M \ (q)_{N-1}} = q^M g(N-1, M; q) \quad \&$$

(***) $g(N,M;q) = g(N,M-1;q) + q^M g(N-1,M;q)$.

Equations (**), (***) uniquely define $g(N,M;q)$ for all $M, N \geq 0$.

The empty partition () of 0 is the only ptn with no parts & the only ptn whose largest part is 0. Hence,

$$p(N,0,n) = p(0,M,n) = \begin{cases} 1 & \text{if } N=M=n=0 \\ 0 & \text{otherwise} \end{cases}$$