Now suppose $0 < z < 1$ and $0 < q < 1$ (11)

$$\alpha_m(q) = f_{3m+2}(q) = \sum_{n=0}^{\infty} c_{3m+2}(n) q^n \geq 0$$

and

$$\lim_{m \to \infty} \alpha_m(q) = f(q).$$

Also $\dfrac{\alpha_{n-1}(q)}{(q^3;q^3)_n} \geq 0$ and

$$\lim_{n \to \infty} \frac{\alpha_{n-1}(q)}{(q^3;q^3)_n} = \frac{f(q)}{(q^3;q^3)_\infty}.$$

Hence

$$\frac{f(q)}{(q^3;q^3)_\infty} = \lim_{z \to 1^-} (1-z) \frac{(-zq;q^3)_\infty(-zq^2;q^3)_\infty}{(1-z)(zq^3;q^3)_\infty}$$

$$= \frac{(-q;q^3)_\infty(-q^2;q^3)_\infty}{(q^3;q^3)_\infty}$$

and

$$f(q) = \sum_{n=0}^{\infty} c(n) q^n = (-q;q^3)_\infty(-q^2;q^3)_\infty$$

for $0 < q < 1$ (and all $|q| < 1$ by analytic continuation).

Hence

$$\sum_{n=0}^{\infty} C(n) q^n = \sum_{n=0}^{\infty} A(n) q^n \quad \text{for } |q| < 1$$

&

$$C(n) = A(n) \quad \text{for all } n \geq 0. \ \square$$