$$\frac{(tq^{-n})_n}{(tq^{-n})_j} = \begin{cases} 1 & \text{if } j=n \\ (1-tq^{-n+j})\cdots(1-tq^{-1}) & \text{if } 0\le j<n \end{cases}$$

Letting $t \to q$ we find that

$$\sum_{j=0}^{n} \frac{(a)_j (q^{-n})_j q^j}{(c)_j (q)_j} = \frac{(q^{-n})_n (aq^{1-n}/c)_n}{(c)_n (q)_n} c^n q^{n^2}$$

$$= \frac{(q)_n \, (c/a)_n \, c^n q^{n^2}}{(c)_n (q)_n \, (-q)^n q^{n(n-1)/2} (-c/a)^n q^{n(n-1)/2}}$$

$$\boxed{\sum_{j=0}^{n} \frac{(a)_j (q^{-n})_j q^j}{(c)_j (q)_j} = \frac{a^n \cancel{(q)_n} (c/a)_n}{(c)_n \cancel{(q)_n}}}$$

Let $\tau(r, 2n+1) = 0$ for $n \ge 0$

and $\tau(r, 2n) = (-1)^n q^{n(n-1)/2} \dfrac{(q^{r+1})_{n-1}(1-q^{r+2n})}{(q)_n}$, for $n \ge 1$

~~[illegible]~~ & $\tau(r, 0) = 1$.

We must show $\tau(r, n)$ satisfies the same recurrence as $T(r, n)$ (***), i.e. (**)', which clearly holds when N is odd.

Let $N = 2n > 0$.