$$\sum_{n=0}^{\infty} \frac{A_n(\theta) z^n}{(q)_n} = \prod_{n=0}^{\infty} \frac{1}{(1 - 2zq^n\cos\theta + z^2 q^{2n})}$$

$$= \prod_{n=0}^{\infty} \frac{1}{(1 - z(e^{i\theta} + e^{-i\theta}) q^n + z^2 q^{2n})}$$

$$= \prod_{n=0}^{\infty} \frac{1}{(1 - ze^{i\theta} q^n)(1 - ze^{-i\theta} q^n)}$$

$$= \frac{1}{(ze^{i\theta})_\infty} \frac{1}{(ze^{-i\theta})_\infty}$$

$$= \sum_{s=0}^{\infty} \frac{(ze^{i\theta})^s}{(q)_s} \sum_{t=0}^{\infty} \frac{(ze^{-i\theta})^t}{(q)_t}$$

$$= \sum_{s=0}^{\infty} \sum_{t=0}^{\infty} \frac{e^{i\theta(s-t)} z^{s+t}}{(q)_s (q)_t}.$$

Hence,

$$A_n(\theta) = \sum_{\substack{s+t=n \\ s,t \geq 0}} \frac{(q)_n}{(q)_s (q)_t} e^{i\theta(s-t)}$$

$$= \sum_{\substack{s+t=n \\ s,t \geq 0}} \frac{(q)_n}{(q)_s (q)_t} \cos(s-t)\theta$$ (by taking real part).

$$= \sum_{t=0}^{n} \frac{(q)_n}{(q)_{n-t} (q)_t} \cos(n-2t)\theta$$