Hence,

$$G(N,0;q) = \sum_{n\geq 0} p(N,0,n)q^n = 1$$

&

$$G(0,M;q) = \sum_{n\geq 0} p(0,M,n)q^n = 1,$$

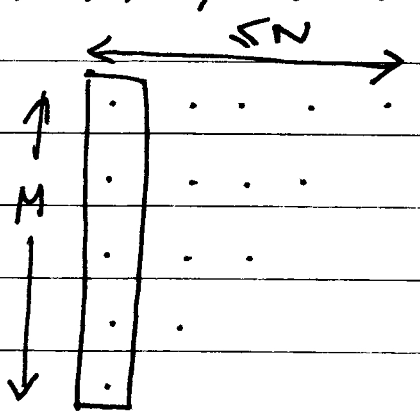
for all $M, N \geq 0$.

Suppose M & $N \geq 1$.

$p(N,M,n)$ = # of ptns of n into at most M parts $\leq N$

$p(N,M-1,n)$ = ... $M-1$ . . . .

$p(N,M,n) - p(N,M-1,n)$ = # of ptns of n into exactly M parts each $\leq N$



If we delete one from each part we obtain a ptn of $n-M$ into at most M parts each part $\leq N-1$. Conversely this is reversible by adding one to each part.

Therefore

$$p(N,M,n) - p(N,M-1,n) = \begin{cases} p(N-1, M, n-M) & \text{if } n\geq M \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n\geq 0} p(N,M,n)q^n - \sum_{n\geq 0} p(N,M-1,n)q^n = \sum_{n\geq M} p(N-1,M,n-M)q^n$$

$$G(N,M;q) - G(N,M-1;q) = q^M \sum_{n\geq M} p(N-1,M,n-M)q^{n-M}$$

$$= q^M \sum_{n\geq 0} p(N-1,M,n)q^n$$

$$= q^M G(N-1,M;q).$$