

(26)

$$= \frac{(-1)^n (1-q^{r+2n}) q^{n(n-1)/2} (q)_{r+1} (q^{r+n})^n (q^{1-n})_n}{(q)_r (q)_n (q^{r+1})_n}$$

($c = q^{r+1}$, $a = q^{r+n}$ in q -Chu-V.)

$$= 0 \quad \text{if } n > 0 \text{ since } (q^{1-n})_n = (1-q^{1-n}) \cdots (1-q^0) = 0.$$

Theorem: If for each $r \geq 0$,

$$b_r = \sum_{j \geq 0} \begin{bmatrix} r+2j \\ j \end{bmatrix} a_{r+2j}$$

Then

$$a_r = \sum_{h \geq 0} \frac{(-1)^h q^{h(h-1)/2} (q^{r+1-q})_h (1-q^{r+2h})}{(q)_n (1-q^{r+h})} b_{r+2h}$$

(assuming convergence conditions).

~~Let~~ Let $F(\lambda, \theta) = \prod_{n=1}^{\infty} (1 + 2\lambda q^n \cos \theta + \lambda^2 q^{2n})$

We find $\{a_r\}, \{b_r\}$ exist that

$$F(\lambda, \theta) = \sum_{r=0}^{\infty} a_r A_r(\theta)$$

and

$$F(\lambda, \theta) = b_0 + \sum_{r=1}^{\infty} 2b_r \cos r\theta.$$